

1.2 Hogeregraads- en modulusvergelijkingen

Opgave 15:

- één oplossing, één oplossing
- twee oplossingen, geen oplossingen

Opgave 16:

a.

x	x^2	x^3	x^4	x^5	x^6
1	1	1	1	1	1
2	4	8	16	32	64
3	9	27	81	243	729
4	16	64	256	1024	
5	25	125	625		
6	36	216			
7	49	343			
8	64				
9	81				

b. ***

Opgave 17:

- $x^6 = 20$
 $x = \sqrt[6]{20} \quad \vee \quad x = -\sqrt[6]{20}$
- $5x^3 = 135$
 $x^3 = 27$
 $x = \sqrt[3]{27} = 3$
- $0,5x^5 = 20$
 $x^5 = 40$
 $x = \sqrt[5]{40}$
- $x^4 + 7 = 88$
 $x^4 = 81$
 $x = \sqrt[4]{81} = 3 \quad \vee \quad x = -3$
- $1 - 3x^5 = 97$
 $-3x^5 = 96$
 $x^5 = -32$
 $x = \sqrt[5]{-32} = -2$
- $\frac{1}{4}x^8 + 3 = 10$
 $\frac{1}{4}x^8 = 7$
 $x^8 = 28$
 $x = \sqrt[8]{28} \quad \vee \quad x = -\sqrt[8]{28}$

Opgave 18:

- a. $5x^4 - 1 = 4$
 $5x^4 = 5$
 $x^4 = 1$
 $x = 1 \vee x = -1$
- b. $5x^4 = -4$
geen oplossingen
- c. $5x^3 - 1 = 9$
 $5x^3 = 10$
 $x^3 = 2$
 $x = \sqrt[3]{2}$
- d. $8x^3 + 2 = 1$
 $8x^3 = -1$
 $x^3 = -\frac{1}{8}$
 $x = \sqrt[3]{-\frac{1}{8}} = -\frac{1}{2}$
- e. $5x^6 + 7 = 97$
 $5x^6 = 90$
 $x^6 = 18$
 $x = \sqrt[6]{18} \vee x = -\sqrt[6]{18}$
- f. $0,1x^7 - 1 = 999$
 $0,1x^7 = 1000$
 $x^7 = 10000$
 $x = \sqrt[7]{10000}$

Opgave 19:

- a. $3(x-2)^4 + 7 = 37$
 $3(x-2)^4 = 30$
 $(x-2)^4 = 10$
 $x-2 = \sqrt[4]{10} \vee x-2 = -\sqrt[4]{10}$
 $x = 2 + \sqrt[4]{10} \vee x = 2 - \sqrt[4]{10}$
- b. $6 - (2x-1)^3 = 1$
 $-(2x-1)^3 = -5$
 $(2x-1)^3 = 5$
 $2x-1 = \sqrt[3]{5}$
 $2x = 1 + \sqrt[3]{5}$
 $x = \frac{1}{2} + \frac{1}{2} \cdot \sqrt[3]{5}$
- c. $\frac{1}{2}(3x-1)^4 = 8$
 $(3x-1)^4 = 16$
 $3x-1 = \sqrt[4]{16} = 2 \vee 3x-1 = -2$
 $3x = 3 \vee 3x = -1$

$$x = 1 \quad \vee \quad x = -\frac{1}{3}$$

d. $100 - \frac{1}{3}(4x - 3)^5 = 19$

$$-\frac{1}{3}(4x - 3)^5 = -81$$

$$(4x - 3)^5 = 243$$

$$4x - 3 = \sqrt[5]{243} = 3$$

$$4x = 6$$

$$x = 1\frac{1}{2}$$

Opgave 20:

a. $5x^4 - 3 = 17$

$$5x^4 = 20$$

$$x^4 = 4$$

$$x = \sqrt[4]{4} \quad \vee \quad x = -\sqrt[4]{4}$$

b. $4x^3 - 5 = 1367$

$$4x^3 = 1372$$

$$x^3 = 343$$

$$x = \sqrt[3]{343} = 7$$

c. $3(4x - 5)^3 = 15$

$$(4x - 5)^3 = 5$$

$$4x - 5 = \sqrt[3]{5}$$

$$4x = 5 + \sqrt[3]{5}$$

$$x = \frac{5}{4} + \frac{1}{4} \cdot \sqrt[3]{5}$$

d. $17 - 2(1 - 3x)^4 = 5$

$$-2(1 - 3x)^4 = -12$$

$$(1 - 3x)^4 = 6$$

$$1 - 3x = \sqrt[4]{6} \quad \vee \quad 1 - 3x = -\sqrt[4]{6}$$

$$-3x = -1 + \sqrt[4]{6} \quad \vee \quad -3x = -1 - \sqrt[4]{6}$$

$$x = \frac{1}{3} - \frac{1}{3} \cdot \sqrt[4]{6} \quad \vee \quad x = \frac{1}{3} + \frac{1}{3} \cdot \sqrt[4]{6}$$

Opgave 21:

a. $x^3 - x^2 - 2x = 0$

$$x(x^2 - x - 2) = 0$$

$$x(x - 2)(x + 1) = 0$$

b. $x = 0 \quad \vee \quad x = 2 \quad \vee \quad x = -1$

Opgave 22:

a. $x^3 - 5x^2 + 6x = 0$

$$x(x^2 - 5x + 6) = 0$$

$$x(x - 2)(x - 3) = 0$$

$$x = 0 \quad \vee \quad x = 2 \quad \vee \quad x = 3$$

- b. $x^3 - 5x^2 = 6x$
 $x^3 - 5x^2 - 6x = 0$
 $x(x^2 - 5x - 6) = 0$
 $x(x - 6)(x + 1) = 0$
 $x = 0 \vee x = 6 \vee x = -1$
- c. $x^3 = 4x^2 + 12x$
 $x^3 - 4x^2 - 12x = 0$
 $x(x^2 - 4x - 12) = 0$
 $x(x - 6)(x + 2) = 0$
 $x = 0 \vee x = 6 \vee x = -2$
- d. $x^4 - 13x^2 + 36 = 0$
 stel $x^2 = p$
 $p^2 - 13p + 36 = 0$
 $(p - 4)(p - 9) = 0$
 $p = 4 \vee p = 9$
 $x^2 = 4 \vee x^2 = 9$
 $x = 2 \vee x = -2 \vee x = 3 \vee x = -3$

Opgave 23:

- a. $x^4 - 10x^2 + 9 = 0$
 stel $x^2 = p$
 $p^2 - 10p + 9 = 0$
 $(p - 1)(p - 9) = 0$
 $p = 1 \vee p = 9$
 $x^2 = 1 \vee x^2 = 9$
 $x = 1 \vee x = -1 \vee x = 3 \vee x = -3$
- b. $x^4 - 8x^2 - 9 = 0$
 stel $x^2 = p$
 $p^2 - 8p - 9 = 0$
 $(p + 1)(p - 9) = 0$
 $p = -1 \vee p = 9$
 $x^2 = -1 \vee x^2 = 9$
 k.n. $x = 3 \vee x = -3$
- c. $x^4 + 16 = 10x^2$
 $x^4 - 10x^2 + 16 = 0$
 stel $x^2 = p$
 $p^2 - 10p + 16 = 0$
 $(p - 2)(p - 8) = 0$
 $p = 2 \vee p = 8$
 $x^2 = 2 \vee x^2 = 8$
 $x = \sqrt{2} \vee x = -\sqrt{2} \vee x = \sqrt{8} \vee x = -\sqrt{8}$

d. $x^3 + 25x = 10x^2$
 $x^3 - 10x^2 + 25x = 0$
 $x(x^2 - 10x + 25) = 0$
 $x(x-5)(x-5) = 0$
 $x = 0 \vee x = 5$

Opgave 24:

a. $2x^4 - 11x^2 + 12 = 0$
 stel $x^2 = p$
 $2p^2 - 11p + 12 = 0$
 $p = \frac{11 \pm \sqrt{121 - 96}}{4} = \frac{11 \pm \sqrt{25}}{4} = \frac{11 \pm 5}{4}$
 $p = \frac{11+5}{4} = 4 \vee p = \frac{11-5}{4} = 1\frac{1}{2}$

b. $x^2 = 4 \vee x^2 = 1\frac{1}{2}$
 $x = 2 \vee x = -2 \vee x = \sqrt{1\frac{1}{2}} \vee x = -\sqrt{1\frac{1}{2}}$

Opgave 25:

a. $6x^4 + 2 = 7x^2$
 $6x^4 - 7x^2 + 2 = 0$
 stel $x^2 = p$
 $6p^2 - 7p + 2 = 0$
 $p = \frac{7 \pm \sqrt{49 - 48}}{12} = \frac{7 \pm 1}{12}$
 $p = \frac{7+1}{12} = \frac{2}{3} \vee p = \frac{7-1}{12} = \frac{1}{2}$
 $x^2 = \frac{2}{3} \vee x^2 = \frac{1}{2}$
 $x = \sqrt{\frac{2}{3}} \vee x = -\sqrt{\frac{2}{3}} \vee x = \sqrt{\frac{1}{2}} \vee x = -\sqrt{\frac{1}{2}}$

b. $2x^4 = x^2 + 3$
 $2x^4 - x^2 - 3 = 0$
 stel $x^2 = p$
 $2p^2 - p - 3 = 0$
 $p = \frac{1 \pm \sqrt{1 + 24}}{4} = \frac{1 \pm 5}{4}$
 $p = \frac{1+5}{4} = 1\frac{1}{2} \vee p = \frac{1-5}{4} = -1$
 $x^2 = 1\frac{1}{2} \vee x^2 = -1$
 $x = \sqrt{1\frac{1}{2}} \vee x = -\sqrt{1\frac{1}{2}}$

c. $4x^4 + 7x^2 = 2$
 $4x^4 + 7x^2 - 2 = 0$
 stel $x^2 = p$

$$4p^2 + 7p - 2 = 0$$

$$p = \frac{-7 \pm \sqrt{49 + 32}}{8} = \frac{-7 \pm 9}{8}$$

$$p = \frac{-7 + 9}{8} = \frac{1}{4} \quad \vee \quad p = \frac{-7 - 9}{8} = -2$$

$$x^2 = \frac{1}{4} \quad \vee \quad x^2 = -2$$

$$x = \frac{1}{2} \quad \vee \quad x = -\frac{1}{2}$$

d. $16x^4 + 225 = 136x^2$

$$16x^4 - 136x^2 + 225 = 0$$

stel $x^2 = p$

$$16p^2 - 136p + 225 = 0$$

$$p = \frac{136 \pm \sqrt{18496 - 14400}}{32} = \frac{136 \pm \sqrt{4096}}{32} = \frac{136 \pm 64}{32}$$

$$p = \frac{136 + 64}{32} = 6\frac{1}{4} \quad \vee \quad p = \frac{136 - 64}{32} = \frac{9}{4}$$

$$x^2 = 6\frac{1}{4} \quad \vee \quad x^2 = \frac{9}{4}$$

$$x = 2\frac{1}{2} \quad \vee \quad x = -2\frac{1}{2} \quad \vee \quad x = 1\frac{1}{2} \quad \vee \quad x = -1\frac{1}{2}$$

Opgave 26:

a. $4x^4 + 153 = 53x^2$

$$4x^4 - 53x^2 + 153 = 0$$

stel $x^2 = p$

$$4p^2 - 53p + 153 = 0$$

$$p = \frac{53 \pm \sqrt{2809 - 2448}}{8} = \frac{53 \pm \sqrt{361}}{8} = \frac{53 \pm 19}{8}$$

$$p = \frac{53 + 19}{8} = 9 \quad \vee \quad p = \frac{53 - 19}{8} = 4\frac{1}{4}$$

$$x^2 = 9 \quad \vee \quad x^2 = 4\frac{1}{4}$$

$$x = 3 \quad \vee \quad x = -3 \quad \vee \quad x = \sqrt{4\frac{1}{4}} \quad \vee \quad x = -\sqrt{4\frac{1}{4}}$$

b. $4x^4 + 21x^2 = 148$

$$4x^4 + 21x^2 - 148 = 0$$

stel $x^2 = p$

$$4p^2 + 21p - 148 = 0$$

$$p = \frac{-21 \pm \sqrt{441 + 2368}}{8} = \frac{-21 \pm \sqrt{2809}}{8} = \frac{-21 \pm 53}{8}$$

$$p = \frac{-21 + 53}{8} = 4 \quad \vee \quad p = \frac{-21 - 53}{8} = -9\frac{1}{4}$$

$$x^2 = 4 \quad \vee \quad x^2 = -9\frac{1}{4}$$

$$x = 2 \quad \vee \quad x = -2$$

c. $4x^6 + 35 = 24x^3$
 $4x^6 - 24x^3 + 35 = 0$
 stel $x^3 = p$
 $4p^2 - 24p + 35 = 0$

$$p = \frac{24 \pm \sqrt{576 - 560}}{8} = \frac{24 \pm \sqrt{16}}{8} = \frac{24 \pm 4}{8}$$

 $p = \frac{24+4}{8} = 3\frac{1}{2} \quad \vee \quad p = \frac{24-4}{8} = 2\frac{1}{2}$
 $x^3 = 3\frac{1}{2} \quad \vee \quad x^3 = 2\frac{1}{2}$
 $x = \sqrt[3]{3\frac{1}{2}} \quad \vee \quad x = \sqrt[3]{2\frac{1}{2}}$

d. $64x^6 + 27 = 224x^3$
 $64x^6 - 224x^3 + 27 = 0$
 stel $x^3 = p$
 $64p^2 - 224p + 27 = 0$

$$p = \frac{224 \pm \sqrt{50176 - 6912}}{128} = \frac{224 \pm \sqrt{43264}}{128} = \frac{224 \pm 208}{128}$$

 $p = \frac{224+208}{128} = 3\frac{3}{8} \quad \vee \quad p = \frac{224-208}{128} = \frac{1}{8}$
 $x^3 = 3\frac{3}{8} \quad \vee \quad x^3 = \frac{1}{8}$
 $x = \sqrt[3]{3\frac{3}{8}} = 1\frac{1}{2} \quad \vee \quad x = \sqrt[3]{\frac{1}{8}} = \frac{1}{2}$

Opgave 27:

a. -7 en 7
 b. $2x-1 = -7 \quad \vee \quad 2x-1 = 7$
 $2x = -6 \quad \vee \quad 2x = 8$
 $x = -3 \quad \vee \quad x = 4$

Opgave 28:

a. $|2x-1| = 8$
 $2x-1 = 8 \quad \vee \quad 2x-1 = -8$
 $2x = 9 \quad \vee \quad 2x = -7$
 $x = 4\frac{1}{2} \quad \vee \quad x = -3\frac{1}{2}$
 b. $|x^2-3| = 1$
 $x^2-3 = 1 \quad \vee \quad x^2-3 = -1$
 $x^2 = 4 \quad \vee \quad x^2 = 2$
 $x = 2 \quad \vee \quad x = -2 \quad \vee \quad x = \sqrt{2} \quad \vee \quad x = -\sqrt{2}$
 c. $|2x^2-5| = 11$
 $2x^2-5 = 11 \quad \vee \quad 2x^2-5 = -11$
 $2x^2 = 16 \quad \vee \quad 2x^2 = -6$
 $x^2 = 8 \quad \text{k.n.}$

$$x = \sqrt{8} \quad \vee \quad x = -\sqrt{8}$$

d. $|5 - x^2| = 11$

$$5 - x^2 = 11 \quad \vee \quad 5 - x^2 = -11$$

$$-x^2 = 6 \quad \vee \quad -x^2 = -16$$

$$x^2 = -6 \quad \vee \quad x^2 = 16$$

$$\text{k.n.} \quad x = 4 \quad \vee \quad x = -4$$

Opgave 29:

a. $|2x^4 - 5| = 15$

$$2x^4 - 5 = 15 \quad \vee \quad 2x^4 - 5 = -15$$

$$2x^4 = 20 \quad \vee \quad 2x^4 = -10$$

$$x^4 = 10 \quad \text{k.n.}$$

$$x = \sqrt[4]{10} \quad \vee \quad x = -\sqrt[4]{10}$$

b. $|2x^3 - 5| = 15$

$$2x^3 - 5 = 15 \quad \vee \quad 2x^3 - 5 = -15$$

$$2x^3 = 20 \quad \vee \quad 2x^3 = -10$$

$$x^3 = 10 \quad \vee \quad x^3 = -5$$

$$x = \sqrt[3]{10} \quad \vee \quad x = \sqrt[3]{-5}$$

c. $|x^4 - 5x^2| = 6$

$$x^4 - 5x^2 = 6 \quad \vee \quad x^4 - 5x^2 = -6$$

$$x^4 - 5x^2 - 6 = 0 \quad \vee \quad x^4 - 5x^2 + 6 = 0$$

$$\text{stel } x^2 = p$$

$$p^2 - 5p - 6 = 0 \quad \vee \quad p^2 - 5p + 6 = 0$$

$$(p-6)(p+1) = 0 \quad \vee \quad (p-2)(p-3) = 0$$

$$p = 6 \quad \vee \quad p = -1 \quad \vee \quad p = 2 \quad \vee \quad p = 3$$

$$x^2 = 6 \quad \vee \quad x^2 = -1 \quad \vee \quad x^2 = 2 \quad \vee \quad x^2 = 3$$

$$x = \sqrt{6} \quad \vee \quad x = -\sqrt{6} \quad \vee \quad x = \sqrt{2} \quad \vee \quad x = -\sqrt{2} \quad \vee \quad x = \sqrt{3} \quad \vee \quad x = -\sqrt{3}$$

d. $|x^6 - 10x^3| = 24$

$$x^6 - 10x^3 = 24 \quad \vee \quad x^6 - 10x^3 = -24$$

$$x^6 - 10x^3 - 24 = 0 \quad \vee \quad x^6 - 10x^3 + 24 = 0$$

$$\text{stel } x^3 = p$$

$$p^2 - 10p - 24 = 0 \quad \vee \quad p^2 - 10p + 24 = 0$$

$$(p-12)(p+2) = 0 \quad \vee \quad (p-4)(p-6) = 0$$

$$p = 12 \quad \vee \quad p = -2 \quad \vee \quad p = 4 \quad \vee \quad p = 6$$

$$x^3 = 12 \quad \vee \quad x^3 = -2 \quad \vee \quad x^3 = 4 \quad \vee \quad x^3 = 6$$

$$x = \sqrt[3]{12} \quad \vee \quad x = \sqrt[3]{-2} \quad \vee \quad x = \sqrt[3]{4} \quad \vee \quad x = \sqrt[3]{6}$$