

## 1.5 Grafisch-numeriek oplossen

### Opgave 59:

I:  $x^4 - x^2 - 2 = 0$

stel  $x^2 = p$

$$p^2 - p - 2 = 0$$

$$(p-2)(p+1) = 0$$

$$p = 2 \quad \vee \quad p = -1$$

$$x^2 = 2 \quad \vee \quad x^2 = -1$$

$$x = \sqrt{2} \quad \vee \quad x = -\sqrt{2}$$

II:  $x^4 - x^3 - 2 = 0$  niet

III:  $x^4 - x^3 - 2x = 0$

$$x(x^3 - x^2 - 2) = 0$$

$$x = 0 \quad \vee \quad x^3 - x^2 - 2 = 0 \text{ niet}$$

IV:  $x^4 - x^3 - 2x^2 = 0$

$$x^2(x^2 - x - 2) = 0$$

$$x^2(x-2)(x+1) = 0$$

$$x = 0 \quad \vee \quad x = 2 \quad \vee \quad x = -1$$

### Opgave 60:

a.  $x = -1 \quad \vee \quad x = 1 \quad \vee \quad x = 2 \quad \vee \quad x = 3$

b.  $(-1)^4 - 5 \cdot (-1)^3 + 5 \cdot (-1)^2 + 5 \cdot (-1) - 6 = 1 + 5 + 5 - 5 - 6 = 0$

$$1^4 - 5 \cdot 1^3 + 5 \cdot 1^2 + 5 \cdot (-1) - 6 = 1 - 5 + 5 + 5 - 6 = 0$$

$$2^4 - 5 \cdot 2^3 + 5 \cdot 2^2 + 5 \cdot 2 - 6 = 16 - 40 + 20 + 10 - 6 = 0$$

$$3^4 - 5 \cdot 3^3 + 5 \cdot 3^2 + 5 \cdot 3 - 6 = 81 - 135 + 45 + 15 - 6 = 0$$

### Opgave 61:

a.  $x = -2 \quad \vee \quad x = 2 \quad \vee \quad x = 4$

b.  $x = -2 \quad \vee \quad x = 2 \quad \vee \quad x = 4$

### Opgave 62:

a.  $y_1 = x^3 - 4x^2 + 3$  met de optie zero:  $x = -0,79 \quad \vee \quad x = 1 \quad \vee \quad x = 3,79$

b.  $y_1 = x^4 - 4x^3 + 2x^2 + x - 1 = 0$  met de optie zero:  $x = -0,58 \quad \vee \quad x = 3,34$

c.  $y_1 = 0,4x^3 + 2x^2 + x - 2$  en  $y_2 = x + 2$  met de optie intersect:

$$x = -4,51 \quad \vee \quad -1,76 \quad \vee \quad x = 1,26$$

d.  $y_1 = 0,2x^5 - x^4 + 4x^2$  en  $y_2 = 0,2x + 3$  met de optie intersect:

$$x = -1,45 \quad \vee \quad x = -1 \quad \vee \quad x = 1 \quad \vee \quad x = 3 \quad \vee \quad x = 3,45$$

### Opgave 63:

a.  $y_1 = 0,5x^3 - 5x^2 + 20$  met de optie zero:  $x = -1,84 \quad \vee \quad x = 2,28 \quad \vee \quad x = 9,56$

b.  $y_1 = 0,1x^4 + 0,1x^3 - 12x^2 + 50$  en  $y_2 = 25x$  met de optie intersect:

$$x = -10 \quad \vee \quad x = -3,53 \quad \vee \quad x = 1,26 \quad \vee \quad x = 11,27$$

**Opgave 64:**

- a.  $y_1 = \text{abs}(x^3 - 9x)$  en  $y_2 = 5$  met de optie intersect:  
 $x = -3,25 \vee x = -2,67 \vee x = -0,58 \vee x = 0,58 \vee x = 2,67 \vee x = 3,25$
- b.  $y_1 = \text{abs}(x^3 - 9x)$  en  $y_2 = x + 5$  met de optie intersect:  
 $x = -3,10 \vee x = -2,87 \vee x = -0,51 \vee x = 0,66 \vee x = 2,44 \vee x = 3,39$

**Opgave 65:**

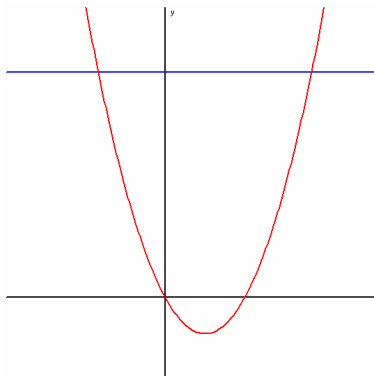
- a.  $y_1 = \text{abs}(x^4 - x^3 + x - 5)$  en  $y_2 = x + 3$  met de optie intersect:  
 $x = -1,48 \vee x = -1,26 \vee x = 1 \vee x = 2$
- b.  $y_1 = \text{abs}(x^3 - 5x^2 - 2x + 24)$  en  $y_2 = 20$  met de optie intersect:  
 $x = -2,55 \vee x = -1 \vee x = 0,76 \vee x = 5,24$
- c.  $y_1 = \text{abs}(x^2 - 4x)$  en  $y_2 = \text{abs}(x^2 + 2x - 3)$  met de optie intersect:  
 $x = -0,82 \vee x = 0,50 \vee x = 1,82$
- d.  $y_1 = \text{abs}(x^3 - 4x^2 - 3x + 10)$  met de optie zero:  
 $x = 1,48 \vee x = 4,14$

**Opgave 66:**

- a.  $y_1 = -x^2 + 6x$  en  $y_2 = x + 4$  met de optie intersect:  $x = 1 \vee x = 4$
- b.  $1 < x < 4$

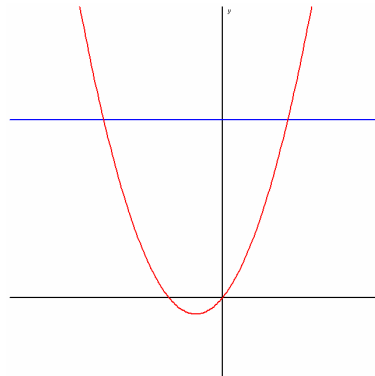
**Opgave 67:**

- a.  $y_1 = x^2 - 3x$  en  $y_2 = 14$   
 met de optie intersect:  
 $x = -2,53 \vee x = 5,53$



$$-2,53 \leq x \leq 5,53$$

- b.  $y_1 = x^2 + 2x$  en  $y_2 = 11$   
 met de optie intersect:  
 $x = -4,46 \vee x = 2,46$



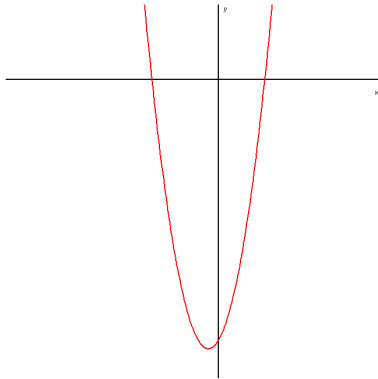
$$x < -4,46 \vee x > 2,46$$

c.  $y_1 = 8x^2 + 6x - 35$

met de optie zero:

$$x = -2,5 \vee x = 1,75$$

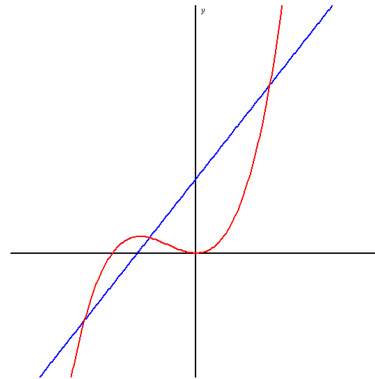
$$x = -6 \vee x = -2,5 \vee x = 4$$



$$x \leq -2,5 \vee x \geq 1,75$$

d.  $y_1 = x^3 + 4,5x^2$  en  $y_2 = 19x + 60$

met de optie intersect:



$$x < -6 \vee -2,5 < x < 4$$

**Opgave 68:**

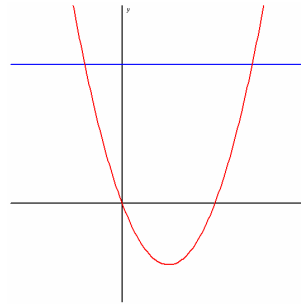
a.  $x^2 - 5x < 14$

$$x^2 - 5x - 14 = 0$$

$$(x - 7)(x + 2) = 0$$

$$x = 7 \vee x = -2$$

$$-2 < x < 7$$



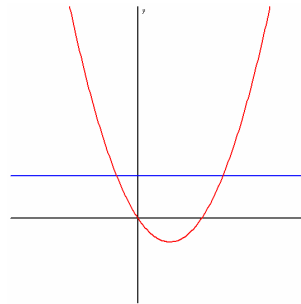
b.  $2x^2 - 3x \geq 2$

$$2x^2 - 3x - 2 = 0$$

$$x = \frac{3 \pm \sqrt{9 + 16}}{4} = \frac{3 \pm 5}{4}$$

$$x = \frac{3 + 5}{4} = 2 \vee x = \frac{3 - 5}{4} = -\frac{1}{2}$$

$$x \leq -\frac{1}{2} \vee x \geq 2$$



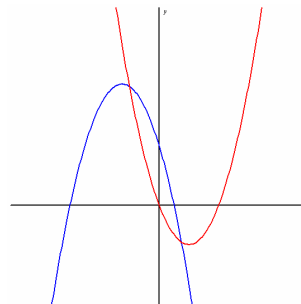
c.  $x^2 - 4x \leq -x^2 - 5x + 6$

$$2x^2 + x - 6 = 0$$

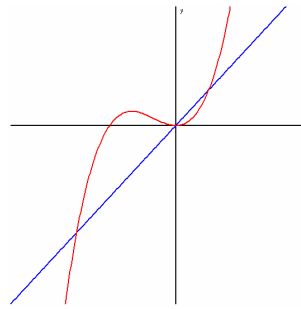
$$x = \frac{-1 \pm \sqrt{1 + 48}}{4} = \frac{-1 \pm 7}{4}$$

$$x = \frac{-1 + 7}{4} = 1\frac{1}{2} \vee x = \frac{-1 - 7}{4} = -2$$

$$-2 \leq x \leq 1\frac{1}{2}$$

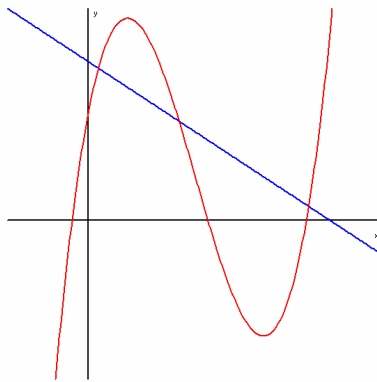


d.  $x^3 + 2x^2 > 3x$   
 $x^3 + 2x^2 - 3x = 0$   
 $x(x^2 + 2x - 3) = 0$   
 $x(x+3)(x-1) = 0$   
 $x = 0 \vee x = -3 \vee x = 1$   
 $-3 < x < 0 \vee x > 1$



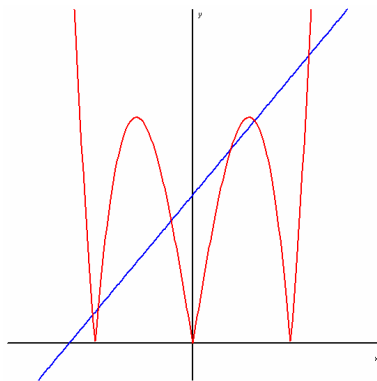
**Opgave 69:**

a.  $y_1 = 0,1x^3 - 2x^2 + 8x + 10$   
 $y_2 = -x + 15$   
 met de optie intersection:  
 $x = 0,65 \vee x = 5,66 \vee x = 13,69$



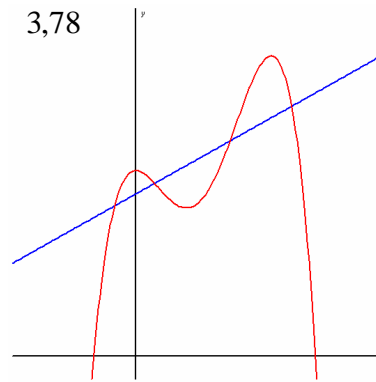
$0,65 \leq x \leq 5,66 \vee x \geq 13,69$

c.  $y_1 = \text{abs}(x^3 - 10x)$   
 $y_2 = 2x + 8$   
 met de optie intersection:  
 $x = -3,24 \vee x = -3,06 \vee x = -0,69$   
 $\vee x = 1,24 \vee x = 2 \vee x = 3,76$



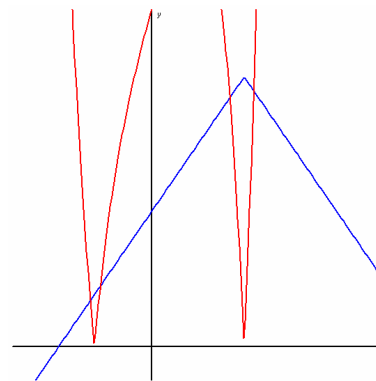
$-3,24 \leq x \leq -3,06 \vee -0,69 \leq x \leq 1,24$   
 $\vee 2 \leq x \leq 3,76$

b.  $y_1 = -0,5x^4 + 3x^3 - 4x^2 + 8$   
 $y_2 = x + 7$   
 met de optie intersection:  
 $x = -0,52 \vee x = 0,45 \vee x = 2,29$   
 $\vee 3,78$



$-0,52 \leq x \leq 0,45 \vee 2,29 \leq x \leq 3,78$

d.  $y_1 = \text{abs}(x^4 + x^2 - 5x - 10)$   
 $y_2 = 8 - \text{abs}(2x - 4)$   
 met de optie intersection:  
 $x = -1,32 \vee x = -1,10 \vee x = 1,69$   
 $\vee x = 2,21$



$-1,32 \leq x \leq -1,10 \vee 1,69 \leq x \leq 2,21$

**Opgave 70:**

a.  $D = p^2 - 4p$   
 b.  $p^2 - 4p > 0$

**Opgave 71:**

a.  $D = p^2 - 12p > 0$

$p(p - 12) = 0$

$p = 0 \vee p = 12$

$p < 0 \vee p > 12$

b.  $p \neq 0 \wedge D = (p - 4)^2 - 2p > 0$

$p^2 - 8p + 16 - 2p > 0$

$p^2 - 10p + 16 > 0$

$(p - 2)(p - 8) = 0$

$p = 2 \vee p = 8$

$p < 2 \vee p > 8$

dus  $p < 0 \vee 0 < p < 2 \vee p > 8$

c. dus  $px^2 + (p - 3)x - 4 = 0$  heeft geen oplossingen

$p \neq 0 \wedge D = (p - 3)^2 + 16p < 0$

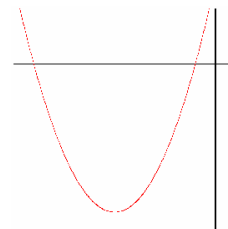
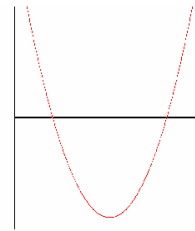
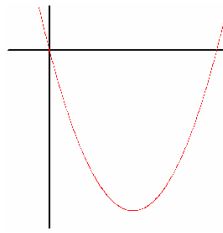
$p^2 - 6p + 9 + 16p < 0$

$p^2 + 10p + 9 < 0$

$(p - 1)(p - 9) = 0$

$p = 1 \vee p = 9$

$1 < p < 9$

**Opgave 72:**

a.  $D = (p^2 - 2)^2 - 49 > 0$

$(p^2 - 2)^2 = 49$

$p^2 - 2 = 7 \vee p^2 - 2 = -7$

$p^2 = 9 \vee p^2 = -5$

$p = 3 \vee p = -3$

$p < -3 \vee p > 3$

b.  $px^3 + p^2x^2 - 16x = 0$

$x(px^2 + p^2x - 16) = 0$

$x = 0 \vee px^2 + p^2x - 16 = 0$  heeft 2 oplossingen

$D = p^4 + 64p > 0$

$p(p^3 + 64) = 0$

$p = 0 \vee p^3 = -64$

$p = 0 \vee p = -4$

$p < -4 \vee p > 0$

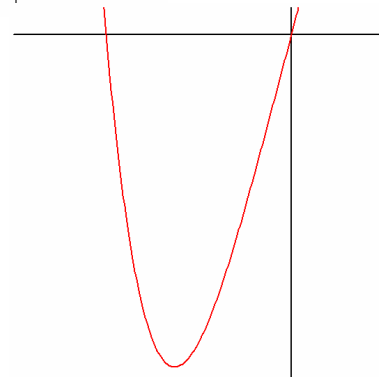
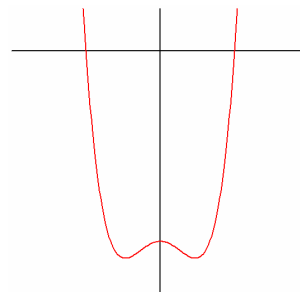
c.  $px^3 + 2px^2 - 3x^2 + \frac{1}{4}x = 0$

$x(px^2 + (2p - 3)x + \frac{1}{4}) = 0$

$x = 0 \vee px^2 + (2p - 3)x + \frac{1}{4} = 0$  heeft geen oplossingen

$D = (2p - 3)^2 - p < 0$

$4p^2 - 12p + 9 - p < 0$



$$4p^2 - 13p + 9 < 0$$

$$p = \frac{13 \pm \sqrt{169 - 144}}{8} = \frac{13 \pm 5}{8}$$

$$p = \frac{13+5}{8} = 2\frac{1}{4} \quad \vee \quad p = \frac{13-5}{8} = 1$$

$$1 < p < 2\frac{1}{4}$$

