

## Hoofdstuk 2: Oppervlakte en inhoud.

### 2.1 Oppervlakte van vlakke figuren

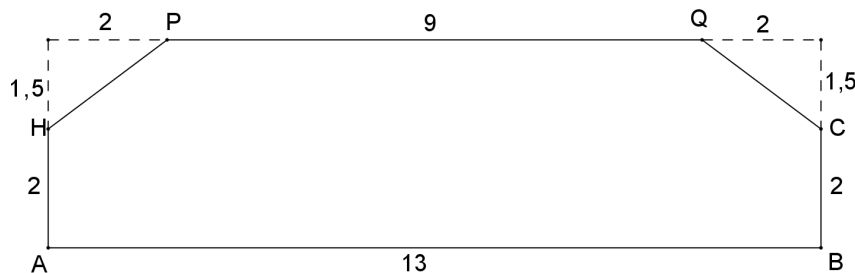
#### Opgave 1:

De oppervlakte van de figuur is precies de oppervlakte van een rechthoek van 7 bij 3, dus

$$Opp = 7 \cdot 3 = 21$$

#### Opgave 2:

a.



$$Opp(ABCQPH) = 13 \cdot 3\frac{1}{2} - 2 \cdot \frac{1}{2} \cdot 2 \cdot 1\frac{1}{2} = 42\frac{1}{2}$$

dus lijnstuk  $PQ$  verdeelt de achthoek niet in twee stukken met gelijke oppervlakte

b.  $Opp(\triangle AIE) = \frac{1}{2} \cdot 9 \cdot 9 = 40\frac{1}{2}$

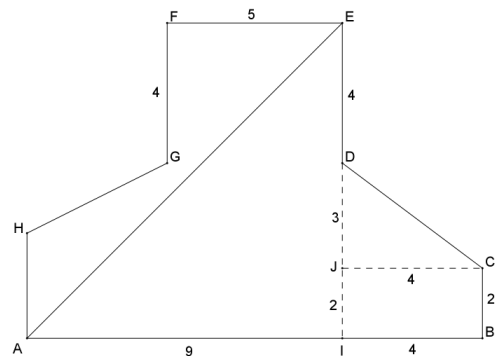
$$Opp(BCJI) = 4 \cdot 2 = 8$$

$$Opp(\triangle CDJ) = \frac{1}{2} \cdot 4 \cdot 3 = 6$$

$$Opp(ABCDE) = 40\frac{1}{2} + 8 + 6 = 54\frac{1}{2}$$

$$Opp(AEFH) = 73 - 54\frac{1}{2} = 18\frac{1}{2}$$

$$Opp(I) : Opp(II) = 54\frac{1}{2} : 18\frac{1}{2} = 109 : 37$$



#### Opgave 3:

a.  $Opp(ABCD) = \frac{1}{2} \cdot Opp(AEFD) = \frac{1}{2} \cdot (a + b) \cdot h$

b.  $Opp(ABCD) = Opp(\triangle ABD) + Opp(\triangle BCD) = \frac{1}{2} \cdot a \cdot h + \frac{1}{2} \cdot b \cdot h = \frac{1}{2} h(a + b)$

#### Opgave 4:

$$Opp(\text{figuur a}) = \frac{1}{2} \cdot 4 \cdot 2 = 4 \text{ cm}^2$$

$$Opp(\text{figuur b}) = 4 \cdot 3 - \frac{1}{2} \cdot 3 \cdot 1 - \frac{1}{2} \cdot 3 \cdot 2 - \frac{1}{2} \cdot 4 \cdot 1 = 5\frac{1}{2} \text{ cm}^2$$

$$Opp(\text{figuur c}) = Opp(\text{parallel log ram}) + Opp(\text{cirkel}) = 4 \cdot 3 + \pi \cdot 1^2 = 12 + \pi = 15,14 \text{ cm}^2$$

$$Opp(\text{figuur d}) = Opp(\text{trapezium}) = \frac{1}{2} \cdot 3 \cdot (5 + 1) = 9 \text{ cm}^2$$

$$Opp(\text{figuur e}) = Opp(\text{rechthoek}) + Opp(\text{halve cirkel}) = 3 \cdot 2 + \frac{1}{2} \cdot \pi \cdot 1^2 = 6 + \frac{1}{2} \pi = 7,57 \text{ cm}^2$$

#### Opgave 5:

$$AB = \sqrt{AM^2 + BM^2} = \sqrt{3^2 + 3^2} = \sqrt{18}$$

$$Opp = Opp(\text{cirkel}) - Opp(\text{vierkant}) = \pi \cdot 3^2 - (\sqrt{18})^2 = 9\pi - 18 = 10,27$$

**Opgave 6:**

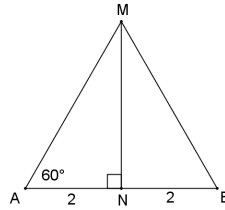
a.  $\angle M = 360^\circ : 6 = 60^\circ$

b.  $\tan 60^\circ = \frac{MN}{AN} = \frac{MN}{2}$

$$MN = 2 \cdot \tan 60^\circ = 3,464$$

$$Opp(\triangle ABM) = \frac{1}{2} \cdot 4 \cdot 3,464 = 6,93$$

c.  $Opp(ABCDEF) = 6 \cdot Opp(\triangle ABM) = 6 \cdot 6,93 = 41,6$

**Opgave 7:**

In  $\triangle ABM$  geldt:  $\angle M = 360^\circ : 5 = 72^\circ$

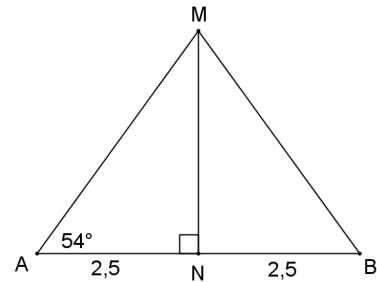
$$\angle A = \angle B = (180 - 72) : 2 = 54^\circ$$

$$\tan 54^\circ = \frac{MN}{AN} = \frac{MN}{2,5}$$

$$MN = 2,5 \cdot \tan 54^\circ = 3,44$$

$$Opp(\triangle ABM) = \frac{1}{2} \cdot 5 \cdot 3,44 = 8,60$$

$$Opp(ABCDE) = 5 \cdot Opp(\triangle ABM) = 5 \cdot 8,60 = 43,01$$

**Opgave 8:**

In  $\triangle ABM$  geldt:  $\angle M = 360^\circ : 8 = 45^\circ$

$$\angle A = \angle B = (180^\circ - 45^\circ) : 2 = 67,5^\circ$$

$$\sin 67,5^\circ = \frac{MN}{6}$$

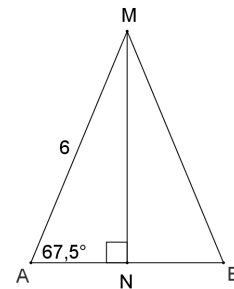
$$MN = 6 \cdot \sin 67,5^\circ = 5,54$$

$$\cos 67,5^\circ = \frac{AN}{6}$$

$$AN = 6 \cdot \cos 67,5^\circ = 2,30 \text{ dus } AB = 4,59$$

$$Opp(\triangle ABM) = \frac{1}{2} \cdot 4,59 \cdot 5,54 = 12,73$$

$$Opp(ABCDEFGH) = 8 \cdot Opp(\triangle ABM) = 8 \cdot 12,73 = 101,82$$

**Opgave 9:**

$$\tan 70^\circ = \frac{CD}{AD} = \frac{10}{AD}$$

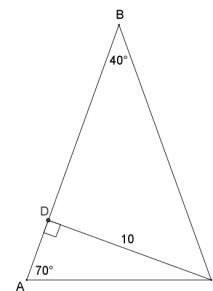
$$AD = \frac{10}{\tan 70^\circ} = 3,64$$

$$\tan 40^\circ = \frac{CD}{BD} = \frac{10}{BD}$$

$$BD = \frac{10}{\tan 40^\circ} = 11,92$$

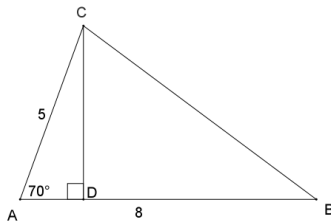
$$AB = 3,64 + 11,92 = 15,56$$

$$Opp(\triangle ABC) = \frac{1}{2} \cdot AB \cdot CD = \frac{1}{2} \cdot 15,56 \cdot 10 = 77,8$$

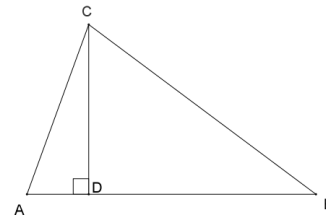


**Opgave 10:**

a.



- b.  $\sin 70^\circ = \frac{CD}{AC} = \frac{CD}{5}$   
 $CD = 5 \cdot \sin 70^\circ = 4,7$   
 $Opp(\triangle ABC) = \frac{1}{2} \cdot AB \cdot CD = \frac{1}{2} \cdot 8 \cdot 4,7 = 18,8$
- c.  $\sin \angle A = \frac{CD}{AC} = \frac{CD}{b}$   
 $CD = b \cdot \sin \angle A$   
 $Opp(\triangle ABC) = \frac{1}{2} \cdot AB \cdot CD = \frac{1}{2} \cdot c \cdot b \cdot \sin \angle A$   
 dus  $Opp(\triangle ABC) = \frac{1}{2} \cdot b \cdot c \cdot \sin \angle A$

**Opgave 11:**

$$Opp(\text{cirkelsegment}) = \frac{80}{360} \cdot \pi \cdot 10^2 = 69,81$$

$$\sin 40^\circ = \frac{AN}{AM} = \frac{AN}{10}$$

$$AN = 10 \cdot \sin 40^\circ = 6,43$$

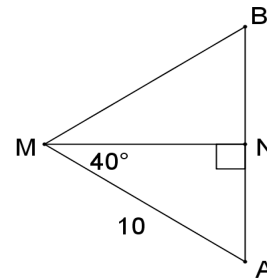
$$AB = 2 \cdot 6,43 = 12,86$$

$$\cos 40^\circ = \frac{MN}{AM} = \frac{MN}{10}$$

$$MN = 10 \cdot \cos 40^\circ = 7,66$$

$$Opp(\triangle ABM) = \frac{1}{2} \cdot AB \cdot MN = \frac{1}{2} \cdot 12,86 \cdot 7,66 = 49,24$$

$$Opp(\text{maantje}) = 69,81 - 49,24 = 20,57$$

**Opgave 12:**

$$Opp(\text{halve cirkel}) = \frac{1}{2} \pi r^2 = \frac{1}{2} \pi \cdot 5^2 = 39,27$$

$$Opp(\triangle CDM) = \frac{1}{2} \cdot 5 \cdot 5 \cdot \sin 60^\circ = 10,83$$

$$Opp(ABCDEF) = Opp(\text{halve cirkel}) + 3 \cdot Opp(\triangle CDM) = 39,27 + 3 \cdot 10,83 = 71,75$$

**Opgave 13:**

$$\sin \angle FMN = \frac{FN}{FM} = \frac{3}{5}$$

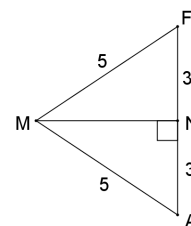
$$\angle FMN = 36,87^\circ$$

$$\angle FMA = 73,74^\circ$$

$$Opp(\triangle AFM) = \frac{1}{2} \cdot 5 \cdot 5 \cdot \sin 73,74^\circ = 12$$

$$Opp(\text{maantje AF}) = Opp(\text{cirkelsegment AFM}) - Opp(\triangle AFM) = \frac{73,74}{360} \cdot \pi \cdot 5^2 - 12 = 4,088$$

$$Opp(ABCDEF) = Opp(\text{cirkel}) - 3 \cdot Opp(\text{maantje}) = \pi \cdot 5^2 - 3 \cdot 4,088 = 66,3$$



**Opgave 14:**

$$\sin \angle PMR = \frac{PR}{PM} = \frac{2\frac{1}{2}}{4} = 0,625$$

$$\angle PMR = 38,68^\circ$$

$$\angle PMQ = 77,36^\circ$$

$$\sin \angle PNR = \frac{PR}{PN} = \frac{2\frac{1}{2}}{6} = \frac{5}{12}$$

$$\angle PNR = 24,62^\circ$$

$$\angle PNQ = 49,25^\circ$$

$$\begin{aligned} \text{Opp}(\text{maantje } PMQ) &= \text{Opp}(\text{cirkelsegment}) - \text{Opp}(\Delta PMQ) \\ &= \frac{77,36}{360} \cdot \pi \cdot 4^2 - \frac{1}{2} \cdot 4 \cdot 4 \cdot \sin 77,36^\circ = 2,995 \end{aligned}$$

$$\begin{aligned} \text{Opp}(\text{maantje } PNQ) &= \text{Opp}(\text{cirkelsegment}) - \text{Opp}(\Delta PNQ) \\ &= \frac{49,25}{360} \cdot \pi \cdot 6^2 - \frac{1}{2} \cdot 6 \cdot 6 \cdot \sin 49,25^\circ = 1,836 \end{aligned}$$

$$\text{Opp} = 2,995 + 1,836 = 4,83$$

