

Hoofdstuk 1: Vergelijkingen en ongelijkheden.

1.1 Tweedegraadsvergelijkingen

Opgave 1:

I, II, IV en V zijn tweedegraads vergelijkingen.

III is een eerstegraads vergelijking en VI is een derdegraads vergelijking.

Opgave 2:

a. $x^2 + 6 = 5x$

$$x^2 - 5x + 6 = 0$$

$$(x-3)(x-2) = 0$$

$$x = 3 \quad \vee \quad x = 2$$

b. $x(x-1) = 12$

$$x^2 - x - 12 = 0$$

$$(x-4)(x+3) = 0$$

$$x = 4 \quad \vee \quad x = -3$$

c. $2x^2 = 5x$

$$2x^2 - 5x = 0$$

$$x(2x-5) = 0$$

$$x = 0 \quad \vee \quad 2x = 5$$

$$x = 0 \quad \vee \quad x = 2\frac{1}{2}$$

d. $x = x^2$

$$-x^2 + x = 0$$

$$-x(x-1) = 0$$

$$x = 0 \quad \vee \quad x = 1$$

e. $x^2 = 11$

$$x = \sqrt{11} \quad \vee \quad x = -\sqrt{11}$$

f. $x^2 + 4 = 1$

$$x^2 = -3$$

geen oplossingen

Opgave 3:

a. $3x^2 - 6x = 24$

$$3x^2 - 6x - 24 = 0$$

$$x^2 - 2x - 8 = 0$$

$$(x-4)(x+2) = 0$$

$$x = 4 \quad \vee \quad x = -2$$

b. $3x^2 - 6x = -3(x-6)$

$$3x^2 - 6x = -3x + 18$$

$$3x^2 - 3x - 18 = 0$$

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

$$x = 3 \quad \vee \quad x = -2$$

- c. $2x^2 - 3x = 2$
 $2x^2 - 3x - 2 = 0$
 $x = \frac{3 \pm \sqrt{9+16}}{4} = \frac{3 \pm 5}{4}$
 $x = \frac{3+5}{4} = 2 \quad \vee \quad x = \frac{3-5}{4} = -\frac{1}{2}$
- d. $0,5x^2 - 2x - 6 = 0$
 $x^2 - 4x - 12 = 0$
 $(x-6)(x+2) = 0$
 $x = 6 \quad \vee \quad x = -2$
- e. $x^2 - 3x = 5(x-3)$
 $x^2 - 3x = 5x - 15$
 $x^2 - 8x + 15 = 0$
 $(x-3)(x-5) = 0$
 $x = 3 \quad \vee \quad x = 5$
- f. $2x^2 - 5x = 3x$
 $2x^2 - 8x = 0$
 $2x(x-4) = 0$
 $x = 0 \quad \vee \quad x = 4$

Opgave 4:

- a. $6 - x^2 = -2$
 $-x^2 = -8$
 $x^2 = 8$
 $x = \sqrt{8} \quad \vee \quad x = -\sqrt{8}$
- b. $2x^2 = 9x + 5$
 $2x^2 - 9x - 5 = 0$
 $x = \frac{9 \pm \sqrt{81+40}}{4} = \frac{9 \pm 11}{4}$
 $x = \frac{9+11}{4} = 5 \quad \vee \quad x = \frac{9-11}{4} = -0,5$
- c. $3(x+2)^2 + 5 = 80$
 $3(x+2)^2 = 75$
 $(x+2)^2 = 25$
 $x+2 = 5 \quad \vee \quad x+2 = -5$
 $x = 3 \quad \vee \quad x = -7$
- d. $\frac{1}{2}(x-3)^2 - 3 = 5$
 $\frac{1}{2}(x-3)^2 = 8$
 $(x-3)^2 = 16$
 $x-3 = 4 \quad \vee \quad x-3 = -4$
 $x = 7 \quad \vee \quad x = -1$

e. $-(2x-1)^2 + 5 = 1$
 $-(2x-1)^2 = -4$
 $(2x-1)^2 = 4$
 $2x-1 = 2 \quad \vee \quad 2x-1 = -2$
 $2x = 3 \quad \vee \quad 2x = -1$
 $x = 1\frac{1}{2} \quad \vee \quad x = -\frac{1}{2}$

f. $8 - 3(4x-5)^2 = 5$
 $-3(4x-5)^2 = -3$
 $(4x-5)^2 = 1$
 $4x-5 = 1 \quad \vee \quad 4x-5 = -1$
 $4x = 6 \quad \vee \quad 4x = 4$
 $x = 1\frac{1}{2} \quad \vee \quad x = 1$

Opgave 5:

a. $x^2 - 5x = 0$
 $x(x-5) = 0$
 $x = 0 \quad \vee \quad x = 5$

b. $x^2 - 5x = 14$
 $x^2 - 5x - 14 = 0$
 $(x-7)(x+2) = 0$
 $x = 7 \quad \vee \quad x = -2$

c. $x^2 - 5 = 14$
 $x^2 = 19$
 $x = \sqrt{19} \quad \vee \quad x = -\sqrt{19}$

d. $x^2 - 5 = 14x$
 $x^2 - 14x - 5 = 0$
 $x = \frac{14 \pm \sqrt{196 + 20}}{2}$
 $x = \frac{14 + \sqrt{216}}{2} \quad \vee \quad x = \frac{14 - \sqrt{216}}{2}$

e. $(2x-1)(3x+6) = 0$
 $2x = 1 \quad \vee \quad 3x = -6$
 $x = \frac{1}{2} \quad \vee \quad x = -2$

f. $(2x-1)(3x+6) = 9x$
 $6x^2 + 12x - 3x - 6 = 9x$
 $6x^2 = 6$
 $x^2 = 1$
 $x = 1 \quad \vee \quad x = -1$

g. $(2x-1) \cdot 3x = 6$
 $6x^2 - 3x - 6 = 0$
 $x = \frac{3 \pm \sqrt{9 + 144}}{12} = \frac{3 \pm \sqrt{153}}{12}$

$$\begin{aligned}
 \text{h. } & (2x-1) \cdot 3x = 6 - 9x \\
 & 6x^2 - 3x + 9x - 6 = 0 \\
 & 6x^2 + 6x - 6 = 0 \\
 & x^2 + x - 1 = 0 \\
 & x = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}
 \end{aligned}$$

Opgave 6:

$$\begin{aligned}
 \text{a. } & (x+3)^2 = 16x \\
 & x^2 + 6x + 9 = 16x \\
 & x^2 - 10x + 9 = 0 \\
 & (x-9)(x-1) = 0 \\
 & x = 9 \quad \vee \quad x = 1 \\
 \text{b. } & (2x+3)^2 = -16 \\
 & \text{geen oplossingen} \\
 \text{c. } & 2(x+3)^2 = -4x \\
 & (x+3)^2 = -2x \\
 & x^2 + 6x + 9 = -2x \\
 & x^2 + 8x + 9 = 0 \\
 & x = \frac{-8 \pm \sqrt{64-36}}{2} = \frac{-8 \pm \sqrt{28}}{2} \\
 \text{d. } & (2x+3)(4-x) = 9 \\
 & 8x - 2x^2 + 12 - 3x = 9 \\
 & -2x^2 + 5x + 3 = 0 \\
 & x = \frac{-5 \pm \sqrt{25+24}}{-4} = \frac{-5 \pm 7}{-4} \\
 & x = \frac{-5-7}{-4} = 3 \quad \vee \quad x = \frac{-5+7}{-4} = -\frac{1}{2} \\
 \text{e. } & (-4x+3)^2 = 36 \\
 & -4x+3 = 6 \quad \vee \quad -4x+3 = -6 \\
 & -4x = 3 \quad \vee \quad -4x = -9 \\
 & x = -\frac{3}{4} \quad \vee \quad x = 2\frac{1}{4} \\
 \text{f. } & -4(x+3)^2 = 4x \\
 & (x+3)^2 = -x \\
 & x^2 + 6x + 9 = -x \\
 & x^2 + 7x + 9 = 0 \\
 & x = \frac{-7 \pm \sqrt{49-36}}{2} = \frac{-7 \pm \sqrt{13}}{2}
 \end{aligned}$$

g. $x^2 - (x+1)^2 = (x+3)^2$
 $x^2 - (x^2 + 2x + 1) = x^2 + 6x + 9$
 $x^2 - x^2 - 2x - 1 = x^2 + 6x + 9$
 $-x^2 - 8x - 10 = 0$
 $x^2 + 8x + 10 = 0$
 $x = \frac{-8 \pm \sqrt{64 - 40}}{2} = \frac{-8 \pm \sqrt{24}}{2}$

h. $(x+3)^2 + (x+2)^2 = 25$
 $x^2 + 6x + 9 + x^2 + 4x + 4 = 25$
 $2x^2 + 10x - 12 = 0$
 $x^2 + 5x - 6 = 0$
 $(x+6)(x-1) = 0$
 $x = -6 \vee x = 1$

Opgave 7:

- a. $x^2 - x - 6 = 0$
 $(x-3)(x+2) = 0$
 $x = 3 \vee x = -2$
- b. $x^2 + 2x - 6 = 0$
 $D = 4 + 24 = 28 > 0$ dus twee oplossingen
- c. $D = p^2 + 24 > 0$ klopt altijd want $p^2 \geq 0$ dus $p^2 + 24 \geq 24$
dus er zijn altijd twee oplossingen

Opgave 8:

- a. $D = (-7)^2 - 4 \cdot 1 \cdot p = 49 - 4p > 0$
 $-4p > -49$
 $p < 12\frac{1}{4}$
- b. $D = (-5)^2 - 4 \cdot 2 \cdot -p = 25 + 8p > 0$
 $8p > -25$
 $p > -3\frac{1}{8}$
- c. $D = 4^2 - 4 \cdot -3 \cdot -p = 16 - 12p > 0$
 $-12p > -16$
 $p < 1\frac{1}{3}$
- d. $D = (-3)^2 - 4 \cdot \frac{1}{4} \cdot p = 9 - p > 0$
 $-p > -9$
 $p < 9$

Opgave 9:

- a. $D = p^2 - 4 \cdot 1 \cdot 25 = p^2 - 100 > 0$
 $p^2 > 100$
 $p < -10 \vee p > 10$

- b. $D = p^2 - 4 \cdot 1 \cdot 4 = p^2 - 16 < 0$
 $p^2 < 16$
 $-4 < p < 4$
- c. $D = p^2 - 4 \cdot -2 \cdot 3 = p^2 + 24 > 0$
dus voor iedere waarde van p zijn er twee oplossingen

Opgave 10:

- a. $1 + 2 + p = 0$
 $p = -3$
 $x^2 + 2x - 3 = 0$
 $(x+3)(x-1) = 0$
 $x = -3 \vee x = 1$
- b. $4p - 22 + 10 = 0$
 $4p = 12$
 $p = 3$
 $3x^2 - 11x + 10 = 0$
 $x = \frac{11 \pm \sqrt{121 - 120}}{6} = \frac{11 \pm 1}{6}$
 $x = \frac{11+1}{6} = 2 \vee x = \frac{11-1}{6} = \frac{5}{3}$

Opgave 11:

- a. $3x + 1 = 0$
 $3x = -1$
 $x = -\frac{1}{3}$ dus er is één oplossing
- b. $D = 3^2 - 4 \cdot p \cdot 1 = 9 - 4p > 0 \wedge p \neq 0$
 $-4p > -9$
 $p < 2\frac{1}{4}$
dus $p < 0 \vee 0 < p < 2\frac{1}{4}$

Opgave 12:

- a. $D = 5^2 - 4 \cdot p \cdot 2 = 25 - 8p > 0 \wedge p \neq 0$
 $-8p > -25$
 $p < 3\frac{1}{8}$
dus $p < 0 \vee 0 < p < 3\frac{1}{8}$
- b. $D = (-3)^2 - 4 \cdot p \cdot -4 = 9 + 16p > 0 \wedge p \neq 0$
 $16p > -9$
 $p > -\frac{9}{16}$
dus $-\frac{9}{16} < p < 0 \vee p > 0$

Opgave 13:

- a. $D = 1^2 - 4 \cdot 2 \cdot p = 1 - 8p < 0$
 $-8p < -1$
 $p > \frac{1}{8}$
- b. $D = 1^2 - 4 \cdot p \cdot p = 1 - 4p^2 > 0 \wedge p \neq 0$
 $-4p^2 > -1$
 $p^2 < \frac{1}{4}$
 $-\frac{1}{2} < p < \frac{1}{2}$
dus $-\frac{1}{2} < p < 0 \vee 0 < p < \frac{1}{2}$
- c. $D = p^2 - 4 \cdot 2 \cdot 1 = p^2 - 8 > 0$
 $p^2 > 8$
 $p < -\sqrt{8} \vee p > \sqrt{8}$

Opgave 14:

- a. $D = 6^2 - 4 \cdot p \cdot 9 = 36 - 36p = 0 \quad \vee \quad \text{als } p = 0 \quad \begin{array}{l} 6x + 9 = 0 \\ 6x = -9 \\ x = -1\frac{1}{2} \end{array}$
 $-36p = -36$
 $p = 1$
 $x^2 + 6x + 9 = 0$
 $(x + 3)(x + 3) = 0$
 $x = -3$
- b. $D = p^2 - 4 \cdot 1 \cdot 1 = p^2 - 4 = 0$
 $p^2 = 4$
 $p = 2 \vee p = -2$
 $x^2 + 2x + 1 = 0 \quad \vee \quad x^2 - 2x + 1 = 0$
 $(x + 1)(x + 1) = 0 \quad \quad \quad (x - 1)(x - 1) = 0$
 $x = -1 \quad \quad \quad x = 1$