

## Gemengde opgave hoofdstuk 4 Algebra en meetkunde.

### Opgave 31:

- a.  $(2a + \sqrt{3})^2 = 4a^2 + 4a\sqrt{3} + 3$
- b.  $(a + 2\sqrt{3})(a - 2\sqrt{3}) = a^2 - 12$
- c.  $(2\sqrt{2} + 3\sqrt{8})^2 = (2\sqrt{2} + 3 \cdot 2\sqrt{2})^2 = (2\sqrt{2} + 6\sqrt{2})^2 = (8\sqrt{2})^2 = 128$
- d.  $\sqrt{\frac{1}{2}} + 6\sqrt{32} = \frac{1}{2}\sqrt{2} + 6 \cdot 4\sqrt{2} = \frac{1}{2}\sqrt{2} + 24\sqrt{2} = 24\frac{1}{2}\sqrt{2}$
- e.  $\frac{\sqrt{2}+1}{\sqrt{2}-1} = \frac{\sqrt{2}+1}{\sqrt{2}-1} \cdot \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{2+2\sqrt{2}+1}{2-1} = \frac{3+2\sqrt{2}}{1} = 3+2\sqrt{2}$
- f.  $\frac{5}{\sqrt{3}+1} = \frac{5}{\sqrt{3}+1} \cdot \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{5\sqrt{3}-5}{3-1} = \frac{5\sqrt{3}-5}{2} = 2\frac{1}{2}\sqrt{3} - 2\frac{1}{2}$
- g.  $\frac{\sqrt{8} + \sqrt{12}}{2\sqrt{3}} = \frac{2\sqrt{2} + 2\sqrt{3}}{2\sqrt{3}} = \frac{2\sqrt{2} + 2\sqrt{3}}{2\sqrt{3}} = \frac{2\sqrt{2} + 2\sqrt{3}}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{6} + 6}{6} = \frac{1}{3}\sqrt{6} + 1$
- h.  $(2a + 3\sqrt{2})(2a + 2\sqrt{3}) = 4a^2 + 4a\sqrt{3} + 6a\sqrt{2} + 6\sqrt{6}$

### Opgave 32:

- a.  $3x + \frac{6}{2x-1} = \frac{3x(2x-1) + 6}{2x-1} = \frac{6x^2 - 3x + 6}{2x-1}$
- b.  $\frac{2x-1}{x+2} - \frac{x+2}{x-4} = \frac{(2x-1)(x-4) - (x+2)(x+2)}{(x+2)(x-4)} = \frac{2x^2 - 9x + 4 - (x^2 + 4x + 4)}{(x+2)(x-4)} =$   
 $= \frac{2x^2 - 9x + 4 - x^2 - 4x - 4}{(x+2)(x-4)} = \frac{x^2 - 13x}{(x+2)(x-4)}$
- c.  $\frac{a^2}{2a+5} + \frac{a^4}{a-3} = \frac{a^2(a-3) + a^4(2a+5)}{(2a+5)(a-3)} = \frac{a^3 - 3a^2 + 2a^5 + 5a^4}{(2a+5)(a-3)}$
- d.  $\frac{3x^2 + 6x}{x^2 + 8x + 12} = \frac{3x(x+2)}{(x+2)(x+6)} = \frac{3x}{x+6}$
- e.  $\frac{x^4 - 9x^2 + 8}{x^4 - 1} = \frac{(x^2 - 1)(x^2 - 8)}{(x^2 - 1)(x^2 + 1)} = \frac{x^2 - 8}{x^2 + 1}$
- f.  $\frac{a^6 - 5a^3 + 4}{6a^3 - 24} = \frac{(a^3 - 1)(a^3 - 4)}{6(a^3 - 4)} = \frac{a^3 - 1}{6} = \frac{1}{6}a^3 - \frac{1}{6}$

### Opgave 33:

- a.  $\frac{1}{x+1} + \frac{3}{2x+1} = \frac{8}{15}$   
 $\frac{2x+1+3(x+1)}{(x+1)(2x+1)} = \frac{8}{15}$   
 $\frac{2x+1+3x+3}{2x^2+3x+1} = \frac{8}{15}$   
 $\frac{5x+4}{2x^2+3x+1} = \frac{8}{15}$   
 $8(2x^2+3x+1) = 15(5x+4)$

$$16x^2 + 24x + 8 = 75x + 60$$

$$16x^2 - 51x - 52 = 0$$

$$x = \frac{51 \pm \sqrt{2601 + 3328}}{32} = \frac{51 \pm \sqrt{5929}}{32} = \frac{51 \pm 77}{32}$$

$$x = \frac{51 + 77}{32} = 4 \quad \vee \quad x = \frac{51 - 77}{32} = -\frac{13}{16}$$

b.  $\frac{x^2 - 4}{x^2 + 4x + 4} = 2x$

$$\frac{(x-2)(x+2)}{(x+2)(x+2)} = 2x$$

$$\frac{x-2}{x+2} = 2x$$

$$2x(x+2) = x-2$$

$$2x^2 + 4x = x - 2$$

$$2x^2 + 3x + 2 = 0$$

$$x = \frac{-2 \pm \sqrt{9 - 16}}{4} = k.n. \text{ dus geen oplossingen}$$

### **Opgave 34:**

a.  $x^4 \cdot \sqrt[3]{x} = x^4 \cdot x^{\frac{1}{3}} = x^{4\frac{1}{3}}$

b.  $\frac{x^{-2}}{x^3} = x^{-5}$

c.  $x \cdot \sqrt{\frac{1}{x^5}} = x \cdot \sqrt{x^{-5}} = x \cdot (x^{-5})^{\frac{1}{2}} = x \cdot x^{-2\frac{1}{2}} = x^{-1\frac{1}{2}}$

d.  $\frac{1}{x} \cdot (\sqrt[4]{x^3})^8 = x^{-1} \cdot (x^{\frac{3}{4}})^8 = x^{-1} \cdot x^6 = x^5$

e.  $\frac{x^3 \cdot x^{-5}}{\sqrt{x}} = \frac{x^{-2}}{x^{\frac{1}{2}}} = x^{-2\frac{1}{2}}$

f.  $(x\sqrt{x})^{-3} = (x^{1\frac{1}{2}})^{-3} = x^{-4\frac{1}{2}}$

### **Opgave 35:**

a.  $F = (2000 - 16,3 \cdot 60)(-5 - -20)^{-1,668} = 11,2$  dus maximaal 11 minuten

b.  $(2000 - 16,3v)(-5 + 18)^{-1,668} = 20$

$$(2000 - 16,3v) \cdot 13^{-1,668} = 20$$

$$2000 - 16,3v = \frac{20}{13^{-1,668}} = 1442,4$$

$$-16,3v = -557,6$$

$$v = 34,2 \text{ dus } 34 \frac{\text{km}}{\text{uur}}$$

c.  $tijd = \frac{10}{40} = 0,25$  uur dus  $F = 15$  en  $v = 0$

$$2000 \cdot (-5 - T)^{-1,668} = 15$$

$$y_1 = 2000 \cdot (-5 - X)^{-1,668} \text{ en } y_2 = 15$$

Intersect geeft  $x = -23,8$  dus  $T < -23,8$

**Opgave 36:**

a.  $A = 136$

$$a \cdot 40^{0,67} = 136$$

$$a = \frac{136}{40^{0,67}} = 11,5$$

b.  $A = 11,5 \cdot 275^{0,67} = 496 \text{ dm}^2$

c.  $A = 1,16$

$$11,5m^{0,67} = 1,16$$

$$m^{0,67} = 0,1$$

$$m = \sqrt[0,67]{0,1} = 0,033 \text{ kg} = 33 \text{ g}$$

**Opgave 37:**

$$\cos 30^\circ = \frac{AK}{5}$$

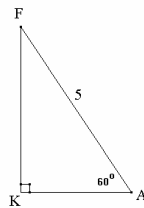
$$AK = 5 \cos 30^\circ = 5 \cdot \frac{1}{2} \sqrt{2} = 2\frac{1}{2} \sqrt{2}$$

$$FK = 5 \sin 30^\circ = 5 \cdot \frac{1}{2} = 2\frac{1}{2}$$

$$KL = 2 \cdot 2\frac{1}{2} + 5 = 10$$

$$KN = 2 \cdot 2\frac{1}{2} \sqrt{3} = 5\sqrt{3}$$

$$Opp = 10 \cdot 5\sqrt{3} = 50\sqrt{3}$$

**Opgave 38:**

$$h = \sqrt{4^2 - 2^2} = \sqrt{12} = 2\sqrt{3}$$

$$Opp\Delta = \frac{1}{2} \cdot 4 \cdot 2\sqrt{3} = 4\sqrt{3}$$

$$Opp(\text{zeshoek}) = 6 \cdot 4\sqrt{3} = 24\sqrt{3}$$

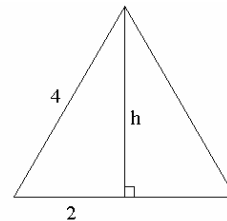
$$Opp(\text{ingeschreven cirkel}) = \pi \cdot (2\sqrt{3})^2 = 12\pi$$

$$Opp(\text{omgeschreven cirkel}) = \pi \cdot 4^2 = 16\pi$$

$$Opp(\text{zeshoek—ingeschreven cirkel}) = 24\sqrt{3} - 12\pi$$

$$Opp(\text{omgeschreven zeshoek—zeshoek}) = 16\pi - 24\sqrt{3}$$

$$Opp = \frac{1}{2}(16\pi - 24\sqrt{3}) + 24\sqrt{3} - 12\pi = 8\pi - 12\sqrt{3} + 24\sqrt{3} - 12\pi = 12\sqrt{3} - 4\pi$$

**Opgave 39:**

$$Opp(ABCD) = 25$$

$$Opp\Delta DRS = \frac{1}{2} \cdot (25 - 15) = 5$$

$$DS = 5 - x$$

$$DR = \frac{1}{2}(5 - x)$$

$$Opp\Delta DRS = \frac{1}{2} \cdot (5 - x) \cdot \frac{1}{2}(5 - x) = \frac{1}{4}(5 - x)^2 = 5$$

$$(5 - x)^2 = 20$$

$$5 - x = \sqrt{20} \quad \vee \quad 5 - x = -\sqrt{20}$$

$$x = 5 - \sqrt{20} \quad \vee \quad x = 5 + \sqrt{20} \text{ (vervalt)}$$

**Opgave 40:**

a.  $\tan 60^\circ = \frac{h}{3}$

$$h = 3 \cdot \tan 60^\circ = 3 \cdot \sqrt{3} = 3\sqrt{3}$$

$$AC = \sqrt{10^2 + (3\sqrt{3})^2} = \sqrt{100 + 27} = \sqrt{127}$$

b.  $\cos 60^\circ = \frac{3}{AD}$

$$AD = \frac{3}{\cos 60^\circ} = \frac{3}{\frac{1}{2}} = 6$$

$$\text{omtrek} = 6 + 7 + 6 + 13 = 32$$

$$\text{Opp} = \frac{1}{2} h (b_1 + b_2) = \frac{1}{2} \cdot 3\sqrt{3} \cdot (13 + 7) = 30\sqrt{3}$$

c.  $\frac{DQ}{3\sqrt{3}} = \frac{x}{3}$

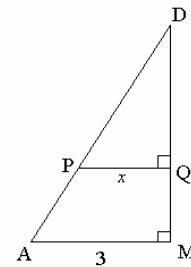
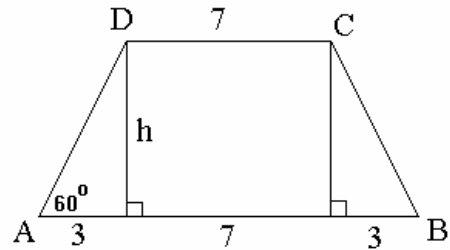
$$DQ = \frac{3\sqrt{3} \cdot x}{3} = x\sqrt{3}$$

$$\text{Opp}_{PRCD} = \frac{1}{2} \cdot x\sqrt{3} \cdot (2x + 7 + 7) = \frac{1}{2} x\sqrt{3} \cdot (2x + 14) = 15\sqrt{3}$$

$$y_1 = \frac{1}{2} x\sqrt{3} \cdot (2x + 14) \text{ en } y_2 = 15\sqrt{3}$$

Intersect geeft:  $x = 1,72$

$$QM = 3\sqrt{3} - DQ = 3\sqrt{3} - 1,72\sqrt{3} = 2,22$$

**Opgave 41:**

a.  $MT = \sqrt{a^2 + (3a)^2} = \sqrt{a^2 + 9a^2} = \sqrt{10a^2} = a\sqrt{10}$

b.  $CM = \sqrt{(2a)^2 + (1\frac{1}{2}a)^2} = \sqrt{4a^2 + 2\frac{1}{4}a^2} = \sqrt{6\frac{1}{4}a^2} = 2\frac{1}{2}a$

c. neem  $P$  het midden van  $SC$

$$AC = \sqrt{(3a)^2 + (2a)^2} = \sqrt{9a^2 + 4a^2} = \sqrt{13a^2} = a\sqrt{13}$$

$$AP = \frac{3}{4} AC = \frac{3}{4} \cdot a\sqrt{13} = \frac{3}{4} a\sqrt{13}$$

$$AN = \sqrt{AP^2 + PN^2} = \sqrt{(\frac{3}{4} a\sqrt{13})^2 + (1\frac{1}{2}a)^2} = \sqrt{29\frac{1}{4}a^2 + 2\frac{1}{4}a^2} = \sqrt{31\frac{1}{2}a^2} = a\sqrt{31\frac{1}{2}}$$

d. van  $M$  naar  $N$  is  $\frac{3}{4}a$  naar rechts,  $1\frac{1}{2}a$  naar achter en  $1\frac{1}{2}a$  omhoog

$$MN = \sqrt{(\frac{3}{4}a)^2 + (1\frac{1}{2}a)^2 + (1\frac{1}{2}a)^2} = \sqrt{\frac{9}{16}a^2 + 2\frac{1}{4}a^2 + 2\frac{1}{4}a^2} = \sqrt{5\frac{1}{16}a^2} = \sqrt{\frac{81}{16}a^2} = \frac{9}{4}a$$

**Opgave 42:**

Stel  $BP = x$  dan is  $PQ = \frac{3}{4}x$  en  $AP = 4 - x$

$$\text{Opp}(APQR) = \frac{3}{4}x(4 - x) = -\frac{3}{4}x^2 + 3x$$

$$\text{Opp}' = -1\frac{1}{2}x + 3 = 0$$

$$-1\frac{1}{2}x = -3$$

$$x = 2$$

$$\text{Opp} = 3$$