

GEMENGDE OPGAVEN: H1 Vergelijkingen en ongelijkheden.

Opgave 1:

- a. $7x^2 = 5x$
 $7x^2 - 5x = 0$
 $x(7x - 5) = 0$
 $x = 0 \quad \vee \quad 7x = 5$
 $x = 0 \quad \vee \quad x = \frac{5}{7}$
- b. $2x^2 + x = 3$
 $2x^2 + x - 3 = 0$
 $x = \frac{-1 \pm \sqrt{1 + 24}}{4} = \frac{-1 \pm 5}{4}$
 $x = \frac{-1 + 5}{4} = 1 \quad \vee \quad x = \frac{-1 - 5}{4} = -1\frac{1}{2}$
- c. $(x + 2)(x - 6) = 9$
 $x^2 - 4x - 12 = 9$
 $x^2 - 4x - 21 = 0$
 $(x - 7)(x + 3) = 0$
 $x = 7 \quad \vee \quad x = -3$
- d. $(x - 3)^2 - (x + 1)^2 = x^2 - 1$
 $x^2 - 6x + 9 - (x^2 + 2x + 1) = x^2 - 1$
 $x^2 - 6x + 9 - x^2 - 2x - 1 = x^2 - 1$
 $-x^2 - 8x + 9 = 0$
 $x^2 + 8x - 9 = 0$
 $(x + 9)(x - 1) = 0$
 $x = -9 \quad \vee \quad x = 1$
- e. $(2x - 3)^2 = 36$
 $2x - 3 = 6 \quad \vee \quad 2x - 3 = -6$
 $2x = 9 \quad \vee \quad 2x = -3$
 $x = 4\frac{1}{2} \quad \vee \quad x = -1\frac{1}{2}$
- f. $4 - (x - 2)^2 = 7x - 3$
 $4 - (x^2 - 4x + 4) = 7x - 3$
 $4 - x^2 + 4x - 4 = 7x - 3$
 $-x^2 - 3x + 3 = 0$
 $x = \frac{3 \pm \sqrt{9 + 12}}{-2} = -1\frac{1}{2} \pm \frac{1}{2}\sqrt{21}$
 $x = -1\frac{1}{2} + \frac{1}{2}\sqrt{21} \quad \vee \quad x = -1\frac{1}{2} - \frac{1}{2}\sqrt{21}$

Opgave 2:

- a. $D = 6^2 - 4 \cdot p \cdot 3p > 0$
 $36 - 12p^2 > 0$
 $-12p^2 > -36$

$$p^2 < 3$$

$$p = \sqrt{3} \quad \vee \quad p = -\sqrt{3}$$

$$-\sqrt{3} < x < \sqrt{3}$$

b. $36 + 6p - 6p^2 = 0$

$$p^2 - p - 6 = 0$$

$$(p-3)(p+2) = 0$$

$$p = 3 \quad \vee \quad p = -2$$

$$x^2 + 3x - 54 = 0 \quad \vee \quad x^2 - 2x - 24 = 0$$

$$(x+9)(x-6) = 0 \quad \vee \quad (x-6)(x+4) = 0$$

$$x = -9 \quad \vee \quad x = 6 \quad \vee \quad x = 6 \quad \vee \quad x = -4$$

c. $D = (-2p)^2 - 4 \cdot p \cdot 4 = 0 \quad \wedge \quad p \neq 0$ (als $p = 0$ dan $4 = 0$ dus geen oplossingen)

$$4p^2 - 16p = 0$$

$$4p(p-4) = 0$$

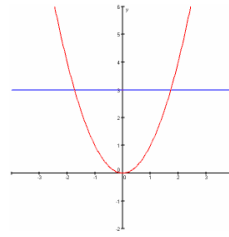
$$p = 0 \quad \vee \quad p = 4$$

k.n. $4x^2 - 8x + 4 = 0$

$$x^2 - 2x + 1 = 0$$

$$(x-1)^2 = 0$$

$$x = 1$$



Opgave 3:

a. $x^6 - 6x^3 + 5 = 0$

stel $p = x^3$ dan $p^2 - 6p + 5 = 0$

$$(p-1)(p-5) = 0$$

$$p = 1 \quad \vee \quad p = 5$$

$$x^3 = 1 \quad \vee \quad x^3 = 5$$

$$x = 1 \quad \vee \quad x = \sqrt[3]{5}$$

b. $|x^4 - 7x^2| = 18$

$$x^4 - 7x^2 = 18 \quad \vee \quad x^4 - 7x^2 = -18$$

stel $p = x^2$

$$p^2 - 7p = 18 \quad \vee \quad p^2 - 7p = -18$$

$$p^2 - 7p - 18 = 0 \quad \vee \quad p^2 - 7p + 18 = 0$$

$$(p+2)(p-9) = 0 \quad \vee \quad p = \frac{7 \pm \sqrt{49 - 72}}{2} = k.n.$$

$$p = -2 \quad \vee \quad p = 9$$

$$x^2 = -2 \quad \vee \quad x^2 = 9$$

k.n. $x = 3 \quad \vee \quad x = -3$

c. $10x^4 = 17x^2 + 657$

$$10x^4 - 17x^2 - 657 = 0$$

stel $p = x^2$

$$10p^2 - 17p - 657 = 0$$

$$p = \frac{17 \pm \sqrt{289 + 26280}}{20} = \frac{17 \pm \sqrt{26569}}{20} = \frac{17 \pm 163}{20}$$

$$p = \frac{17 + 163}{20} = 9 \quad \vee \quad p = \frac{17 - 163}{20} = -7,3$$

$$x^2 = 9 \quad \vee \quad x^2 = -7,3$$

$$x = 3 \quad \vee \quad x = -3$$

d. $10 - (2x - 1)^4 = 8$

$$-(2x - 1)^4 = -2$$

$$(2x - 1)^4 = 2$$

$$2x - 1 = \sqrt[4]{2} \quad \vee \quad 2x - 1 = -\sqrt[4]{2}$$

$$2x = 1 + \sqrt[4]{2} \quad \vee \quad 2x = 1 - \sqrt[4]{2}$$

$$x = \frac{1}{2} + \frac{1}{2} \cdot \sqrt[4]{2} \quad \vee \quad x = \frac{1}{2} - \frac{1}{2} \cdot \sqrt[4]{2}$$

e. $x^5 - 16x^3 + 28x = 0$

$$x(x^4 - 16x^2 + 28) = 0$$

$$x = 0 \quad \vee \quad x^4 - 16x^2 + 28 = 0$$

stel $p = x^2$

$$x = 0 \quad \vee \quad p^2 - 16p + 28 = 0$$

$$x = 0 \quad \vee \quad (p - 2)(p - 14) = 0$$

$$x = 0 \quad \vee \quad p = 2 \quad \vee \quad p = 14$$

$$x = 0 \quad \vee \quad x^2 = 2 \quad \vee \quad x^2 = 14$$

$$x = 0 \quad \vee \quad x = \sqrt{2} \quad \vee \quad x = -\sqrt{2} \quad \vee \quad x = \sqrt{14} \quad \vee \quad x = -\sqrt{14}$$

f. $x^3 - 3x\sqrt{x} - 108 = 0$

stel $p = x\sqrt{x}$

$$p^2 - 3p - 108 = 0$$

$$(p + 9)(p - 12) = 0$$

$$p = -9 \quad \vee \quad p = 12$$

$$x\sqrt{x} = -9 \quad \vee \quad x\sqrt{x} = 12$$

k.n. $x^3 = 144$
 $x = \sqrt[3]{144}$

g. $6x^5 + 10x^2 \cdot \sqrt{x} - 464 = 0$

stel $p = x^2 \cdot \sqrt{x}$

$$6p^2 + 10p - 464 = 0$$

$$p = \frac{-10 \pm \sqrt{100 + 11136}}{12} = \frac{-10 \pm \sqrt{11236}}{12} = \frac{-10 \pm 106}{12}$$

$$p = \frac{-10 - 106}{12} = -9\frac{2}{3} \quad \vee \quad p = \frac{-10 + 106}{12} = 8$$

$$x^2 \cdot \sqrt{x} = -9\frac{2}{3} \quad \vee \quad x^2 \cdot \sqrt{x} = 8$$

k.n. $x^5 = 64$
 $x = \sqrt[5]{64}$

h. $(2x-1)^4 - 5(2x-1)^2 + 4 = 0$
 stel $p = 2x-1$
 $p^4 - 5p^2 + 4 = 0$
 $(p^2 - 1)(p^2 - 4) = 0$
 $p^2 = 1 \vee p^2 = 4$
 $p = 1 \vee p = -1 \vee p = 2 \vee p = -2$
 $2x-1 = 1 \vee 2x-1 = -1 \vee 2x-1 = 2 \vee 2x-1 = -2$
 $2x = 2 \vee 2x = 0 \vee 2x = 3 \vee 2x = -1$
 $x = 1 \vee x = 0 \vee x = 1\frac{1}{2} \vee x = -\frac{1}{2}$

Opgave 4:

a. $\frac{2x}{x-1} - 2 = 4$

$$\frac{2x}{x-1} = 6$$

$$6(x-1) = 2x$$

$$6x - 6 = 2x$$

$$4x = 6$$

$$x = 1\frac{1}{2}$$

b. $3\sqrt{2-3x} = 21$

$$\sqrt{2-3x} = 7$$

$$2-3x = 49$$

$$-3x = 47$$

$$x = -15\frac{2}{3}$$

c. $\frac{x+2}{x-1} = \frac{x}{x+5}$

$$(x+2)(x+5) = x(x-1)$$

$$x^2 + 7x + 10 = x^2 - x$$

$$8x = -10$$

$$x = -1\frac{1}{4}$$

d. $\frac{2x-1}{x+2} = \frac{x+4}{x-2}$

$$(2x-1)(x-2) = (x+2)(x+4)$$

$$2x^2 - 5x + 2 = x^2 + 6x + 8$$

$$x^2 - 11x - 6 = 0$$

$$x = \frac{11 \pm \sqrt{121 + 24}}{2} = \frac{11 \pm \sqrt{145}}{2}$$

$$x = 5\frac{1}{2} + \frac{1}{2}\sqrt{145} \vee x = 5\frac{1}{2} - \frac{1}{2}\sqrt{145}$$

e. $2\sqrt{x-1} + 8 = 15$

$$2\sqrt{x-1} = 7$$

$$\sqrt{x-1} = 3\frac{1}{2}$$

$$x-1 = 12\frac{1}{4}$$

$$x = 13\frac{1}{4}$$

f. $\frac{4+2x}{x} = \frac{12}{x+1}$
 $(4+2x)(x+1) = 12x$
 $4x+4+2x^2+2x = 12x$
 $2x^2-6x+4 = 0$
 $x^2-3x+2 = 0$
 $(x-1)(x-2) = 0$
 $x = 1 \vee x = 2$

g. $\sqrt{2-2x} = -2x$
 $2-2x = 4x^2$
 $-4x^2-2x+2 = 0$
 $x = \frac{2 \pm \sqrt{4+32}}{-8} = \frac{2 \pm 6}{-8}$
 $x = \frac{2+6}{-8} = -1 \vee x = \frac{2-6}{-8} = \frac{1}{2}$ (vervalt)

h. $3x = \sqrt{8x+1}$
 $9x^2 = 8x+1$
 $9x^2-8x-1 = 0$
 $x = \frac{8 \pm \sqrt{64+36}}{18} = \frac{8 \pm 10}{18}$
 $x = \frac{8+10}{18} = 1 \vee x = \frac{8-10}{18} = -\frac{1}{9}$ (vervalt)

Opgave 5:

a.
$$\begin{cases} 3x - 2y = -5 & \times 5 \\ -x + 5y = 32 & \times 2 \end{cases}$$

$$\begin{cases} 15x - 10y = -25 \\ -2x + 10y = 64 & + \end{cases}$$

$$13x = 39$$

$$x = 3$$

$$9 - 2y = -5$$

$$-2y = -14$$

$$y = 7$$

dus $x = 3 \wedge y = 7$

b.
$$\begin{cases} 4x + 2y = 14 & \times 3 \\ 5x - 3y = 45 & \times 2 \end{cases}$$

$$\begin{cases} 12x + 6y = 42 \\ 10x - 6y = 90 & + \end{cases}$$

$$22x = 132$$

$$x = 6$$

$$24 + 2y = 14$$

$$2y = -10$$

$$y = -5$$

$$\text{dus } x = 6 \wedge y = -5$$

Opgave 6:

(-4,42) en (2,12) invullen geeft:

$$\begin{cases} -32 + 16a - 4b + 6 = 42 \\ 4 + 4a + 2b + 6 = 12 \end{cases}$$

$$\begin{cases} 16a - 4b = 68 & | \times 1 \\ 4a + 2b = 2 & | \times 2 \end{cases}$$

$$\begin{cases} 16a - 4b = 68 \\ 8a + 4b = 4 & + \end{cases}$$

$$\hline 24a = 72$$

$$a = 3$$

$$12 + 2b = 2$$

$$2b = -10$$

$$b = -5$$

$$\text{dus } a = 3 \wedge b = -5$$

Opgave 7:

a.
$$\begin{cases} a + b = 150 \\ 8,6a + 7b = 7,9 \cdot 150 \end{cases}$$

$$\begin{cases} a + b = 150 & | \times 7 \\ 8,6a + 7b = 1185 & | \times 1 \end{cases}$$

$$\begin{cases} 7a + 7b = 1050 \\ 8,6a + 7b = 1185 & - \end{cases}$$

$$\hline -1,6a = 135$$

$$a = 84,375$$

$$b = 150 - 84,375 = 65,625$$

b. stel a ml van 15% oplossing en b ml van 30% oplossing

$$\begin{cases} a + b = 600 \\ 0,15a + 0,3b = 0,22 \cdot 600 \end{cases}$$

$$\begin{cases} a + b = 600 & | \times 0,3 \\ 0,15a + 0,3b = 132 & | \times 1 \end{cases}$$

$$\begin{cases} 0,3a + 0,3b = 180 \\ 0,15a + 0,3b = 132 & - \end{cases}$$

$$\hline 0,15a = 48$$

$$a = 320$$

$$b = 600 - 320 = 280$$

Dus 320 ml van de 15% oplossing en 280 ml van de 30% oplossing.

Opgave 8:

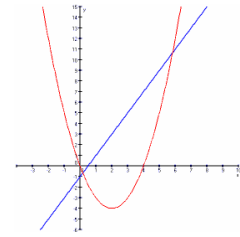
a. $x^2 - 4x > 2x - 1$

$y_1 = x^2 - 4x$ en $y_2 = 2x - 1$

calc-menu intersection geeft:

$x = 0,17 \quad \vee \quad x = 5,83$

$x < 0,17 \quad \vee \quad x > 5,83$



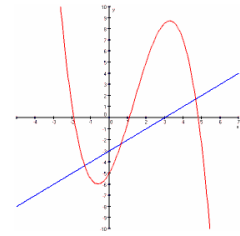
b. $-0,5x^3 + 2x^2 + 3x - 5 \leq x - 3$

$y_1 = -0,5x^3 + 2x^2 + 3x - 5$ en $y_2 = x - 3$

calc-menu intersection geeft:

$x = -1,32 \quad \vee \quad x = 0,65 \quad \vee \quad x = 4,67$

$-1,32 \leq x \leq 0,65 \quad \vee \quad x \geq 4,67$



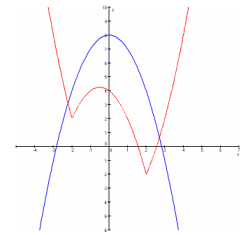
c. $|x^2 - 4| - x < 8 - x^2$

$y_1 = \text{abs}(x^2 - 4) - x$ en $y_2 = 8 - x^2$

calc-menu intersection geeft:

$x = -2,21 \quad \vee \quad x = 2,71$

$-2,21 < x < 2,71$



d. $\sqrt{x^2 - 4x + 4} > |x^3 - 6x|$

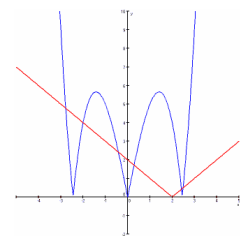
$y_1 = \sqrt{x^2 - 4x + 4}$ en $y_2 = \text{abs}(x^3 - 6x)$

calc-menu intersection geeft:

$x = -2,78 \quad \vee \quad x = -2 \quad \vee \quad x = -0,41 \quad \vee \quad x = 0,29 \quad \vee$

$x = 2,41 \quad \vee \quad x = 2,49$

$-2,78 < x < -2 \quad \vee \quad -0,41 < x < 0,29 \quad \vee \quad 2,41 < x < 2,49$

**Opgave 9:**

a. $px^3 + 2px^2 + x^2 + 2\frac{1}{4}x = 0$

$x(px^2 + 2px + x + 2\frac{1}{4}) = 0$

$x = 0 \quad \vee \quad px^2 + 2px + x + 2\frac{1}{4} = 0$ heeft twee oplossingen (dus $p \neq 0$)

$D = (2p+1)^2 - 4 \cdot p \cdot 2\frac{1}{4} > 0$

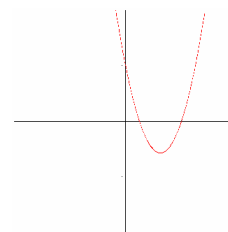
$4p^2 + 4p + 1 - 9p > 0$

$4p^2 - 5p + 1 > 0$

$p = \frac{5 \pm \sqrt{25 - 16}}{8} = \frac{5 \pm 3}{8}$

$p = \frac{5+3}{8} = 1 \quad \vee \quad p = \frac{5-3}{8} = \frac{1}{4}$

$\frac{1}{4} < p < 1$



b. $2px^4 - px^3 + 5x^3 + 2x^2 = 0$

$x^2(2px^2 - px + 5x + 2) = 0$

$x = 0 \quad \vee \quad 2px^2 - px + 5x + 2 = 0$ heeft geen oplossingen (behalve $x = 0$)

$D = (-p+5)^2 - 4 \cdot 2p \cdot 2 < 0$

$p^2 - 10p + 25 - 16p < 0$

$$p^2 - 26p + 25 < 0$$
$$(p - 1)(p - 25) < 0$$
$$p = 1 \vee p = 25$$
$$1 < p < 25$$

