12.2 De afgeleide van $y = ax^n$

Opgave 14:

a. $\frac{10}{x^3} = 10x^{-3}$

b. $8 \cdot \sqrt[3]{x} = 8x^{\frac{1}{3}}$

c. $\frac{2}{x} = 2x^{-1}$

d. $\frac{7}{\sqrt{x}} = \frac{7}{x^{\frac{1}{2}}} = 7x^{-\frac{1}{2}}$

e. $4x\sqrt{x} = 4x^1 \cdot x^{\frac{1}{2}} = 4x^{1\frac{1}{2}}$

f. $\frac{3}{4x^2\sqrt{x}} = \frac{3}{4x^2 \cdot x^{\frac{1}{2}}} = \frac{3}{4x^{2\frac{1}{2}}} = \frac{3}{4}x^{-2\frac{1}{2}}$

Opgave 15:

a. $f(x) = \frac{6}{x^3} + 6x^3 = 6x^{-3} + 6x^3$

$f'(x) = -18x^{-4} + 18x^2 = -\frac{18}{x^4} + 18x^2$

b. $g(x) = 3x^6 - \frac{4}{x} = 3x^6 - 4x^{-1}$

$g'(x) = 18x^5 + 4x^{-2} = 18x^5 + \frac{4}{x^2}$

c. $k(q) = \frac{1}{q} = q^{-1}$

$k'(q) = -q^{-2} = -\frac{1}{q^2}$

d. $l(p) = \frac{2}{p} + \frac{1}{2p^2} = 2p^{-1} + \frac{1}{2}p^{-2}$

$l'(p) = -2p^{-2} - p^{-3} = -\frac{1}{p^2} - \frac{1}{p^3}$

e. $h(x) = 3x\sqrt{x} - 2\sqrt{x} = 3x^{1\frac{1}{2}} - 2x^{\frac{1}{2}}$

$h'(x) = 4\frac{1}{2}x^{\frac{1}{2}} - x^{-\frac{1}{2}} = 4\frac{1}{2}\sqrt{x} - \frac{1}{x^{\frac{1}{2}}} = 4\frac{1}{2}\sqrt{x} - \frac{1}{\sqrt{x}}$

f. $m(t) = 12t \cdot \sqrt[4]{t} = 12t^{1\frac{1}{4}}$

$m'(t) = 15t^{\frac{1}{4}} = 15 \cdot \sqrt[4]{t}$

Opgave 16:

a. $\frac{d}{dx}(x + \frac{4}{\sqrt{x}}) = \frac{d}{dx}(x + 4x^{-\frac{1}{2}}) = 1 - 2x^{-1\frac{1}{2}} = 1 - \frac{2}{x\sqrt{x}}$

b. $\frac{d}{dx}(3x^2 \cdot \sqrt{x}) = \frac{d}{dx}(3x^{2\frac{1}{2}}) = 7\frac{1}{2}x^{1\frac{1}{2}} = 7\frac{1}{2}x\sqrt{x}$

c. $\frac{d}{dx}(3x - x^{-1,65}) = 3 + 1{,}65x^{-2,65}$

d. $\frac{d}{dx}(10 \cdot \sqrt[5]{x} + \frac{5}{x}) = \frac{d}{dx}(10x^{\frac{1}{5}} + 5x^{-1}) = 2x^{-\frac{4}{5}} - 5x^{-2} = \frac{2}{\sqrt[5]{x^4}} - \frac{5}{x^2}$

e. $\frac{d}{dx}(\frac{2}{5x} + \frac{5x}{2}) = \frac{d}{dx}(\frac{2}{5}x^{-1} + 2\frac{1}{2}x) = -\frac{2}{5}x^{-2} + 2\frac{1}{2} = -\frac{2}{5x^2} + 2\frac{1}{2}$

f. $\frac{d}{dx}((1+x)\sqrt{x}) = \frac{d}{dx}(\sqrt{x} + x\sqrt{x}) = \frac{d}{dx}(x^{\frac{1}{2}} + x^{1\frac{1}{2}}) = \frac{1}{2}x^{-\frac{1}{2}} + 1\frac{1}{2}x^{\frac{1}{2}} = \frac{1}{2\sqrt{x}} + 1\frac{1}{2}\sqrt{x}$

Opgave 17:

a. $y_A = \frac{1}{2}$

$y = \frac{2}{x} = 2x^{-1}$

$y' = -2x^{-2} = -\frac{2}{x^2}$

$y'(4) = -\frac{1}{8}$

b. $y'(1) = -2$

c. $y'(5) = -\frac{2}{25}$

Opgave 18:

a. $10 \cdot \sqrt{x} - 5x = 0$

$10 \cdot \sqrt{x} = 5x$

$100x = 25x^2$

$100x - 25x^2 = 0$

$25x(4 - x) = 0$

$x = 0 \quad \vee \quad x = 4$

$f'(x) = 5x^{-\frac{1}{2}} - 5 = \frac{5}{\sqrt{x}} - 5$

$f'(4) = -2\frac{1}{2}$

b. $f'(\frac{1}{4}) = \frac{5}{\sqrt{\frac{1}{4}}} - 5 = \frac{5}{\frac{1}{2}} - 5 = 10 - 5 = 5$

c. $f'(1) = \frac{5}{\sqrt{1}} - 5 = 5 - 5 = 0$ dus de raaklijn loopt horizontaal

Opgave 19:

a. $y = \frac{2}{x} + 2x = 2x^{-1} + 2x$

$y'(x) = -2x^{-2} + 2 = -\frac{2}{x^2} + 2$

$y'(4) = -\frac{2}{16} + 2 = 1\frac{7}{8}$

b. $y'(0{,}5) = -\frac{2}{0{,}25} + 2 = -8 + 2 = -6 < 0$

c. $y'(1) = -\frac{2}{1} + 2 = 0$ dus de raaklijn loopt horizontaal

Opgave 20:

a. $f(x) = x + \frac{1}{x} = x + x^{-1}$

$f'(x) = 1 - x^{-2} = 1 - \frac{1}{x^2} = 0$

$1 = \frac{1}{x^2}$

$x^2 = 1$

$x = 1 \quad \vee \quad x = -1$

max $f(-1) = -2$

min $f(1) = 2$

b. $y_A = 4\frac{1}{4}$

$f'(4) = \frac{15}{16}$

$k: \quad y = \frac{15}{16}x + b$ door $(4, 4\frac{1}{4})$

$4\frac{1}{4} = \frac{15}{4} + b$

$b = \frac{1}{2}$

$k: \quad y = \frac{15}{16}x + \frac{1}{2}$

c. $f'(0{,}5) = 1 - \frac{1}{0{,}25} = 1 - 4 = -3$

d. $f'(0{,}2) = 1 - \frac{1}{0{,}04} = 1 - 25 = -24 < 0$

e. $a = 2 \quad \vee \quad a = -2$

Opgave 21:

a. $y = 8 - \frac{16}{x} + \frac{24}{x^2} = 8 - 16x^{-1} + 24x^{-2}$

$y' = 16x^{-2} - 48x^{-3} = \frac{16}{x^2} - \frac{48}{x^3} = 0$

$\frac{16}{x^2} = \frac{48}{x^3}$

$16x^3 = 48x^2$

$16x^3 - 48x^2 = 0$

$16x^2(x - 3) = 0$

$x = 0 \quad \vee \quad x = 3$

k.n. $y = 5\frac{1}{3}$

dus $A(3, 5\frac{1}{3})$

b. $y'(-2) = 10$

c. $\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(16x^{-2} - 48x^{-3}\right) = -32x^{-3} + 144x^{-4} = -\frac{32}{x^3} + \frac{144}{x^4}$

$-\frac{32}{x^3} + \frac{144}{x^4} = 0$

$\frac{144}{x^4} = \frac{32}{x^3}$

$32x^4 = 144x^3$
$32x^4 - 144x^3 = 0$
$32x^3(x - 4\frac{1}{2}) = 0$
$x = 0 \quad \vee \quad x = 4\frac{1}{2}$
k.n. dus $x = 4\frac{1}{2}$

d. de snelheid waarmee y verandert wordt gegeven door y' ofwel $\frac{dy}{dx}$
dus de snelheid is maximaal als $\left(\frac{dy}{dx}\right)' = 0$
dat is voor $x = 4\frac{1}{2}$, dan $y = 5{,}63$
dus $B(4{,}5;5{,}63)$

e. de grafiek moet 3 naar links verschoven worden, dus $a = -3$

Opgave 22:

a. $y_A = 6{,}19$
$y' = 3{,}6x^{-0,7} - 3$
$y'(4) = -1{,}64$

b. $y'(0{,}3) = 5{,}36$

c. $3{,}6x^{-0,7} - 3 = 0$
$3{,}6x^{-0,7} = 3$
$x^{-0,7} = \frac{5}{6}$
$x = \sqrt[-0,7]{\frac{5}{6}} = 1{,}30$

Opgave 23:

a. $R' = 1{,}6q^{-0,6} - 1$
$R'(0{,}5) = 1{,}43$ dus € 1,43 per stuk

b. $R'(4{,}5) = -0{,}35$ dus € 0,35 per stuk

c. $1{,}6q^{-0,6} - 1 = 0$
$1{,}6q^{-0,6} = 1$
$q^{-0,6} = 0{,}625$
$q = 2{,}19$
$R(2{,}19) = 3{,}283$ dus € 3283,-

Opgave 24:

a. $B = 4\sqrt{t} - t = 4t^{\frac{1}{2}} - t$
$B' = 2t^{-\frac{1}{2}} - 1 = \frac{2}{\sqrt{t}} - 1$
$B'(3) = 0{,}15$ dus 0,15 dm² ofwel 15 cm² per maand

b. $B'(4) = \frac{2}{\sqrt{4}} - 1 = 1 - 1 = 0$
$B(4) = 4$ dus 4 dm²

Opgave 25:

a. $t = 8$ geeft $q = 25{,}373$ dus 25373

b. $q' = 6 - 1{,}5t^{0{,}5}$

$t = 20$ geeft $q' = -0{,}708$ dus een afname van 708 stuks per dag

c. $t = 4$ geeft $q = 16$

$t = 16$ geeft $q = 32$

dus $\frac{32-16}{16} \cdot 100\% = 100\%$ meer

d. als $t = 36$ dan $q = 0$

e. $6 - 1{,}5t^{0{,}5} = 0$

$-1{,}5t^{0{,}5} = -6$

$t^{0{,}5} = 4$

$t = 16$ dus op 28 mei was de verkoop maximaal

$q = 32$ dus 32000 stuks