

Diagnostische toets hoofdstuk 1.

Opgave 1:

- a. $3x^2 - x = 0$
 $x(3x - 1) = 0$
 $x = 0 \vee 3x = 1$
 $x = 0 \vee x = \frac{1}{3}$
- b. $3x^2 - 9x = 12$
 $3x^2 - 9x - 12 = 0$
 $x^2 - 3x - 4 = 0$
 $(x - 4)(x + 1) = 0$
 $x = 4 \vee x = -1$
- c. $3x^2 - x = 2$
 $3x^2 - x - 2 = 0$
 $x = \frac{1 \pm \sqrt{1 + 24}}{6} = \frac{1 \pm 5}{6}$
 $x = \frac{1 + 5}{6} = 1 \vee x = \frac{1 - 5}{6} = -\frac{2}{3}$
- d. $x^2 + 14 = 16$
 $x^2 = 2$
 $x = \sqrt{2} \vee x = -\sqrt{2}$
- e. $(2x - 3)^2 = 81$
 $2x - 3 = 9 \vee 2x - 3 = -9$
 $2x = 12 \vee 2x = -6$
 $x = 6 \vee x = -3$
- f. $(3x + 2)(x - 1) = 0$
 $3x = -2 \vee x = 1$
 $x = -\frac{2}{3} \vee x = 1$
- g. $x^2 = 7x + 13$
 $x^2 - 7x - 13 = 0$
 $x = \frac{7 \pm \sqrt{49 + 52}}{2} = \frac{7 \pm \sqrt{101}}{2}$
 $x = \frac{7 + \sqrt{101}}{2} \vee x = \frac{7 - \sqrt{101}}{2}$
- h. $(3x + 2)(x - 1) = (x + 5) \cdot x$
 $3x^2 - 3x + 2x - 2 = x^2 + 5x$
 $2x^2 - 6x - 2 = 0$
 $x^2 - 3x - 1 = 0$
 $x = \frac{3 \pm \sqrt{9 + 4}}{2} = \frac{3 \pm \sqrt{13}}{2}$
 $x = \frac{3 + \sqrt{13}}{2} \vee x = \frac{3 - \sqrt{13}}{2}$

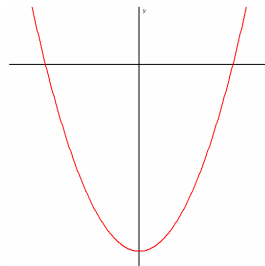
i. $(x+2)^2 = 3x+7$
 $x^2 + 4x + 4 = 3x + 7$
 $x^2 + x - 3 = 0$
 $x = \frac{-1 \pm \sqrt{1+12}}{2} = \frac{-1 \pm \sqrt{13}}{2}$
 $x = \frac{-1 + \sqrt{13}}{2} \quad \vee \quad x = \frac{-1 - \sqrt{13}}{2}$

j. $(x-3)^2 - (x+1)^2 = (x-4)^2$
 $x^2 - 6x + 9 - (x^2 + 2x + 1) = x^2 - 8x + 16$
 $x^2 - 6x + 9 - x^2 - 2x - 1 - x^2 + 8x - 16 = 0$
 $-x^2 - 8 = 0$
 $-x^2 = 8$
 $x^2 = -8$ dus geen oplossingen

Opgave 2:

a. $D = 4^2 - 4 \cdot 2 \cdot p = 16 - 8p < 0$
 $-8p < -16$
 $p > 2$

b. $D = p^2 - 4 \cdot 3 \cdot 27 = p^2 - 324 > 0$
 $p^2 > 324$
 $p = 18 \quad \vee \quad p = -18$
 $p < -18 \quad \vee \quad p > 18$



c. als $p = 0$ heb je een eerstegraads vergelijking
 $2x + 5 = 0$
 $2x = -5$
 $x = -2\frac{1}{2}$

als $p \neq 0$ heb je een tweedegraads vergelijking dus moet gelden $D = 0$

$D = (-6)^2 - 4 \cdot p \cdot 12 = 36 - 48p = 0$
 $-48p = -36$
 $p = \frac{3}{4}$

$\frac{3}{4}x^2 - 6x + 12 = 0$
 $x^2 - 8x + 16 = 0$
 $(x-4)(x-4) = 0$
 $x = 4$

dus één oplossing als $p = 0 \quad \vee \quad p = \frac{3}{4}$

Opgave 3:

a. $x = 2$ invullen geeft $4 + 8 + p = 0$
dus $p = -12$
 $x^2 + 4x - 12 = 0$
 $(x+6)(x-2) = 0$
 $x = -6 \quad \vee \quad x = 2$

$$\begin{aligned}
\text{b. } & p \neq 0 \quad \wedge \quad D = 2^2 - 4 \cdot p \cdot 5 = 4 - 20p > 0 \\
& p \neq 0 \quad \wedge \quad -20p > -4 \\
& p \neq 0 \quad \wedge \quad p < \frac{1}{5} \\
& p < 0 \quad \vee \quad 0 < p < \frac{1}{5}
\end{aligned}$$

Opgave 4:

$$\begin{aligned}
\text{a. } & 3x^3 + 5 = 86 \\
& 3x^3 = 81 \\
& x^3 = 27 \\
& x = \sqrt[3]{27} = 3
\end{aligned}$$

$$\begin{aligned}
\text{b. } & 5x^4 - 6 = 9 \\
& 5x^4 = 15 \\
& x^4 = 3 \\
& x = \sqrt[4]{3} \quad \vee \quad x = -\sqrt[4]{3}
\end{aligned}$$

$$\begin{aligned}
\text{c. } & 2x^3 + 19 = 5 \\
& 2x^3 = -14 \\
& x^3 = -7 \\
& x = \sqrt[3]{-7}
\end{aligned}$$

$$\begin{aligned}
\text{d. } & \frac{1}{2}(x+2)^4 = \frac{1}{32} \\
& (x+2)^4 = \frac{1}{16} \\
& x+2 = \sqrt[4]{\frac{1}{16}} = \frac{1}{2} \quad \vee \quad x+2 = -\sqrt[4]{\frac{1}{16}} = -\frac{1}{2} \\
& x = -1\frac{1}{2} \quad \vee \quad x = -2\frac{1}{2}
\end{aligned}$$

$$\begin{aligned}
\text{e. } & 100 - (2x+1)^5 = 68 \\
& -(2x+1)^5 = -32 \\
& (2x+1)^5 = 32 \\
& 2x+1 = \sqrt[5]{32} = 2 \\
& 2x = 1 \\
& x = \frac{1}{2}
\end{aligned}$$

$$\begin{aligned}
\text{f. } & (2x+4)^3 = 10 \\
& 2x+4 = \sqrt[3]{10} \\
& 2x = -4 + \sqrt[3]{10} \\
& x = -2 + \frac{1}{2} \cdot \sqrt[3]{10}
\end{aligned}$$

Opgave 5:

$$\begin{aligned}
\text{a. } & x^4 - 6x^2 + 5 = 0 \\
& \text{stel } x^2 = p \\
& p^2 - 6p + 5 = 0 \\
& (p-1)(p-5) = 0 \\
& p = 1 \quad \vee \quad p = 5 \\
& x^2 = 1 \quad \vee \quad x^2 = 5 \\
& x = -1 \quad \vee \quad x = 1 \quad \vee \quad x = -\sqrt{5} \quad \vee \quad x = \sqrt{5}
\end{aligned}$$

- b. $5x^4 - 6x^2 + 1 = 0$
 stel $x^2 = p$
 $5p^2 - 6p + 1 = 0$

$$p = \frac{6 \pm \sqrt{36 - 20}}{10} = \frac{6 \pm 4}{10}$$

$$p = \frac{6+4}{10} = 1 \quad \vee \quad p = \frac{6-4}{10} = \frac{1}{5}$$

$$x^2 = 1 \quad \vee \quad x^2 = \frac{1}{5}$$

$$x = 1 \quad \vee \quad x = -1 \quad \vee \quad x = \sqrt{\frac{1}{5}} \quad \vee \quad x = -\sqrt{\frac{1}{5}}$$
- c. $x^4 - 6x^3 + 5x^2 = 0$
 $x^2(x^2 - 6x + 5) = 0$
 $x^2(x-1)(x-5) = 0$
 $x = 0 \quad \vee \quad x = 1 \quad \vee \quad x = 5$
- d. $x^3 + 6x^2 + 2x = 0$
 $x(x^2 + 6x + 2) = 0$
 $x = 0 \quad \vee \quad x^2 + 6x + 2 = 0$

$$x = 0 \quad \vee \quad x = \frac{-6 \pm \sqrt{36 - 8}}{2}$$

$$x = 0 \quad \vee \quad x = \frac{-6 + \sqrt{24}}{2} \quad \vee \quad x = \frac{-6 - \sqrt{24}}{2}$$
- e. $3x^6 + 3 = 10x^3$
 $3x^6 - 10x^3 + 3 = 0$
 stel $x^3 = p$
 $3p^2 - 10p + 3 = 0$

$$p = \frac{10 \pm \sqrt{100 - 36}}{6} = \frac{10 \pm 8}{6}$$

$$p = \frac{10+8}{6} = 3 \quad \vee \quad p = \frac{10-8}{6} = \frac{1}{3}$$

$$x^3 = 3 \quad \vee \quad x^3 = \frac{1}{3}$$

$$x = \sqrt[3]{3} \quad \vee \quad x = \sqrt[3]{\frac{1}{3}}$$
- f. $x^8 + x^4 = 42$
 $x^8 + x^4 - 42 = 0$
 stel $x^4 = p$
 $p^2 + p - 42 = 0$
 $(p+7)(p-6) = 0$
 $p = -7 \quad \vee \quad p = 6$
 $x^4 = -7 \quad \vee \quad x^4 = 6$
 $x = \sqrt[4]{-7} \quad \vee \quad x = \sqrt[4]{6}$

Opgave 6:

a. $|x^2 - 4| = 21$

$x^2 - 4 = 21 \quad \vee \quad x^2 - 4 = -21$

$x^2 = 25 \quad \vee \quad x^2 = -17$

$x = 5 \quad \vee \quad x = -5$

b. $|4x^3 - 5| = 17$

$4x^3 - 5 = 17 \quad \vee \quad 4x^3 - 5 = -17$

$4x^3 = 22 \quad \vee \quad 4x^3 = -12$

$x^3 = 5\frac{1}{2} \quad \vee \quad x^3 = -3$

$x = \sqrt[3]{5\frac{1}{2}} \quad \vee \quad x = \sqrt[3]{-3}$

Opgave 7:

a. $\sqrt{3x+5} + 1 = 5$

$\sqrt{3x+5} = 4$

$3x+5 = 16$

$3x = 11$

$x = 3\frac{2}{3}$

b. $3x = 5\sqrt{x+4}$

$9x^2 = 25(x+4)$

$9x^2 = 25x + 100$

$9x^2 - 25x - 100 = 0$

$$x = \frac{25 \pm \sqrt{625 + 3600}}{18} = \frac{25 \pm \sqrt{4225}}{18} = \frac{25 \pm 65}{18}$$

$$x = \frac{25 + 65}{18} = 5 \quad \vee \quad x = \frac{25 - 65}{18} = -2\frac{2}{9} \text{ (vervalt)}$$

c. $x = \sqrt{x+6}$

$x - 6 = \sqrt{x}$

$x^2 - 12x + 36 = x$

$x^2 - 13x + 36 = 0$

$(x-4)(x-9) = 0$

$x = 4 \text{ (vervalt)} \quad \vee \quad x = 9$

d. $2x + 3\sqrt{x} = 2$

$3\sqrt{x} = 2 - 2x$

$9x = 4 - 8x + 4x^2$

$-4x^2 + 17x - 4 = 0$

$$x = \frac{-17 \pm \sqrt{289 - 64}}{-8} = \frac{-17 \pm \sqrt{225}}{-8} = \frac{-17 \pm 15}{-8}$$

$$x = \frac{-17 - 15}{-8} = 4 \text{ (vervalt)} \quad \vee \quad x = \frac{-17 + 15}{-8} = \frac{1}{4}$$

Opgave 8:

- a. $x^3 - 189 = 20x\sqrt{x}$
 $x^3 - 20x\sqrt{x} - 189 = 0$
 stel $x\sqrt{x} = p$
 $p^2 - 20p - 189 = 0$
 $(p - 27)(p + 7) = 0$
 $p = 27 \quad \vee \quad p = -7$
 $x\sqrt{x} = 27 \quad \vee \quad x\sqrt{x} = -7$
 $x^3 = 729 \quad \vee \quad x^3 = 49$
 $x = \sqrt[3]{729} = 9 \quad \vee \quad x = \sqrt[3]{49}$ (vervalt)
- b. $x^5 + 12 = 8x^2 \cdot \sqrt{x}$
 $x^5 - 8x^2 \cdot \sqrt{x} + 12 = 0$
 stel $x^2 \cdot \sqrt{x} = p$
 $p^2 - 8p + 12 = 0$
 $(p - 6)(p - 2) = 0$
 $p = 6 \quad \vee \quad p = 2$
 $x^2 \cdot \sqrt{x} = 6 \quad \vee \quad x^2 \cdot \sqrt{x} = 2$
 $x^5 = 36 \quad \vee \quad x^5 = 4$
 $x = \sqrt[5]{36} \quad \vee \quad x = \sqrt[5]{4}$

Opgave 9:

- a. $\frac{6x - 18}{x + 1} = 0$
 $6x - 18 = 0$
 $6x = 18$
 $x = 3$
- b. $\frac{x^2 - 5x + 6}{2x + 4} = 0$
 $x^2 - 5x + 6 = 0$
 $(x - 2)(x - 3) = 0$
 $x = 2 \quad \vee \quad x = 3$
- c. $\frac{3x - 5}{x + 1} = \frac{x + 2}{x + 1}$
 $3x - 5 = x + 2$
 $2x = 7$
 $x = 3\frac{1}{2}$
- d. $\frac{x^2 - 4}{2x + 1} = \frac{x^2 - 4}{x - 4}$
 $2x + 1 = x - 4$
 $x = -5$

e. $\frac{2x-1}{x+1} = \frac{x+3}{x-4}$
 $(2x-1)(x-4) = (x+1)(x+3)$
 $2x^2 - 8x - x + 4 = x^2 + 3x + x + 3$
 $x^2 - 13x + 1 = 0$
 $x = \frac{13 \pm \sqrt{169-4}}{2} = \frac{13 \pm \sqrt{165}}{2}$
 $x = \frac{13 + \sqrt{165}}{2} \quad \vee \quad x = \frac{13 - \sqrt{165}}{2}$

f. $\frac{2x^2-4}{x+5} = 1\frac{3}{4}$
 $\frac{2x^2-4}{x+5} = \frac{7}{4}$
 $4(2x^2-4) = 7(x+5)$
 $8x^2 - 16 = 7x + 35$
 $8x^2 - 7x - 51 = 0$
 $x = \frac{7 \pm \sqrt{49+1632}}{16} = \frac{7 \pm \sqrt{1681}}{16} = \frac{7 \pm 41}{16}$
 $x = \frac{7+41}{16} = 3 \quad \vee \quad x = \frac{7-41}{16} = -2\frac{1}{8}$

Opgave 10:

a. $\begin{cases} 4x + 5y = 27 & | \times 1 \\ -2x + 3y = 25 & | \times 2 \end{cases}$
 $\begin{cases} 4x + 5y = 27 \\ -4x + 6y = 50 & + \end{cases}$
 $\hline 11y = 77$
 $y = 7$
 $4x + 35 = 27$
 $4x = -8$
 $x = -2$
 $x = -2 \quad \wedge \quad y = 7$

b. $\begin{cases} 2x + 3y = 7 & | \times 2 \\ 5x - 2y = 8 & | \times 3 \end{cases}$
 $\begin{cases} 4x + 6y = 14 \\ 15x - 6y = 24 & + \end{cases}$
 $\hline 19x = 38$
 $x = 2$
 $4 + 3y = 7$
 $3y = 3$
 $y = 1$
 $x = 2 \quad \wedge \quad y = 1$

Opgave 11:

$$\begin{cases} 4a + 2b = 18 & \times 2 \\ 16a - 4b = 0 & \times 1 \end{cases}$$

$$\begin{cases} 8a + 4b = 36 \\ 16a - 4b = 0 & + \end{cases}$$

$$24a = 36$$

$$a = 1\frac{1}{2}$$

$$6 + 2b = 18$$

$$2b = 12$$

$$b = 6$$

$$y = 1\frac{1}{2}x^2 + 6x$$

Opgave 12:

a. $5x - 3y = 3 \quad \wedge \quad y = \frac{2}{3}x - 4$

$$5x - 3(\frac{2}{3}x - 4) = 3$$

$$5x - 2x + 12 = 3$$

$$3x = -9$$

$$x = -3$$

$$y = -6$$

dus $x = -3 \quad \wedge \quad y = -6$

b. $2x + 3y = 10 \quad \wedge \quad y = x^2 - 4x + 6$

$$2x + 3(x^2 - 4x + 6) = 10$$

$$2x + 3x^2 - 12x + 18 = 10$$

$$3x^2 - 10x + 8 = 0$$

$$x = \frac{10 \pm \sqrt{100 - 96}}{6} = \frac{10 \pm 2}{6}$$

$$x = \frac{10 + 2}{6} = 2 \quad \vee \quad x = \frac{10 - 2}{6} = 1\frac{1}{3}$$

$$y = 2 \quad \vee \quad y = 2\frac{4}{9}$$

$$(x = 2 \quad \wedge \quad y = 2) \quad \vee \quad (x = 1\frac{1}{3} \quad \wedge \quad y = 2\frac{4}{9})$$

Opgave 13:

a. $y_1 = x^4 - 4x^2$

$$y_2 = 0,5x - 2$$

De optie intersection geeft: $x = -1,75 \quad \vee \quad x = -0,86 \quad \vee \quad x = 0,69 \quad \vee \quad x = 1,93$

b. $y_1 = abs(x^3 - 3x)$

$$y_2 = -\frac{1}{2}x + 2$$

De optie intersection geeft: $x = -2,11 \quad \vee \quad x = 0,65 \quad \vee \quad x = 1,89 \quad \vee \quad x = 1,46$

Opgave 14:

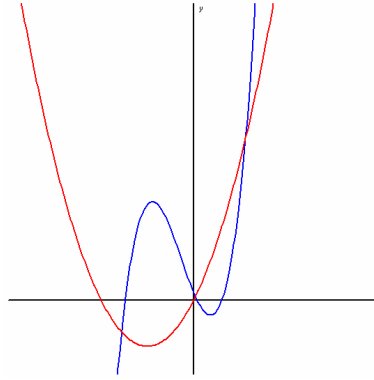
a. $y_1 = x^2 + 5x$

$y_2 = x^3 + 2x^2 - 6x + 1$

De optie intersection geeft:

$x = -3,89 \vee x = 0,09 \vee x = 2,80$

$-3,89 \leq x \leq 0,09 \vee x \geq 2,80$



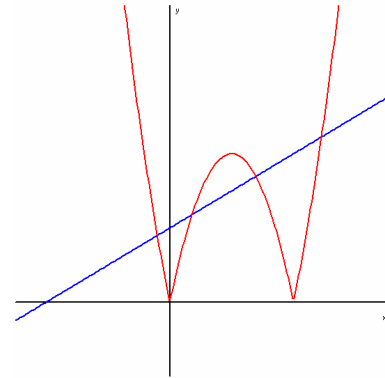
b. $y_1 = \text{abs}(x^2 - 4x)$

$y_2 = \frac{1}{2}x + 2$

De optie intersection geeft:

$x = -0,41 \vee x = 0,72 \vee x = 2,78 \vee x = 4,91$

$x < -0,41 \vee 0,72 < x < 2,78 \vee x > 4,91$

**Opgave 15:**

a. $3x^2 + 2x \geq 33$

$3x^2 + 2x - 33 = 0$

$$x = \frac{-2 \pm \sqrt{4 + 396}}{6} = \frac{-2 \pm 20}{6}$$

$$x = \frac{-2 + 20}{6} = 3 \vee x = \frac{-2 - 20}{6} = -3\frac{2}{3}$$

$x \leq -3\frac{2}{3} \vee x \geq 3$

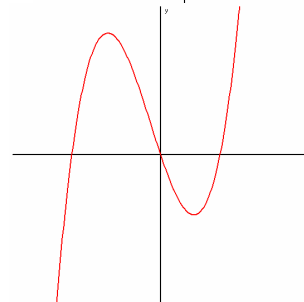
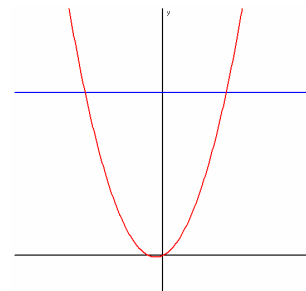
b. $x^3 + x^2 - 6x < 0$

$x(x^2 + x - 6) = 0$

$x(x+3)(x-2) = 0$

$x = 0 \vee x = -3 \vee x = 2$

$x < -3 \vee 0 < x < 2$

**Opgave 16:**

a. $D = (p-2)^2 - 4 \cdot p \cdot 4p < 0$

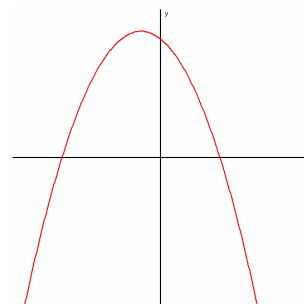
$p^2 - 4p + 4 - 16p^2 < 0$

$-15p^2 - 4p + 4 < 0$

$$p = \frac{4 \pm \sqrt{16 + 240}}{-30} = \frac{4 \pm 16}{-30}$$

$$p = \frac{4+16}{-30} = -\frac{2}{3} \vee p = \frac{4-16}{-30} = \frac{2}{5}$$

$p < -\frac{2}{3} \vee p > \frac{2}{5}$



b. $x(px^2 + 2px - 2) = 0$

$x = 0 \vee px^2 + 2px - 2 = 0$ heeft geen oplossingen

$$D = (2p)^2 - 4 \cdot p \cdot -2 < 0$$

$$4p^2 + 8p < 0$$

$$4p(p+2) = 0$$

$$p = 0 \vee p = -2$$

$$-2 < p < 0$$

voor $p = 0$ heb je de eerstegraads vergelijking $-2x = 0$ dus $x = 0$

dus $-2 < p \leq 0$

