

1.3 Wortel- en gebroken vergelijkingen.

Opgave 30:

- a. links en rechts kwadrateren

$$\sqrt{2x-5} = 3$$

$$2x - 5 = 9$$

$$2x = 14$$

$$x = 7$$

- b. de uitkomst van een wortel kan nooit negatief zijn

Opgave 31:

a. $x = \sqrt{5x+14}$

$$x^2 = 5x + 14$$

$$x^2 - 5x - 14 = 0$$

$$(x-7)(x+2) = 0$$

$$x = 7 \quad \vee \quad x = -2 \text{ (vervalt)}$$

$$\text{dus } x = 7$$

b. $3x = \sqrt{8x+20}$

$$9x^2 = 8x + 20$$

$$9x^2 - 8x - 20 = 0$$

$$x = \frac{8 \pm \sqrt{64 + 720}}{18} = \frac{8 \pm \sqrt{784}}{18} = \frac{8 \pm 28}{18}$$

$$x = \frac{8+28}{18} = 2 \quad \vee \quad x = \frac{8-28}{18} = -1\frac{1}{9} \text{ (vervalt)}$$

c. $5\sqrt{x} = x$

$$25x = x^2$$

$$-x^2 + 25x = 0$$

$$-x(x-25) = 0$$

$$x = 0 \quad \vee \quad x = 25$$

d. $3x = \sqrt{18x+72}$

$$9x^2 = 18x + 72$$

$$9x^2 - 18x - 72 = 0$$

$$x^2 - 2x - 8 = 0$$

$$(x-4)(x+2) = 0$$

$$x = 4 \quad \vee \quad x = -2 \text{ (vervalt)}$$

Opgave 32:

a. $4 - 3\sqrt{x} = 2$

$$-3\sqrt{x} = -2$$

$$\sqrt{x} = \frac{2}{3}$$

$$x = \frac{4}{9}$$

b. $5\sqrt{x} - 2x = 0$

$$5\sqrt{x} = 2x$$

$$25x = 4x^2$$

$$-4x^2 + 25x = 0$$

$$-4x(x - 6\frac{1}{4}) = 0$$

$$x = 0 \quad \vee \quad x = 6\frac{1}{4}$$

c. $2x - 5\sqrt{x} = 3$

$$2x - 3 = 5\sqrt{x}$$

$$4x^2 - 12x + 9 = 25x$$

$$4x^2 - 37x + 9 = 0$$

$$x = \frac{37 \pm \sqrt{1369 - 144}}{8} = \frac{37 \pm \sqrt{1225}}{8} = \frac{37 \pm 35}{8}$$

$$x = 9 \quad \vee \quad x = \frac{1}{4} \text{ (vervalt)}$$

d. $5 - 2\sqrt{x} = 3$

$$-2\sqrt{x} = -2$$

$$\sqrt{x} = 1$$

$$x = 1$$

Opgave 33:

a. $2x + \sqrt{x} = 10$

$$2x - 10 = -\sqrt{x}$$

$$4x^2 - 40x + 100 = x$$

$$4x^2 - 41x + 100 = 0$$

$$x = \frac{41 \pm \sqrt{1681 - 1600}}{8} = \frac{41 \pm \sqrt{81}}{8} = \frac{41 \pm 9}{8}$$

$$x = \frac{41 + 9}{8} = 6\frac{1}{4} \text{ (vervalt)} \quad \vee \quad x = \frac{41 - 9}{8} = 4$$

b. $\sqrt{x+12} = x$

$$x + 12 = x^2$$

$$-x^2 + x + 12 = 0$$

$$x^2 - x - 12 = 0$$

$$(x - 4)(x + 3) = 0$$

$$x = 4 \quad \vee \quad x = -3 \text{ (vervalt)}$$

c. $2x + \sqrt{x} = 6$

$$2x - 6 = -\sqrt{x}$$

$$4x^2 - 24x + 36 = x$$

$$4x^2 - 25x + 36 = 0$$

$$x = \frac{25 \pm \sqrt{625 - 576}}{8} = \frac{25 \pm \sqrt{49}}{8} = \frac{25 \pm 7}{8}$$

$$x = \frac{25 + 7}{8} = 4 \text{ (vervalt)} \quad \vee \quad x = \frac{25 - 7}{8} = 2\frac{1}{4}$$

d. $10 - x\sqrt{x} = 2$

$$-x\sqrt{x} = -8$$

$$x^3 = 64$$

$$x = \sqrt[3]{64} = 4$$

Opgave 34:

- a. $p^2 + p - 6 = 0$
 $(p+3)(p-2) = 0$
 $p = -3 \vee p = 2$
- b. $x\sqrt{x} = -3 \vee x\sqrt{x} = 2$
k.n. $x^3 = 4$
 $x = \sqrt[3]{4}$

Opgave 35:

- a. $x^3 - 9x\sqrt{x} + 8 = 0$
stel $x\sqrt{x} = p$
 $p^2 - 9p + 8 = 0$
 $(p-1)(p-8) = 0$
 $p = 1 \vee p = 8$
 $x\sqrt{x} = 1 \vee x\sqrt{x} = 8$
 $x^3 = 1 \vee x^3 = 64$
 $x = \sqrt[3]{1} = 1 \vee x = \sqrt[3]{64} = 4$
- b. $x^3 + 27 = 28x\sqrt{x}$
 $x^3 - 28x\sqrt{x} + 27 = 0$
stel $x\sqrt{x} = p$
 $p^2 - 28p + 27 = 0$
 $(p-1)(p-27) = 0$
 $p = 1 \vee p = 27$
 $x\sqrt{x} = 1 \vee x\sqrt{x} = 27$
 $x^3 = 1 \vee x^3 = 729$
 $x = \sqrt[3]{1} = 1 \vee x = \sqrt[3]{729} = 9$
- c. $8x^3 + 8 = 65x\sqrt{x}$
 $8x^3 - 65x\sqrt{x} + 8 = 0$
stel $x\sqrt{x} = p$
 $8p^2 - 65p + 8 = 0$

$$p = \frac{65 \pm \sqrt{4225 - 256}}{16} = \frac{65 \pm \sqrt{3969}}{16} = \frac{65 \pm 63}{16}$$

$$p = \frac{65 + 63}{16} = 8 \vee p = \frac{65 - 63}{16} = \frac{1}{8}$$
 $x\sqrt{x} = 8 \vee x\sqrt{x} = \frac{1}{8}$
 $x^3 = 64 \vee x^3 = \frac{1}{64}$
 $x = \sqrt[3]{64} = 4 \vee x = \sqrt[3]{\frac{1}{64}} = \frac{1}{4}$

d. $x^5 - 33x^2\sqrt{x} + 32 = 0$
 stel $x^2\sqrt{x} = p$
 $p^2 - 33p + 32 = 0$
 $(p-1)(p-32) = 0$
 $p = 1 \quad \vee \quad p = 32$
 $x^2\sqrt{x} = 1 \vee x^2\sqrt{x} = 32$
 $x^5 = 1 \quad \vee \quad x^5 = 1024$
 $x = \sqrt[5]{1} = 1 \quad \vee \quad x = \sqrt[5]{1024} = 4$

Opgave 36:

a. $x^3 + 30 = 11x\sqrt{x}$
 $x^3 - 11x\sqrt{x} + 30 = 0$
 stel $x\sqrt{x} = p$
 $p^2 - 11p + 30 = 0$
 $(p-5)(p-6) = 0$
 $p = 5 \quad \vee \quad p = 6$
 $x\sqrt{x} = 5 \quad \vee \quad x\sqrt{x} = 6$
 $x^3 = 25 \quad \vee \quad x^3 = 36$
 $x = \sqrt[3]{25} \quad \vee \quad x = \sqrt[3]{36}$

b. $x^3 + 125 = 126x\sqrt{x}$
 $x^3 - 126x\sqrt{x} + 125 = 0$
 stel $x\sqrt{x} = p$
 $p^2 - 126p + 125 = 0$
 $(p-1)(p-125) = 0$
 $p = 1 \quad \vee \quad p = 125$
 $x\sqrt{x} = 1 \quad \vee \quad x\sqrt{x} = 125$
 $x^3 = 1 \quad \vee \quad x^3 = 15625$
 $x = \sqrt[3]{1} = 1 \quad \vee \quad x = \sqrt[3]{15625} = 25$

c. $x^5 + 10 = 7x^2\sqrt{x}$
 $x^5 - 7x^2\sqrt{x} + 10 = 0$
 stel $x^2\sqrt{x} = p$
 $p^2 - 7p + 10 = 0$
 $(p-2)(p-5) = 0$
 $p = 2 \quad \vee \quad p = 5$
 $x^2\sqrt{x} = 2 \quad \vee \quad x^2\sqrt{x} = 5$
 $x^5 = 4 \quad \vee \quad x^5 = 25$
 $x = \sqrt[5]{4} \quad \vee \quad x = \sqrt[5]{25}$

d. $32x^5 + 32 = 1025x^2\sqrt{x}$
 $32x^5 - 1025x^2\sqrt{x} + 32 = 0$

$$\text{stel } x^2\sqrt{x} = p$$

$$32p^2 - 1025p + 32 = 0$$

$$p = \frac{1025 \pm \sqrt{1050625 - 4096}}{64} = \frac{1025 \pm \sqrt{1046529}}{64} = \frac{1025 \pm 1023}{64}$$

$$p = \frac{1025 + 1023}{64} = 32 \quad \vee \quad p = \frac{1025 - 1023}{64} = \frac{1}{32}$$

$$x^2\sqrt{x} = 32 \quad \vee \quad x^2\sqrt{x} = \frac{1}{32}$$

$$x^5 = 1024 \quad \vee \quad x^5 = \frac{1}{1024}$$

$$x = \sqrt[5]{1024} = 4 \quad \vee \quad x = \sqrt[5]{\frac{1}{1024}} = \frac{1}{4}$$

Opgave 37:

Isoleren, kwadrateren en controleren.

Opgave 38:

a. kruiselings vermenigvuldigen

$$x^2 = 2(x + 4)$$

$$x^2 = 2x + 8$$

$$x^2 - 2x - 8 = 0$$

b. $(x - 4)(x + 2) = 0$

$$x = 4 \quad \vee \quad x = -2$$

Opgave 39:

a. $\frac{x+3}{x-1} = \frac{10}{x}$

$$x(x+3) = 10(x-1)$$

$$x^2 + 3x = 10x - 10$$

$$x^2 - 7x + 10 = 0$$

$$(x-2)(x-5) = 0$$

$$x = 2 \quad \vee \quad x = 5$$

b. $\frac{2x+3}{x+1} = \frac{2x+2}{x-1}$

$$(2x+3)(x-1) = (2x+2)(x+1)$$

$$2x^2 - 2x + 3x - 3 = 2x^2 + 2x + 2x + 2$$

$$-3x = 5$$

$$x = -1\frac{2}{3}$$

c. $\frac{x-3}{x+1} = 1\frac{1}{2}$

$$\frac{x-3}{x+1} = \frac{3}{2}$$

$$2(x-3) = 3(x+1)$$

$$2x - 6 = 3x + 3$$

$$-x = 9$$

$$x = -9$$

$$d. \frac{x-1}{x} + 1 = 3$$

$$\frac{x-1}{x} = 2$$

$$x-1 = 2x$$

$$-x = 1$$

$$x = -1$$

$$e. \frac{3x+4}{x-1} = \frac{x+18}{x}$$

$$x(3x+4) = (x+18)(x-1)$$

$$3x^2 + 4x = x^2 - x + 18x - 18$$

$$2x^2 - 13x + 18 = 0$$

$$x = \frac{13 \pm \sqrt{169 - 144}}{4} = \frac{13 \pm \sqrt{25}}{4} = \frac{13 \pm 5}{4}$$

$$x = \frac{13+5}{4} = 4\frac{1}{2} \quad \vee \quad x = \frac{13-5}{4} = 2$$

$$f. \frac{2x-5}{4-x} = \frac{x+2}{3x-4}$$

$$(2x-5)(3x-4) = (x+2)(4-x)$$

$$6x^2 - 8x - 15x + 20 = 4x - x^2 + 8 - 2x$$

$$7x^2 - 25x + 12 = 0$$

$$x = \frac{25 \pm \sqrt{625 - 336}}{14} = \frac{25 \pm \sqrt{289}}{14} = \frac{25 \pm 17}{14}$$

$$x = \frac{25+17}{14} = 3 \quad \vee \quad x = \frac{25-17}{14} = \frac{4}{7}$$

Opgave 40:

$$a. \frac{5x^2 - 15}{x^2 + 5} = 0$$

$$5x^2 - 15 = 0$$

$$5x^2 = 15$$

$$x^2 = 3$$

$$x = \sqrt{3} \quad \vee \quad x = -\sqrt{3}$$

$$b. \frac{x^2 - 3}{x^2 + 1} = \frac{x-1}{x^2 + 1}$$

$$x^2 - 3 = x - 1$$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

$$x = 2 \quad \vee \quad x = -1$$

$$c. \frac{x^2 - 4}{2x+5} = \frac{x^2 - 4}{x+4}$$

$$x^2 - 4 = 0 \quad \vee \quad 2x+5 = x+4$$

$$x^2 = 4 \quad \vee \quad x = -1$$

$$x = 2 \quad \vee \quad x = -2 \quad \vee \quad x = -1$$

d. $\frac{x^2 + 1}{x + 1} = \frac{x + 3}{x + 1}$

$$x^2 + 1 = x + 3$$

$$x^2 - x - 2 = 0$$

$$(x - 2)(x + 1) = 0$$

$$x = 2 \quad \vee \quad x = -1 \text{ (vervalt)}$$

Opgave 41:

a. $\frac{3x^2 - 10}{x^2 + 1} = 2$

$$3x^2 - 10 = 2(x^2 + 1)$$

$$3x^2 - 10 = 2x^2 + 2$$

$$x^2 = 12$$

$$x = \sqrt{12} \quad \vee \quad x = -\sqrt{12}$$

b. $\frac{x^3 - 8}{x^2 + 2} = \frac{x^3 - 8}{x + 8}$

$$x^3 = 8 \quad \vee \quad x^2 + 2 = x + 8$$

$$x = \sqrt[3]{8} = 2 \quad \vee \quad x^2 - x - 6 = 0$$

$$x = 2 \quad \vee \quad (x - 3)(x + 2) = 0$$

$$x = 2 \quad \vee \quad x = 3 \quad \vee \quad x = -2$$

c. $\frac{3x^2 - 10}{(x^2 + 1)^2} = \frac{2}{25}$

$$25(3x^2 - 10) = 2(x^2 + 1)^2$$

$$75x^2 - 250 = 2(x^4 + 2x^2 + 1)$$

$$75x^2 - 250 = 2x^4 + 4x^2 + 2$$

$$-2x^4 + 71x^2 - 252 = 0$$

stel $x^2 = p$

$$-2p^2 + 71p - 252 = 0$$

$$p = \frac{-71 \pm \sqrt{5041 - 2016}}{-4} = \frac{-71 \pm \sqrt{3025}}{-4} = \frac{-71 \pm 55}{-4}$$

$$p = \frac{-71 + 55}{-4} = 4 \quad \vee \quad p = \frac{-71 - 55}{-4} = 31\frac{1}{2}$$

$$x^2 = 4 \quad \vee \quad x^2 = 31\frac{1}{2}$$

$$x = 2 \quad \vee \quad x = -2 \quad \vee \quad x = \sqrt{31\frac{1}{2}} \quad \vee \quad x = -\sqrt{31\frac{1}{2}}$$

d. $\frac{6x^2 - 12}{(x^2 - 1)^2} = 1\frac{1}{3}$

$$\frac{6x^2 - 12}{(x^2 - 1)^2} = \frac{4}{3}$$

$$3(6x^2 - 12) = 4(x^2 - 1)^2$$

$$18x^2 - 36 = 4(x^4 - 2x^2 + 1)$$

$$18x^2 - 36 = 4x^4 - 8x^2 + 4$$

$$-4x^4 + 26x^2 - 40 = 0$$

$$\text{stel } x^2 = p$$

$$-4p^2 + 26p - 40 = 0$$

$$p = \frac{-26 \pm \sqrt{676 - 640}}{-8} = \frac{-26 \pm \sqrt{36}}{-8} = \frac{-26 \pm 6}{-8}$$

$$p = \frac{-26 + 6}{-8} = 2\frac{1}{2} \quad \vee \quad p = \frac{-26 - 6}{-8} = 4$$

$$x^2 = 2\frac{1}{2} \quad \vee \quad x^2 = 4$$

$$x = \sqrt{2\frac{1}{2}} \quad \vee \quad x = -\sqrt{2\frac{1}{2}} \quad \vee \quad x = 2 \quad \vee \quad x = -2$$