

## 16.3 Optimaliseren

### Opgave 34:

a.  $f(x) = \frac{x-1}{x+2}$

$$f'(x) = \frac{(x+2) \cdot 1 - (x-1) \cdot 1}{(x+2)^2} = \frac{x+2-x+1}{(x+2)^2} = \frac{3}{(x+2)^2}$$

b.  $(x+2)^2 > 0$  op het domein dus  $\frac{3}{(x+2)^2} > 0$  op het domein

c. de grafiek van  $f$  blijft stijgen

### Opgave 35:

a.  $y = \frac{x^2}{x-2}$

$$y' = \frac{(x-2) \cdot 2x - x^2 \cdot 1}{(x-2)^2} = \frac{2x^2 - 4x - x^2}{(x-2)^2} = \frac{x^2 - 4x}{(x-2)^2} = 0$$

$$x^2 - 4x = 0$$

$$x(x-4) = 0$$

$$x = 0 \quad \vee \quad x = 4$$

$$\max f(0) = 0$$

$$\min f(4) = 8$$

b.  $y_A = y(3) = 9$

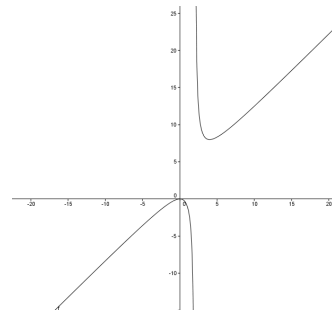
$$rc = y'(3) = -3$$

$$y = -3x + b \text{ door } (3,9)$$

$$9 = -9 + b$$

$$18 = b$$

$$y = -3x + 18$$



### Opgave 36:

a.  $y = \frac{4x}{x^2 + 1}$

$$y' = \frac{(x^2 + 1) \cdot 4 - 4x \cdot 2x}{(x^2 + 1)^2} = \frac{4x^2 + 4 - 8x^2}{(x^2 + 1)^2} = \frac{4 - 4x^2}{(x^2 + 1)^2} = 0$$

$$4 - 4x^2 = 0$$

$$-4x^2 = -4$$

$$x^2 = 1$$

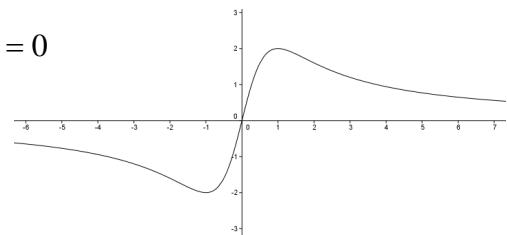
$$x = 1 \quad \vee \quad x = -1$$

$$\min y(-1) = -2$$

$$\max y(1) = 2$$

b.  $y = 2x + \frac{8}{x} = 2x + 8x^{-1}$

$$y' = 2 - 8x^{-2} = 2 - \frac{8}{x^2} = 0$$



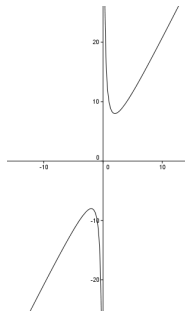
$$-\frac{8}{x^2} = -2$$

$$x^2 = 4$$

$$x = 2 \quad \vee \quad x = -2$$

$$\max y(-2) = -8$$

$$\min y(2) = 8$$



c.  $y = 1,5x^2 + \frac{24}{x^2} = 1,5x^2 + 24x^{-2}$

$$y' = 3x - 48x^{-3} = 3x - \frac{48}{x^3} = 0$$

$$-\frac{48}{x^3} = -3x$$

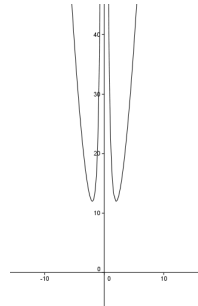
$$-3x^4 = -48$$

$$x^4 = 16$$

$$x = 2 \quad \vee \quad x = -2$$

$$\min y(-2) = 12$$

$$\min y(2) = 12$$



**Opgave 37:**

a.  $C = \frac{0,16t}{t^2 + 4t + 4}$

$$C' = \frac{(t^2 + 4t + 4) \cdot 0,16 - 0,16t \cdot (2t + 4)}{(t^2 + 4t + 4)^2}$$

$$= \frac{0,16t^2 + 0,64t + 0,64 - 0,32t^2 - 0,64t}{(t^2 + 4t + 4)^2}$$

$$= \frac{-0,16t^2 + 0,64}{(t^2 + 4t + 4)^2}$$

$C'(0) = 0,04 > 0$  dus de concentratie neemt toe

b.  $C' = \frac{-0,16t^2 + 0,64}{(t^2 + 4t + 4)^2} = 0$

$$-0,16t^2 + 0,64 = 0$$

$$-0,16t^2 = -0,64$$

$$t^2 = 4$$

$$t = 2$$

c.  $C'(5) = -0,00140$

$$C'(10) = -0,00074$$

$$\frac{-0,00074}{-0,00140} = 1,9 \text{ dus inderdaad ongeveer 2 keer zo groot}$$

d.  $C(24) = 0,0057 > 0,005$  dus ja

**Opgave 38:**

a.  $V(0) = 55$

$$\begin{aligned}
 \text{b. } V &= \frac{1000t + 3000}{t^2 + 16t + 64} + 8 \\
 V' &= \frac{(t^2 + 16t + 64) \cdot 1000 - (1000t + 3000)(2t + 16)}{(t^2 + 16t + 64)^2} \\
 &= \frac{1000t^2 + 16000t + 64000 - (2000t^2 + 16000t + 6000t + 48000)}{(t^2 + 16t + 64)^2} \\
 &= \frac{1000t^2 + 16000t + 64000 - 2000t^2 - 22000t - 48000}{(t^2 + 16t + 64)^2} \\
 &= \frac{-1000t^2 - 6000t + 16000}{(t^2 + 16t + 64)^2}
 \end{aligned}$$

$V'(0) = 3,9 > 0$  dus  $V$  stijgt op  $t = 0$

$$\text{c. } V' = \frac{-1000t^2 - 6000t + 16000}{(t^2 + 16t + 64)^2} = 0$$

$$-1000t^2 - 6000t + 16000 = 0$$

$$t^2 + 6t - 16 = 0$$

$$(t + 8)(t - 2) = 0$$

$$t = -8 \quad \vee \quad t = 2$$

dus op  $t = 2$

d. vul een groot getal in voor  $t$ , dan geldt:  $V \approx 8$

### **Opgave 39:**

$$\text{a. } T(4) - T(3) = 39,09 - 38,71 = 0,38^\circ \text{C}$$

$$\text{b. } T = 37 + \frac{45t}{t^2 + 70}$$

$$T' = \frac{(t^2 + 70) \cdot 45 - 45t \cdot 2t}{(t^2 + 70)^2} = \frac{45t^2 + 3150 - 90t^2}{(t^2 + 70)^2} = \frac{3150 - 45t^2}{(t^2 + 70)^2}$$

2 mei 17.30 uur is  $t = 29,5$

$T'(29,5) = -0,04$  dus de temperatuur daalt  $0,04^\circ \text{C}$

$$\text{c. } T' = \frac{3150 - 45t^2}{(t^2 + 70)^2} = 0$$

$$3150 - 45t^2 = 0$$

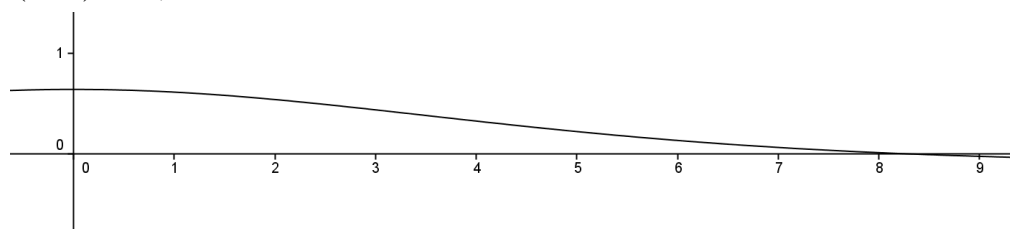
$$-45t^2 = -3150$$

$$t^2 = 70$$

$$t = \sqrt{70}$$

$$T(\sqrt{70}) = 39,7^\circ \text{C}$$

d.



op het eerste stuk is er sprake van een toenemende daling, vanaf ongeveer  $t = 4,5$  is er sprake van een afnemende daling

**Opgave 40:**

- a.  $K_{\text{bodem}} = 24x^2$
- b.  $K_{\text{zijkanten}} = 4 \cdot x \cdot y \cdot 18 = 72xy$
- c.  $K_{\text{totaal}} = 24x^2 + 72xy = 288$   
 $72xy = 288 - 24x^2$   
 $3xy = 12 - x^2$   
 $y = \frac{12 - x^2}{3x}$
- d.  $I = x^2 y = x^2 \cdot \frac{12 - x^2}{3x} = \frac{12x^2 - x^4}{3x} = 4x - \frac{1}{3}x^3$
- e.  $I' = 4 - x^2 = 0$   
 $-x^2 = -4$   
 $x^2 = 4$   
 $x = 2$   
 $y = 1\frac{1}{3}$   
 dus 2 bij 2 bij  $1\frac{1}{3}$  m  
 $I = 5\frac{1}{3}$

**Opgave 41:**

- a. er kunnen zo gevaarlijke situaties ontstaan. Voor de veiligheid moeten de auto's meer afstand nemen als de snelheid groter is.
- b.  $t = \frac{12,5 + 4}{10} = 1,65$  sec
- c. per auto  $t = \frac{18 + 4}{12} = \frac{11}{6}$  sec  
 per uur:  $\frac{3600}{\frac{11}{6}} = 1964$  auto's
- d. per auto:  $4 + r$  meter  
 per auto:  $\frac{4 + r}{v}$  sec  
 per uur:  $\frac{3600}{\frac{4+r}{v}} = \frac{3600v}{4 + r}$  auto's
- e.  $r = 0,125 \cdot 16^2 = 32$  meter  
 $Q = \frac{3600 \cdot 16^2}{4 + 32} = 1600$
- f.  $v = 54 \frac{\text{km}}{\text{uur}} = 15 \frac{\text{m}}{\text{s}}$   
 $r = 0,125 \cdot 15^2 = 28,125$   
 $Q = \frac{3600 \cdot 15}{32,125} = 1681$
- g.  $Q = \frac{3600v}{4 + 0,125v^2}$

$$Q' = \frac{(4 + 0,125v^2) \cdot 3600 - 3600v \cdot 0,25v}{(4 + 0,125v^2)^2}$$

$$= \frac{14400 + 450v^2 - 900v^2}{(4 + 0,125v^2)^2}$$

$$= \frac{14400 - 450v^2}{(4 + 0,125v^2)^2}$$

h.  $\frac{14400 - 450v^2}{(4 + 0,125v^2)^2} = 0$

$$14400 - 450v^2 = 0$$

$$-450v^2 = -14400$$

$$v^2 = 32$$

$$v = \sqrt{32} = 5,66 \text{ m/s} = 20,4 \text{ km/uur}$$

$$Q = 2546$$

### **Opgave 42:**

a.  $W_{\text{ totaal}} = 3v^2 + 7,5 \cdot 10^{-4} \cdot \frac{(4 \cdot 10^6)^2}{v^2} = 3v^2 + \frac{1,2 \cdot 10^{10}}{v^2} = 3v^2 + 1,2 \cdot 10^{10} \cdot v^{-2}$

$$\frac{dW}{dv} = 6v - 2,4 \cdot 10^{10} \cdot v^{-3} = 6v - \frac{2,4 \cdot 10^{10}}{v^3} = \frac{6v^4 - 2,4 \cdot 10^{10}}{v^3}$$

b.  $\frac{6v^4 - 2,4 \cdot 10^{10}}{v^3} = 0$

$$6v^4 - 2,4 \cdot 10^{10} = 0$$

$$6v^4 = 2,4 \cdot 10^{10}$$

$$v^4 = 0,4 \cdot 10^{10}$$

$$v = 251,5 \text{ m/s} = 905 \text{ km/uur}$$

c.  $3v^2 = \frac{1,2 \cdot 10^{10}}{v^2}$

$$3v^4 = 1,2 \cdot 10^{10}$$

$$v^4 = 0,4 \cdot 10^{10}$$

$$v = 251,5 \text{ m/s} = 905 \text{ km/uur}$$

### **Opgave 43:**

a. boven zee heeft een vogel te maken met een neerwaartse luchtstroom en het kost een vogel dan ook meer energie om over zee te vliegen.

b.  $EB = \sqrt{5^2 + 12^2} = 13$   
dus  $13 \cdot 50 = 650 \text{ kJ}$

### **Opgave 44:**

a.  $EC = \sqrt{5^2 + x^2} = \sqrt{25 + x^2}$

$$BC = 12 - x$$

$$\text{Energie} = 50\sqrt{25 + x^2} + 32(12 - x)$$

- b.  $50\sqrt{25+x^2} + 32(12-x) = 640$   
 $y_1 = 50\sqrt{25+x^2} + 32(12-x)$  en  $y_2 = 640$   
 intersect geeft  $x = 11,28$
- c. de optie minimum geeft:  $x = 4,16$   
 $E = 576$

**Opgave 45:**

- a.  $y = 6\sqrt{x^2 + 288} + 10 - 2x$   
 $\frac{dy}{dx} = 6 \cdot \frac{1}{2\sqrt{x^2 + 288}} \cdot 2x - 2 = \frac{6x}{\sqrt{x^2 + 288}} - 2$
- b.  $\frac{6x}{\sqrt{x^2 + 288}} - 2 = 0$   
 $\frac{6x}{\sqrt{x^2 + 288}} = 2$   
 $6x = 2\sqrt{x^2 + 288}$   
 $3x = \sqrt{x^2 + 288}$   
 $9x^2 = x^2 + 288$   
 $8x^2 = 288$   
 $x^2 = 36$   
 $x = 6$   
 $y = 106$

**Opgave 46:**

- a.  $BE = \sqrt{5^2 + 10^2} = \sqrt{125} = 11,180 \text{ km} = 11180 \text{ m}$   
 $K = 11180 \cdot 140 = 1565200$
- b.  $K = 10000 \cdot 100 + 5000 \cdot 140 = 1700000$
- c.  $CE = \sqrt{5^2 + 2^2} = \sqrt{29} = 5,385 \text{ km} = 5385 \text{ m}$   
 $K = 5385 \cdot 140 + 8000 \cdot 100 = 1553900$
- d.  $BP = 10000 - 1000x$   
 $K = (10000 - 1000x) \cdot 100 = 1000000 - 100000x$
- e.  $PE = \sqrt{5^2 + x^2} = \sqrt{25 + x^2}$   
 $K = 140 \cdot 1000\sqrt{25 + x^2} = 140000\sqrt{25 + x^2}$
- f.  $TK = 140000\sqrt{25 + x^2} + 1000000 - 100000x$
- g.  $TK' = 140000 \cdot \frac{1}{2\sqrt{25 + x^2}} \cdot 2x - 100000 = \frac{140000x}{\sqrt{25 + x^2}} - 100000 = 0$   
 $y_1 = \frac{140000x}{\sqrt{25 + x^2}} - 100000$  de optie zero geeft  $x = 5,103$

**Opgave 47:**

- a.  $PE = \sqrt{x^2 + 200^2} = \sqrt{x^2 + 40000}$   
 $PF = 2000 - x$

$$K = 65\sqrt{x^2 + 40000} + 50(2000 - x) = 65\sqrt{x^2 + 40000} + 100000 - 50x$$

$$\text{b. } K' = 65 \cdot \frac{1}{2\sqrt{x^2 + 40000}} \cdot 2x - 50 = \frac{65x}{\sqrt{x^2 + 40000}} - 50 = 0$$

$$y_1 = \frac{65}{\sqrt{x^2 + 40000}} - 50 \text{ de optie zero geeft } x = 241$$

**Opgave 48:**

$$\text{a. } p(q) = 1560 - a\sqrt{q}$$

$$R = p \cdot q = (1560 - a\sqrt{q}) \cdot q = 1560q - aq\sqrt{q} = 1560q - aq^{1\frac{1}{2}}$$

$$R' = 1560 - 1\frac{1}{2}aq^{\frac{1}{2}} = 1560 - 1\frac{1}{2}a\sqrt{q}$$

$$R'(169) = 1560 - 1\frac{1}{2}a\sqrt{169} = 1560 - 19,5a = 0$$

$$-19,5a = -1560$$

$$a = 80$$

$$\text{b. } p = 1560 - 92\sqrt{q} = 548$$

$$-92\sqrt{q} = -1012$$

$$\sqrt{q} = 11$$

$$q = 121$$

$$W = R - K = 1560q - 92q\sqrt{q} - (250 + bq) = 1560q - 92q\sqrt{q} - 250 - bq$$

$$W' = 1560 - 138q^{\frac{1}{2}} - b = 1560 - 138\sqrt{q} - b$$

$$W'(121) = 1560 - 138\sqrt{121} - b = 0$$

$$-b = -1560 + 138\sqrt{121} = -42$$

$$b = 42$$