HOOFDSTUK 3: De afgeleide functie.

3.4 Limiet en afgeleide

Opgave 31:

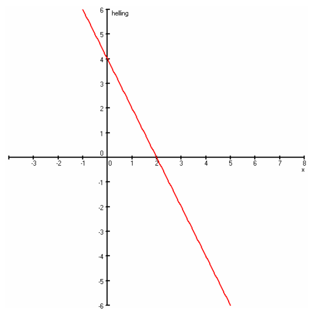
a. positief ; negatief

b. 0

c.

x-coördinaat	-1	0	1	2	3	4
helling in punt	6	4	2	0	-2	-4

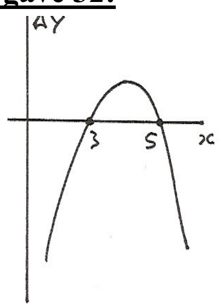
d.



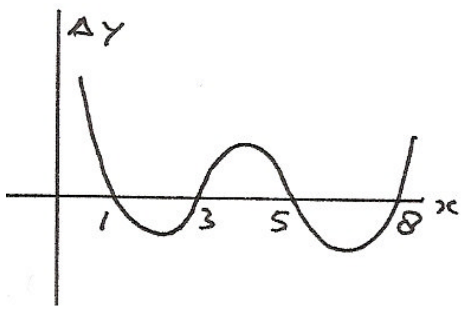
e. Deze grafiek geeft voor iedere x de helling van de grafiek van f in het punt $(x, f(x))$

Opgave 32:

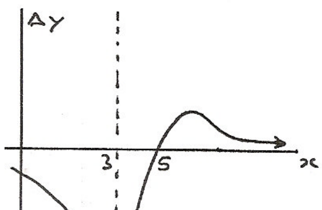
f



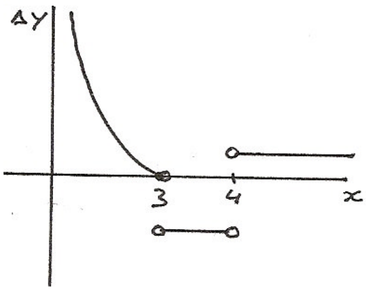
g



h



k



Opgave 33:

a. de hellinggrafiek ligt boven de x-as en is toenemend stijgend

b. de hellinggrafiek ligt onder de x-as en is afnemend stijgend.

c. voor $x = p$ snijdt de hellinggrafiek de x-as, voor $x < p$ ligt de hellinggrafiek boven de x-as, voor $x > p$ ligt de hellinggrafiek onder de x-as.

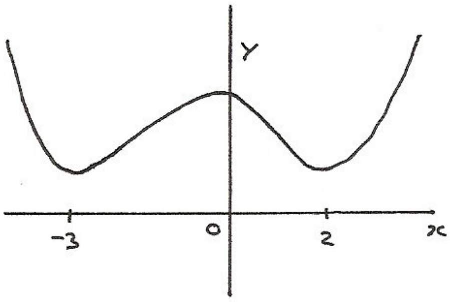
d. de hellinggrafiek ligt onder de x-as en heeft een laagste punt voor $x = q$.

Opgave 34:

a.

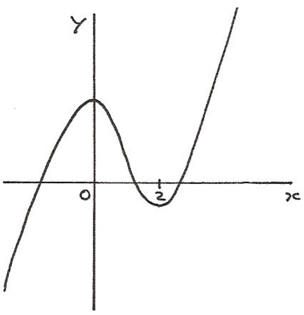
	hellinggrafiek van f	grafiek van f
$\langle -4.-3 \rangle$	onder de x-as	dalend
$x = -3$	snijdt de x-as	top
$\langle -3,0 \rangle$	boven de x-as	stijgend
$x = 0$	snijdt de x-as	top
$\langle 0,2 \rangle$	onder de x-as	dalend
$x = 2$	snijdt de x-as	top
$\langle 2,4 \rangle$	boven de x-as	stijgend

b.

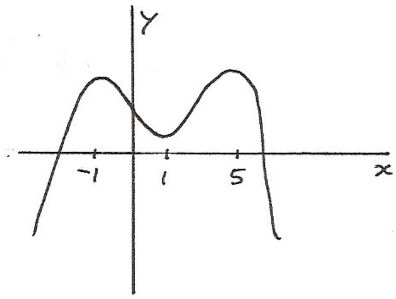


Opgave 35:

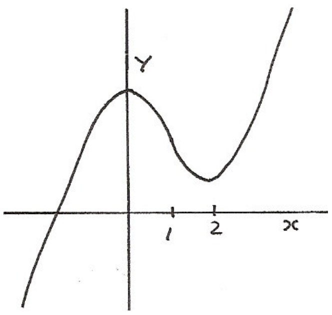
a.



b.



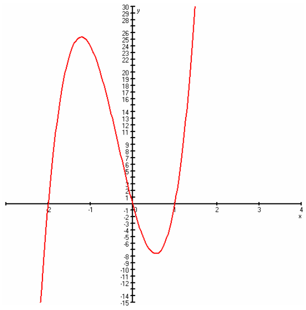
c.



Opgave 36:

a. $y_1 = 3x^4 + 4x^3 - 12x^2 + 2$ calcmenu optie minimum en maximum geeft:
$(-2,-30)$ $(0,2)$ $(1,-3)$

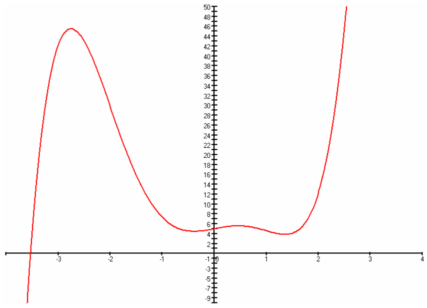
b.



c. calcmenu optie 6 geeft: $\left[\frac{dy}{dx}\right]_{x=-1} = 24$

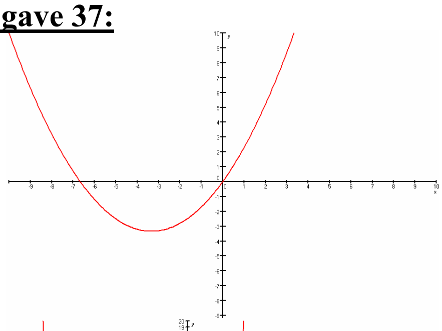
d. calcmenu optie zero geeft: $x = -2{,}8$, $x = -0{,}4$, $x = 0{,}5$, $x = 1{,}4$

e.

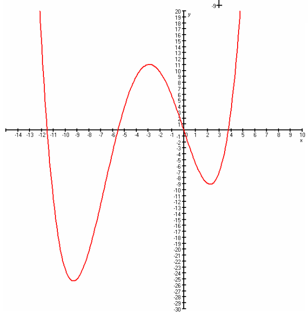


Opgave 37:

a.

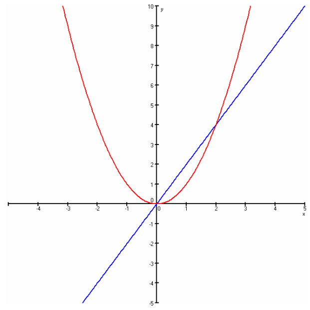


b.



Opgave 38:

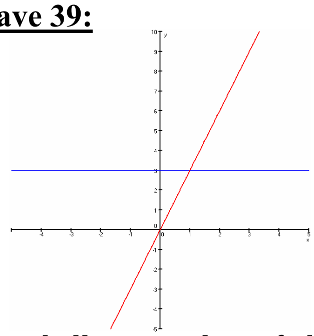
a.



b. $y = 2x$

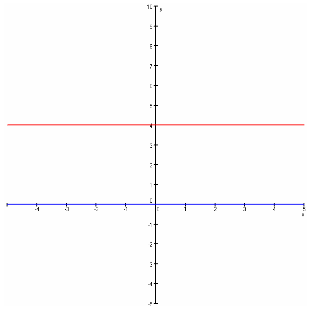
Opgave 39:

a.



b. De helling van de grafiek van f is in ieder punt gelijk aan 3 (=richtingscoëfficiënt).
hellingfunctie: $y = 3$

c.



de hellinggrafiek is de x-as

d. $y = 0$

Opgave 40:

Anders wordt de noemer 0 en je kunt niet door 0 delen.

Opgave 41:

a. $$f'(4) = \lim_{h \to 0} \frac{f(4+h) - f(4)}{h}$$
$$= \lim_{h \to 0} \frac{1{,}5(4+h)^2 - 24}{h}$$
$$= \lim_{h \to 0} \frac{1{,}5(16 + 8h + h^2) - 24}{h}$$
$$= \lim_{h \to 0} \frac{14 + 12h + 1{,}5h^2 - 24}{h}$$
$$= \lim_{h \to 0} \frac{12h + 1{,}5h^2}{h}$$

$$= \lim_{h \to 0} (12 + 1{,}5h) = 12 + 0 = 12$$

b. $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{1{,}5(x+h)^2 - 1{,}5x^2}{h}$$

$$= \lim_{h \to 0} \frac{1{,}5(x^2 + 2hx + h^2) - 1{,}5x^2}{h}$$

$$= \lim_{h \to 0} \frac{1{,}5x^2 + 3hx + 1{,}5h^2 - 1{,}5x^2}{h}$$

$$= \lim_{h \to 0} \frac{3hx + 1{,}5h^2}{h}$$

$$= \lim_{h \to 0} (3x + 1{,}5h) = 3x + 0 = 3x$$

Opgave 42:

a. $$f'(3) = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$$

$$= \lim_{h \to 0} \frac{(3+h)^2 - 4(3+h) - (9-12)}{h}$$

$$= \lim_{h \to 0} \frac{9 + 6h + h^2 - 12 - 4h + 3}{h}$$

$$= \lim_{h \to 0} \frac{2h + h^2}{h}$$

$$= \lim_{h \to 0} (2 + h) = 2 + 0 = 2$$

b. $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - 4(x+h) - (x^2 - 4x)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2hx + h^2 - 4x - 4h - x^2 + 4x}{h}$$

$$= \lim_{h \to 0} \frac{2hx - 4h + h^2}{h}$$

$$= \lim_{h \to 0} (2x - 4 + h) = 2x - 4 + 0 = 2x - 4$$

Opgave 43:

a. $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{a(x+h) - ax}{h}$$

$$= \lim_{h \to 0} \frac{ax + ah - ax}{h}$$

$$= \lim_{h \to 0} \frac{ah}{h} = \lim_{h \to 0} a = a$$

b. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{a - a}{h} = \lim_{h \to 0} \frac{0}{h} = \lim_{h \to 0} 0 = 0$

Opgave 44:

a. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{a(x+h)^3 - ax^3}{h}$

$= \lim_{h \to 0} \frac{a(x^3 + 3hx^2 + 3h^2x + h^3) - ax^3}{h}$

$= \lim_{h \to 0} \frac{ax^3 + 3ahx^2 + 3ah^2x + ah^3 - ax^3}{h}$

$= \lim_{h \to 0} \frac{3ahx^2 + 3ah^2x + ah^3}{h}$

$= \lim_{h \to 0} (3ax^2 + 3ahx + ah^2)$

$= \lim_{h \to 0} (3ax^2 + 0 + 0) = 0$

b. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{a(x+h)^2 + b(x+h) + c - (ax^2 + bx + c)}{h}$

$= \lim_{h \to 0} \frac{a(x^2 + 2hx + h^2) + bx + bh + c - ax^2 - bx - c}{h}$

$= \lim_{h \to 0} \frac{ax^2 + 2ahx + ah^2 + bx + bh + c - ax^2 - bx - c}{h}$

$= \lim_{h \to 0} \frac{2ahx + ah^2 + bh}{h}$

$= \lim_{h \to 0} (2ax + ah + b)$

$= \lim_{h \to 0} (2ax + 0 + b) = 2ax + b$

Opgave 45:

a. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{c \cdot g(x+h) - c \cdot g(x)}{h}$

$= \lim_{h \to 0} \frac{c \cdot (g(x+h) - g(x))}{h}$

$= c \cdot \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$

$= c \cdot g'(x)$

b. $s'(x)=\lim_{h\to 0}\frac{s(x+h)-s(x)}{h}$

$=\lim_{h\to 0}\frac{f(x+h)+g(x+h)-(f(x)+g(x))}{h}$

$=\lim_{h\to 0}\frac{f(x+h)-f(x)+g(x+h)-g(x)}{h}$

$=\lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}+\frac{g(x+h)-g(x)}{h}\right)$

$=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}+\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}$

$=f'(x)+g'(x)$

Opgave 46:

a. $f'(x)=30x^5-15x^4+2$

b. $g'(x)=-16x^7-16x^3$

c. $h'(x)=-x^2-x-1$

d. $k'(q)=3-6q-35q^6$

Opgave 47:

a. $f(x)=(5x+7)(4-3x)=20x-15x^2+28-21x=-15x^2-x+28$

$f'(x)=-30x-1$

b. $g(x)=(3x+6)^2-8x=9x^2+36x+36-8x=9x^2+28x+36$

$g'(x)=18x+28$

c. $h(x)=5(x-3)^2+5(2x-1)=5(x^2-6x+9)+10x-5=5x^2-30x+45+10x-5=$

$5x^2-20x+40$

$h'(x)=10x-20$

d. $k(x)=-3(x-1)(5-9x)-8(x-7)=-3(5x-9x^2-5+9x)-8x+56=$

$-15x+27x^2+15-27x-8x+56=27x^2-50x+71$

$k'(x)=54x-50$

Opgave 48:

a. $f(x)=(3x-1)(x^2+5x)=3x^3+15x^2-x^2-5x=3x^3+14x^2-5x$

$f'(x)=9x^2+28x-5$

b. $g(x)=(3x^3-1)^2=9x^6-6x^3+1$

$g'(x)=54x^5-18x^2$

c. $h(x)=(5x^5-3)(3x-2)=15x^6-10x^5-9x+6$

$h'(x)=90x^5-50x^4-9$

d. $k(x)=5-3(x^4-x)(x+1)=5-3(x^5+x^4-x^2-x)=5-3x^5-3x^4+3x^2+3x$

$k'(x)=-15x^4-12x^3+6x+3$

e. $l(t)=(5t^3-t)(3t^5+t)=15t^8-3t^6+5t^4-t^2$

$l'(t)=120t^7-18t^5+20t^3-2t$

f. $m(q) = 1 - (3q^2 - 2)^2 = 1 - (9q^4 - 12q^2 + 4) = 1 - 9q^4 + 12q^2 - 4$

$m'(q) = -36q^3 + 24q$