16.2 Regels voor het differentiëren

Opgave 17:

a. $p'(x) = 6x^2 + 2x$

b. $f'(x) = 2x$

$g'(x) = 2$

$f'(x) \cdot g'(x) = 2x \cdot 2 = 4x$

c. nee

Opgave 18:

a. $f(x) = x(2x+1)^3$

$f'(x) = 1 \cdot (2x+1)^3 + x \cdot 3(2x+1)^2 \cdot 2 = (2x+1)^3 + 6x(2x+1)^2$

b. $g(x) = x\sqrt{1-3x}$

$g'(x) = 1 \cdot \sqrt{1-3x} + x \cdot \dfrac{1}{2\sqrt{1-3x}} \cdot -3 = \sqrt{1-3x} - \dfrac{3x}{2\sqrt{1-3x}}$

c. $h(x) = 5x + x^2(x^2+1)^{1,8}$

$h'(x) = 5 + 2x \cdot (x^2+1)^{1,8} + x^2 \cdot 1{,}8(x^2+1)^{0,8} \cdot 2x = 5 + 2x(x^2+1)^{1,8} + 3{,}6x^3(x^2+1)^{0,8}$

d. $k(x) = 6(2x-1)\sqrt{2x-1} = 6(2x-1)^{1\frac{1}{2}}$

$k'(x) = 9(2x-1)^{\frac{1}{2}} \cdot 2 = 18\sqrt{2x-1}$

Opgave 19:

a. $y = (x+3)(2x-5)^2$

$y' = 1 \cdot (2x-5)^2 + (x+3) \cdot 2(2x-5) \cdot 2 = (2x-5)^2 + 4(x+3)(2x-5)$

b. $y = (x^2-1)\sqrt{x^2+4}$

$y' = 2x \cdot \sqrt{x^2+4} + (x^2-1) \cdot \dfrac{1}{2\sqrt{x^2+4}} \cdot 2x = 2x\sqrt{x^2+4} + \dfrac{x(x^2-1)}{\sqrt{x^2+4}}$

c. $K = 7q\sqrt{100-q}$

$K' = 7 \cdot \sqrt{100-q} + 7q \cdot \dfrac{1}{2\sqrt{100-q}} \cdot -1 = 7\sqrt{100-q} - \dfrac{7q}{2\sqrt{100-q}}$

d. $A = 5t^3 - t\sqrt{2-3t}$

$A' = 15t^2 - 1 \cdot \sqrt{2-3t} - t \cdot \dfrac{1}{2\sqrt{2-3t}} \cdot -3 = 15t^2 - \sqrt{2-3t} + \dfrac{3t}{2\sqrt{2-3t}}$

Opgave 20:

a. $\left[5x^3\right]' = \left[5 \cdot x^3\right]' = 0 \cdot x^3 + 5 \cdot \left[x^3\right]' = 5 \cdot \left[x^3\right]'$

b. $\left[a \cdot f(x)\right]' = [a]' \cdot f(x) + a \cdot \left[f(x)\right]' = 0 \cdot f(x) + a \cdot \left[f(x)\right]' = a \cdot \left[f(x)\right]'$

Opgave 21:

a. $8(x+1)^2 + 3(x+1) = (x+1)\big(8(x+1)+3\big) = (x+1)(8x+8+3) = (x+1)(8x+11)$

b. $6(2x-1)^5 - 2(2x-1)^4 = (2x-1)^4\big(6(2x-1)-2\big) = (2x-1)^4(12x-6-2)$

$= (2x-1)^4(12x-8)$

c. $(x+3)\sqrt{5-x}-2\sqrt{5-x}=\sqrt{5-x}(x+3-2)=(x+1)\sqrt{5-x}$

d. $(2x-3)(x-7)^5-(x-7)^3=(x-7)^3\left((2x-3)(x-7)^2-1\right)$

$$=(x-7)^3\left((2x-3)(x^2-14x+49)-1\right)$$
$$=(x-7)^3(2x^3-28x^2+98x-3x^2+42x-147-1)$$
$$=(x-7)^3(2x^3-31x^2+140x-148)$$

Opgave 22:

a. $\sqrt{x}+\dfrac{3}{\sqrt{x}}=\dfrac{x}{\sqrt{x}}+\dfrac{3}{\sqrt{x}}=\dfrac{x+3}{\sqrt{x}}$

b. $\sqrt{1-x}+\dfrac{3}{\sqrt{1-x}}=\dfrac{1-x}{\sqrt{1-x}}+\dfrac{3}{\sqrt{1-x}}=\dfrac{4-x}{\sqrt{1-x}}$

c. $\sqrt{x^2+3}+\dfrac{5x}{\sqrt{x^2+3}}=\dfrac{x^2+3}{\sqrt{x^2+3}}+\dfrac{5x}{\sqrt{x^2+3}}=\dfrac{x^2+5x+3}{\sqrt{x^2+3}}$

d. $2\sqrt{2x-1}-\dfrac{4}{\sqrt{2x-1}}=\dfrac{2(2x-1)}{\sqrt{2x-1}}-\dfrac{4}{\sqrt{2x-1}}=\dfrac{4x-2-4}{\sqrt{2x-1}}=\dfrac{4x-6}{\sqrt{2x-1}}$

Opgave 23:

a. $f(x)=4x(3x-1)^5$

$f'(x)=4\cdot(3x-1)^5+4x\cdot 5(3x-1)^4\cdot 3=4(3x-1)^5+60x(3x-1)^4$

$=(3x-1)^4\left(4(3x-1)+60x\right)=(3x-1)^4(12x-4+60x)=(3x-1)^4(72x-4)$

b. $g(x)=x(1-x^2)^4$

$g'(x)=1\cdot(1-x^2)^4+x\cdot 4(1-x^2)^3\cdot -2x=(1-x^2)^3\left(1-x^2-8x^2\right)=(1-x^2)^3(1-9x^2)$

c. $h(x)=x\sqrt{2-2x}$

$h'(x)=1\cdot\sqrt{2-2x}+x\cdot\dfrac{1}{2\sqrt{2-2x}}\cdot -2=\sqrt{2-2x}-\dfrac{x}{\sqrt{2-2x}}=\dfrac{2-2x}{\sqrt{2-2x}}-\dfrac{x}{\sqrt{2-2x}}$

$=\dfrac{2-3x}{\sqrt{2-2x}}$

Opgave 24:

a. $f(x)=x\sqrt{3+2x}$

$f'(x)=1\cdot\sqrt{3+2x}+x\cdot\dfrac{1}{2\sqrt{3+2x}}\cdot 2=\sqrt{3+2x}+\dfrac{x}{\sqrt{3+2x}}=\dfrac{3+2x}{\sqrt{3+2x}}+\dfrac{x}{\sqrt{3+2x}}$

$=\dfrac{3+3x}{\sqrt{3+2x}}$

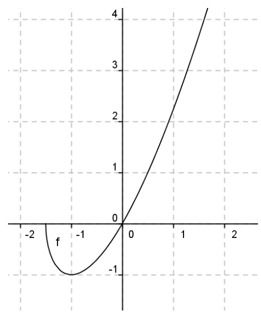
b. $\dfrac{3+3x}{\sqrt{3+2x}}=0$

$3+3x=0$

$3x=-3$

$x=-1$

min $f(-1)=-1$



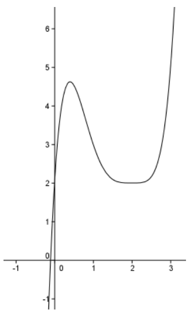
c. $y_A=9$

$f'(3)=4$

$y = 4x + b$ door $(3,9)$
$9 = 12 + b$
$b = -3$
$k: \quad y = 4x - 3$

Opgave 25:

a. $g(x) = x(2-x)^4 + 2$
$g'(x) = 1 \cdot (2-x)^4 + x \cdot 4(2-x)^3 \cdot -1$
$= (2-x)^4 - 4x(2-x)^3$
$= (2-x)^3(2-x-4x)$
$= (2-x)^3(2-5x)$



b. $g'(x) = (2-x)^3(2-5x) = 0$
$2 - x = 0 \quad \vee \quad 2 - 5x = 0$
$-x = -2 \quad \vee \quad -5x = -2$
$x = 2 \quad \vee \quad x = \frac{2}{5}$
max $g(\frac{2}{5}) = 4{,}62144$
min $g(2) = 2$

c. $P(0,2)$
$g'(0) = 16$
$y = 16x + b$ door $(0,2)$
$2 = b$
$l: \quad y = 16x + 2$

Opgave 26:

a. $t'(x) = 3$
$n'(x) = 1$
$\frac{t'(x)}{n'(x)} = \frac{3}{1} = 3$

b. als $\frac{t'(x)}{n'(x)} = 3$ dan is de helling van de grafiek constant, dus zou de grafiek een rechte lijn zijn, wat niet het geval is.

Opgave 27:

a. $y = \frac{x+1}{x-4}$
$y' = \frac{(x-4)\cdot 1 - (x+1)\cdot 1}{(x-4)^2} = \frac{x-4-x-1}{(x-4)^2} = \frac{-5}{(x-4)^2}$

b. $y = 7x + \frac{-2x+3}{x+7}$
$y' = 7 + \frac{(x+7)\cdot -2 - (-2x+3)\cdot 1}{(x+7)^2} = 7 + \frac{-2x-14+2x-3}{(x+7)^2} = 7 - \frac{17}{(x+7)^2}$

c. $y = \frac{3x^2}{6-x} + 4x^3$

$$y' = \frac{(6-x)\cdot 6x - 3x^2 \cdot -1}{(6-x)^2} + 12x^2 = \frac{36x - 6x^2 + 3x^2}{(6-x)^2} + 12x^2 = \frac{36x - 3x^2}{(6-x)^2} + 12x^2$$

d. $y = \frac{4x + x^3}{8x^2 - 5}$

$$y' = \frac{(8x^2 - 5)\cdot(4 + 3x^2) - (4x + x^3)\cdot 16x}{(8x^2 - 5)^2}$$

$$= \frac{32x^2 + 24x^4 - 20 - 15x^2 - 64x^2 - 16x^4}{(8x^2 - 5)^2}$$

$$= \frac{8x^4 - 47x^2 - 20}{(8x^2 - 5)^2}$$

Opgave 28:

a. $A = \frac{-2}{3 + 2t} + 5t$

$$A' = \frac{(3 + 2t)\cdot 0 - -2 \cdot 2}{(3 + 2t)^2} + 5 = \frac{4}{(3 + 2t)^2} + 5$$

b. $K = 2q - \frac{1}{q + 8}$

$$K' = 2 - \frac{(q + 8)\cdot 0 - 1\cdot 1}{(q + 8)^2} = 2 + \frac{1}{(q + 8)^2}$$

c. $P = \left(\frac{2}{1+q}\right)^3 = u^3$ met $u = \frac{2}{1+q}$ dus $u' = \frac{(1+q)\cdot 0 - 2\cdot 1}{(1+q)^2} = \frac{-2}{(1+q)^2}$

$$P' = 3u^2 \cdot u' = 3\cdot\left(\frac{2}{1+q}\right)^2 \cdot \frac{-2}{(1+q)^2} = 3\cdot\frac{4}{(1+q)^2}\cdot\frac{-2}{(1+q)^2}\cdot = \frac{-24}{(1+q)^4}$$

d. $N = \left(\frac{2t}{t-1}\right)^4 = u^4$ met $u = \frac{2t}{t-1}$ dus $u' = \frac{(t-1)\cdot 2 - 2t\cdot 1}{(t-1)^2} = \frac{2t - 2 - 2t}{(t-1)^2} = \frac{-2}{(t-1)^2}$

$$N' = 4u^3 \cdot u' = 4\cdot\left(\frac{2t}{t-1}\right)^3 \cdot \frac{-2}{(t-1)^2} = 4\cdot\frac{8t^3}{(t-1)^3}\cdot\frac{-2}{(t-1)^2} = \frac{-64t^3}{(t-1)^5}$$

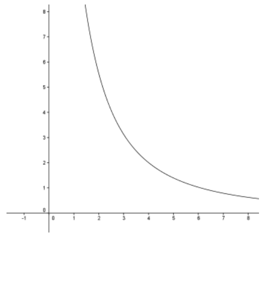
Opgave 29:

a. $P(4{,}5) = 140{,}9$

$P(6{,}5) = 143{,}3$

$$\frac{143{,}3 - 140{,}9}{140{,}9}\cdot 100\% = 1{,}7\%$$

b. $P' = -\frac{(1+x)\cdot 0 - 50\cdot 1}{(1+x)^2} = \frac{50}{(1+x)^2}$



de grafiek van P' ligt boven de x-as dus de grafiek van P is stijgend
de grafiek van P' daalt, dus er is sprake van een afnemende stijging bij de grafiek van P

c. $P' = \dfrac{50}{(1+x)^2} < 0{,}8$

neem $y_1 = \dfrac{50}{(1+x)^2}$ en $y_2 = 0{,}8$

intersect geeft $x = 6{,}9$, dus voor $x > 6{,}9$

Opgave 30:

a. $\dfrac{20p+1600}{4p+5} = 20$

$20(4p+5) = 20p+1600$

$80p+100 = 20p+1600$

$60p = 1500$

$p = 25$ dus 25 euro

b. $q' = \dfrac{(4p+5)\cdot 20-(20p+1600)\cdot 4}{(4p+5)^2} = \dfrac{80p+100-80p-6400}{(4p+5)^2} = \dfrac{-6300}{(4p+5)^2}$

voor iedere waarde van p geldt $q' < 0$ dus de grafiek van q daalt

dus bij een verhoging van de prijs zal de verkoop afnemen

c. $q'(18) = -1{,}06$ dus met 1,06 euro per kist

Opgave 31:

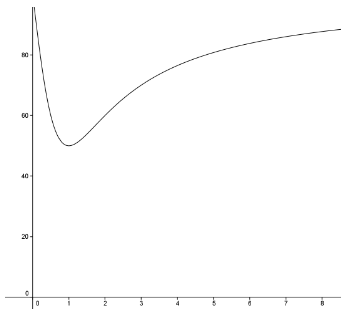
a. $\left[\dfrac{dP}{dt}\right]_{t=4} = 5{,}19$

$\dfrac{5{,}19}{7} = 0{,}7\%$ per dag

b. $P' = \dfrac{(t^2+1)\cdot(200t-100)-100(t^2-t+1)\cdot 2t}{(t^2+1)^2}$

$= \dfrac{200t^3-100t^2+200t-100-200t^3+200t^2-200t}{(t^2+1)^2}$

$= \dfrac{100t^2-100}{(t^2+1)^2}$



$P'(1) = 0$ in de grafiek zie je dat je te maken hebt met een minimum

c. $y_1 = \dfrac{100(x^2-x+1)}{x^2+1}$ en $y_2 = 98$

intersect geeft 49,98 dus $49{,}98\cdot 7 = 350$ dagen

d. $\left[\dfrac{dP}{dt}\right]_{t=8} = 1{,}49$

$P(8) = 87{,}69$

$87{,}69 + 1{,}49t = 100$

$1{,}49t = 12{,}31$

$t = 8{,}26$ dus $8{,}26\cdot 7 = 58$

dus na $8\cdot 7 + 58 = 114$ dagen

Opgave 32:

a. $y = 4(3x-2)^3 = 4u^3$ met $u = 3x-2$ dus $u' = 3$

$y' = 12u^2 \cdot u' = 12(3x-2)^2 \cdot 3 = 36(3x-2)^2$

b. $y = \frac{x+8}{x^2} = \frac{x}{x^2} + \frac{8}{x^2} = x^{-1} + 8x^{-2}$

$y' = -x^{-2} - 16x^{-3} = \frac{-1}{x^2} - \frac{16}{x^3}$

c. $y = 4\sqrt{x^2-3} = 4\sqrt{u} = 4u^{\frac{1}{2}}$ met $u = x^2 - 3$ dus $u' = 2x$

$y' = 2u^{-\frac{1}{2}} \cdot u' = \frac{2}{\sqrt{u}} \cdot u' = \frac{2}{\sqrt{x^2-3}} \cdot 2x = \frac{4x}{\sqrt{x^2-3}}$

d. $y = 6x\sqrt{5-4x}$

$y' = 6 \cdot \sqrt{5-4x} + 6x \cdot \frac{1}{2\sqrt{5-4x}} \cdot -4 = 6\sqrt{5-4x} - \frac{12}{\sqrt{5-4x}}$

e. $y = \frac{3}{(2x-7)^4} = 3(2x-7)^{-4} = 3u^{-4}$ met $u = 2x-7$ dus $u' = 2$

$y' = -12u^{-5} \cdot u' = \frac{-12}{u^5} \cdot u' = \frac{-12}{(2x-7)^5} \cdot 2 = \frac{-24}{(2x-7)^5}$

f. $y = 4x^2 \cdot \sqrt{x} = 4x^2 \cdot x^{\frac{1}{2}} = 4x^{2\frac{1}{2}}$

$y' = 10x^{1\frac{1}{2}} = 10x\sqrt{x}$

g. $y = 5x + \sqrt{1-x}$

$y' = 5 + \frac{1}{2\sqrt{1-x}} \cdot -1 = 5 - \frac{1}{2\sqrt{1-x}}$

h. $y = \frac{3}{x^2} + \frac{2}{3x} = 3x^{-2} + \tfrac{2}{3}x^{-1}$

$y' = -6x^{-3} - \tfrac{2}{3}x^{-2} = \frac{-6}{x^3} - \frac{2}{3x^2}$

i. $y = 5x\sqrt{5x} = (5x)^{1\frac{1}{2}} = u^{1\frac{1}{2}}$ met $u = 5x$ dus $u' = 5$

$y' = 1\tfrac{1}{2}u^{\frac{1}{2}} \cdot u' = 1\tfrac{1}{2}\sqrt{5x} \cdot 5 = 7\tfrac{1}{2}\sqrt{5x}$

Opgave 33:

a. $f(x) = x^2(1-x)^6$

$f'(x) = 2x \cdot (1-x)^6 + x^2 \cdot 6(1-x)^5 \cdot -1 = 2x(1-x)^6 - 6x^2(1-x)^5$

$= (1-x)^5\left(2x(1-x) - 6x^2\right) = (1-x)^5(2x - 2x^2 - 6x^2) = (1-x)^5(2x - 8x^2)$

b. $g(x) = 2x^2\sqrt{1-x^2}$

$g'(x) = 4x \cdot \sqrt{1-x^2} + 2x^2 \cdot \frac{1}{2\sqrt{1-x^2}} \cdot -2x = 4x\sqrt{1-x^2} - \frac{2x^3}{\sqrt{1-x^2}}$

$= \frac{4x(1-x^2)}{\sqrt{1-x^2}} - \frac{2x^3}{\sqrt{1-x^2}} = \frac{4x - 4x^3 - 2x^3}{\sqrt{1-x^2}} = \frac{4x - 6x^3}{\sqrt{1-x^2}}$

c. $h(x) = \frac{x}{(2x-1)^3}$

$$h'(x) = \frac{(2x-1)^3 \cdot 1 - x \cdot 3(2x-1)^2 \cdot 2}{(2x-1)^6}$$

$$= \frac{(2x-1)^3 - 6x(2x-1)^2}{(2x-1)^6}$$

$$= \frac{(2x-1)^2(2x-1-6x)}{(2x-1)^6}$$

$$= \frac{-4x-1}{(2x-1)^4}$$