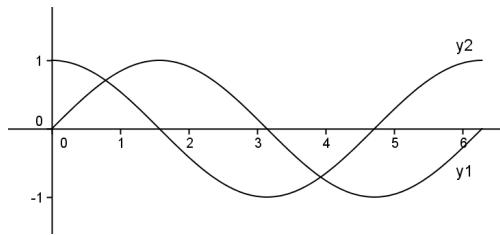


12.5 Goniometrische functies differentiëren.

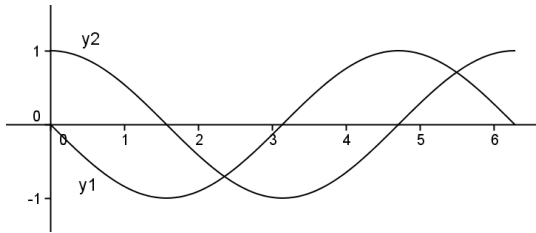
Opgave 49:

a.



b. $y = \cos x$

c.



$$y = -\sin x$$

d. $y = \sin(x - 2)$

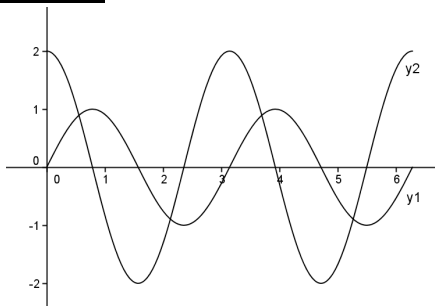
$$y' = \cos(x - 2)$$

$$y = \cos(x + 1)$$

$$y' = -\sin(x + 1)$$

Opgave 50:

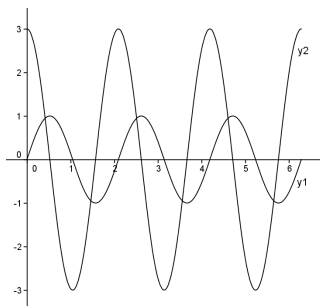
a.



$$y_1 : \text{per} = \pi \quad \text{amp} = 1$$

$$y_2 : \text{per} = \pi \quad \text{amp} = 2$$

b.



$$y_1 : \text{per} = \frac{2}{3}\pi \quad \text{amp} = 1$$

$$y_2 : \text{per} = \frac{2}{3}\pi \quad \text{amp} = 3$$

c. $a = 2: y_2 = 2 \cos 2x$

$$a = 3: y_2 = 3 \cos 3x$$

Opgave 51:

$$y = \cos 3x$$

$$y' = -3 \sin 3x$$

Opgave 52:

a. $f(x) = \cos 2x = \cos u$ met $u = 2x$

$$\text{dus } u' = 2$$

$$f'(x) = -\sin u \cdot u' = -\sin 2x \cdot 2 = -2 \sin 2x$$

b. $g(x) = x \cdot \cos x$

$$g'(x) = 1 \cdot \cos x + x \cdot (-\sin x) = \cos x - x \cdot \sin x$$

c. $h(x) = 3 + 4 \sin(2x - \frac{1}{3}\pi) = 3 + 4 \sin u$ met $u = 2x - \frac{1}{3}\pi$

$$\text{dus } u' = 2$$

$$h'(x) = 4 \cos u \cdot u' = 4 \cos(2x - \frac{1}{3}\pi) \cdot 2 = 8 \cos(2x - \frac{1}{3}\pi)$$

d. $j(x) = 10 + 16 \sin(\frac{1}{2}(x-1)) = 10 + 16 \sin u$ met $u = \frac{1}{2}(x-1)$

$$\text{dus } u' = \frac{1}{2}$$

$$j'(x) = 16 \cos u \cdot u' = 16 \cos(\frac{1}{2}(x-1)) \cdot \frac{1}{2} = 8 \cos(\frac{1}{2}(x-1))$$

Opgave 53:

a. $f(x) = \sin(ax+b) = \sin u$ met $u = ax+b$

$$\text{dus } u' = a$$

$$f'(x) = \cos u \cdot u' = \cos(ax+b) \cdot a = a \cdot \cos(ax+b)$$

b. $g(x) = \cos(ax+b) = \cos u$ met $u = ax+b$

$$\text{dus } u' = a$$

$$g'(x) = -\sin u \cdot u' = -\sin(ax+b) \cdot a = -a \cdot \sin(ax+b)$$

Opgave 54:

a. $f(x) = x \cdot \sin 2x$

$$f'(x) = 1 \cdot \sin 2x + x \cdot \cos 2x \cdot 2 = \sin 2x + 2x \cdot \cos 2x$$

b. *

Opgave 55:

a. $f(x) = x \cdot \cos 2x$

$$f'(x) = 1 \cdot \cos 2x + x \cdot (-\sin 2x) \cdot 2 = \cos 2x - 2x \cdot \sin 2x$$

b. $g(x) = x^2 \cdot \sin 3x$

$$g'(x) = 2x \cdot \sin 3x + x^2 \cdot \cos 3x \cdot 3 = 2x \cdot \sin 3x + 3x^2 \cdot \cos 3x$$

c. $h(x) = 2x \cdot \sin(3x-1)$

$$h'(x) = 2 \cdot \sin(3x-1) + 2x \cdot \cos(3x-1) \cdot 3 = 2 \sin(3x-1) + 6x \cdot \cos(3x-1)$$

d. $j(x) = 1 + 3x \cdot \cos \frac{1}{2}x$

$$j'(x) = 3 \cdot \cos \frac{1}{2}x + 3x \cdot (-\sin \frac{1}{2}x) \cdot \frac{1}{2} = 3 \cos \frac{1}{2}x - 1 \frac{1}{2}x \cdot \sin \frac{1}{2}x$$

Opgave 56:

a. I: $f(x) = \sin^2 x = \sin x \cdot \sin x$

$$f'(x) = \cos x \cdot \sin x + \sin x \cdot \cos x = 2 \sin x \cos x$$

$$\text{II: } f(x) = \sin^2 x = u^2 \text{ met } u = \sin x$$

$$\text{dus } u' = \cos x$$

$$f'(x) = 2u \cdot u' = 2 \sin x \cdot \cos x$$

b. *

Opgave 57:

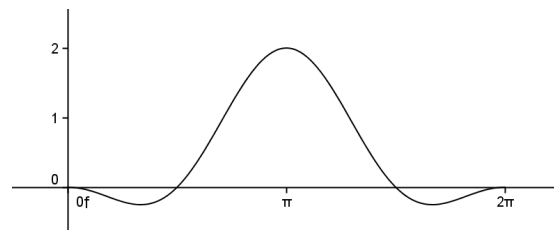
- a. $f(x) = \cos^2 x = u^2$ met $u = \cos x$
dus $u' = -\sin x$
 $f'(x) = 2u \cdot u' = 2 \cos x \cdot -\sin x = -2 \cos x \sin x$
- b. $g(x) = 2 \sin^2 x = 2u^2$ met $u = \sin x$
dus $u' = \cos x$
 $g'(x) = 4u \cdot u' = 4 \sin x \cdot \cos x$
- c. $h(x) = 1 + 2 \cos^2 x = 1 + 2u^2$ met $u = \cos x$
dus $u' = -\sin x$
 $h'(x) = 4u \cdot u' = 4 \cos x \cdot -\sin x = -4 \cos x \sin x$
- d. $j(x) = x + 3 \sin^2 x = x + 3u^2$ met $u = \sin x$
dus $u' = \cos x$
 $j'(x) = 1 + 6u \cdot u' = 1 + 6 \sin x \cdot \cos x$

Opgave 58:

- a. $f(x) = \sin^3 x = u^3$ met $u = \sin x$
dus $u' = \cos x$
 $f'(x) = 3u^2 \cdot u' = 3 \sin^2 x \cdot \cos x$
- b. $g(x) = x \cdot \sin^2 x$
 $g'(x) = 1 \cdot \sin^2 x + x \cdot 2 \sin x \cos x = \sin^2 x + 2x \sin x \cos x$
- c. $h(x) = \sqrt{2 + \sin x} = \sqrt{u}$ met $u = 2 + \sin x$
dus $u' = \cos x$
 $h'(x) = \frac{1}{2\sqrt{u}} \cdot u' = \frac{1}{2\sqrt{2 + \sin x}} \cdot \cos x = \frac{\cos x}{2\sqrt{2 + \sin x}}$
- d. $j(x) = 2x \cdot \cos(x^2)$
 $j'(x) = 2 \cdot \cos(x^2) + 2x \cdot -\cos(x^2) \cdot 2x = 2 \cos(x^2) - 4x^2 \sin(x^2)$

Opgave 59:

- a. $f(x) = \cos^2 x - \cos x$
 $f'(x) = 2 \cos x \cdot -\sin x + \sin x$
 $= -2 \cos x \sin x + \sin x = 0$
 $\sin x(-2 \cos x + 1) = 0$
 $\sin x = 0 \quad \vee \quad -2 \cos x = -1$
 $\sin x = 0 \quad \vee \quad \cos x = \frac{1}{2}$
 $x = 0 \quad \vee \quad x = \pi \quad \vee \quad x = 2\pi \quad \vee \quad x = \frac{1}{3}\pi \quad \vee \quad x = 1\frac{2}{3}\pi$
 $\min f(\frac{1}{3}\pi) = -\frac{1}{4}$
 $\max f(\pi) = 2$
 $\min f(1\frac{2}{3}\pi) = -\frac{1}{4}$



b. $y_B = f\left(\frac{2}{3}\pi\right) = \frac{3}{4}$
 $f'\left(\frac{2}{3}\pi\right) = \sqrt{3}$
 $y = \sqrt{3} \cdot x + b$ door $\left(\frac{2}{3}\pi, \frac{3}{4}\right)$
 $\frac{3}{4} = \sqrt{3} \cdot \frac{2}{3}\pi + b$
 $\frac{3}{4} - \frac{2}{3}\pi\sqrt{3} = b$
 $y = \sqrt{3} \cdot x + \frac{3}{4} - \frac{2}{3}\pi\sqrt{3}$

Opgave 60:

a. $y_A = f\left(\frac{1}{2}\pi\right) = 0$
 $f'(x) = 1 \cdot \cos x + x \cdot -\sin x = \cos x - x \sin x$
 $f'\left(\frac{1}{2}\pi\right) = -\frac{1}{2}\pi$
 $y = -\frac{1}{2}\pi \cdot x + b$ door $\left(\frac{1}{2}\pi, 0\right)$
 $0 = -\frac{1}{4}\pi^2 + b$
 $b = \frac{1}{4}\pi^2$
 $y = -\frac{1}{2}\pi \cdot x + \frac{1}{4}\pi^2$

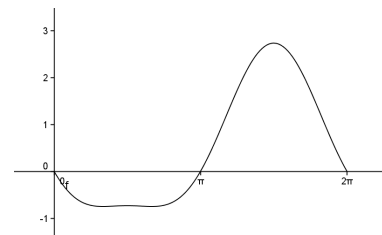
b. $y_B = f(\pi) = -\pi$
 $f'(\pi) = -1$
 $y = -x + b$ door $(\pi, -\pi)$
 $-\pi = -\pi + b$
 $b = 0$
 $y = -x$ en deze lijn gaat door $(0,0)$

c. $f'(1) = \cos(1) - \sin(1) = -0,30 \neq 0$ dus geen top voor $x = 1$

Opgave 61:

a. $f(x) = \sin^2 x - \sqrt{3} \cdot \sin x$
 $f'(x) = 2 \sin x \cos x - \sqrt{3} \cdot \cos x = 0$
 $\cos x(2 \sin x - \sqrt{3}) = 0$
 $\cos x = 0 \quad \vee \quad 2 \sin x = \sqrt{3}$
 $\cos x = 0 \quad \vee \quad \sin x = \frac{1}{2}\sqrt{3}$
 $x = \frac{1}{2}\pi \quad \vee \quad x = 1\frac{1}{2}\pi \quad \vee \quad x = \frac{1}{3}\pi \quad \vee \quad x = \frac{2}{3}\pi$
 $\min f\left(\frac{1}{3}\pi\right) = -\frac{3}{4}$
 $\max f\left(\frac{1}{2}\pi\right) = 1 - \sqrt{3}$
 $\min f\left(\frac{2}{3}\pi\right) = -\frac{3}{4}$
 $\max f\left(1\frac{1}{2}\pi\right) = 1 + \sqrt{3}$

b. $y_A = f\left(\frac{1}{6}\pi\right) = \frac{1}{4} - \frac{1}{2}\sqrt{3}$
 $f'\left(\frac{1}{6}\pi\right) = \frac{1}{2}\sqrt{3} - 1\frac{1}{2}$
 $y = \left(\frac{1}{2}\sqrt{3} - 1\frac{1}{2}\right) \cdot x + b$ door $\left(\frac{1}{6}\pi, \frac{1}{4} - \frac{1}{2}\sqrt{3}\right)$
 $\frac{1}{4} - \frac{1}{2}\sqrt{3} = \left(\frac{1}{2}\sqrt{3} - 1\frac{1}{2}\right) \cdot \frac{1}{6}\pi + b$
 $\frac{1}{4} - \frac{1}{2}\sqrt{3} - \left(\frac{1}{2}\sqrt{3} - 1\frac{1}{2}\right) \cdot \frac{1}{6}\pi = b$



$$y = \left(\frac{1}{2}\sqrt{3} - 1\frac{1}{2}\right) \cdot x + \frac{1}{4} - \frac{1}{2}\sqrt{3} - \left(\frac{1}{2}\sqrt{3} - 1\frac{1}{2}\right) \cdot \frac{1}{6}\pi$$

c. $f'(x) = 2 \sin x \cos x - \sqrt{3} \cdot \cos x = \sqrt{3}$

$y_1 = 2 \sin x \cos x - \sqrt{3} \cdot \cos x$ en $y_2 = \sqrt{3}$ intersect geeft:

$$x = 3,14 = \pi \quad \vee \quad x = 4,19 = \frac{4}{3}\pi$$

$$f'(\pi) = \sqrt{3}$$

$$f'\left(1\frac{1}{3}\pi\right) = \sqrt{3}$$

Opgave 62:

$$y = a + b \sin(c(x - d))$$

beginpunt: $x = d$

periode: $\frac{2\pi}{c}$

max: $x = \text{beginpunt} + \frac{1}{4} \text{periode} = d + \frac{\pi}{2c}$

$$y = a + b$$

min: $x = \text{beginpunt} + \frac{3}{4} \text{periode} = d + \frac{3\pi}{2c}$

$$y = a - b$$

$$y = a + b \cos(c(x - d))$$

beginpunt: $x = d$

periode: $\frac{2\pi}{c}$

max: $x = d$

$$y = a + b$$

min: $x = \text{beginpunt} + \frac{1}{2} \text{periode} = d + \frac{\pi}{c}$

$$y = a - b$$