

## HOOFDSTUK 4: Algebra en meetkunde.

### 4.6 Diagnostische toets

#### Opgave 1:

- a.  $4\sqrt{5} \cdot 3\sqrt{2} = 12\sqrt{10}$
- b.  $\sqrt{16\frac{1}{3}} = \sqrt{\frac{49}{3}} = \sqrt{\frac{147}{9}} = \frac{7}{3}\sqrt{3}$
- c.  $\frac{6}{\sqrt{2}} + \sqrt{8} = \frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} + \sqrt{8} = \frac{6\sqrt{2}}{2} + 2\sqrt{2} = 3\sqrt{2} + 2\sqrt{2} = 5\sqrt{2}$
- d.  $\sqrt{\frac{1}{3}} + \sqrt{3} = \sqrt{\frac{3}{9}} + \sqrt{3} = \frac{1}{3}\sqrt{3} + \sqrt{3} = 1\frac{1}{3}\sqrt{3}$
- e.  $\sqrt{8a^2} + \sqrt{32a^2} = 2 \cdot |a| \cdot \sqrt{2} + 4 \cdot |a| \cdot \sqrt{2} = 6 \cdot |a| \cdot \sqrt{2}$
- f.  $a\sqrt{48} - 2a\sqrt{12} = 4a\sqrt{3} - 2a \cdot 2\sqrt{3} = 4a\sqrt{3} - 4a\sqrt{3} = 0$

#### Opgave 2:

- a.  $(3 + \sqrt{2})^2 = 9 + 6\sqrt{2} + 2 = 11 + 6\sqrt{2}$
- b.  $\frac{\sqrt{3}}{\sqrt{5} + \sqrt{7}} = \frac{\sqrt{3}}{\sqrt{5} + \sqrt{7}} \cdot \frac{\sqrt{5} - \sqrt{7}}{\sqrt{5} - \sqrt{7}} = \frac{\sqrt{15} - \sqrt{21}}{5 - 7} = \frac{\sqrt{15} - \sqrt{21}}{-2} = -\frac{1}{2}\sqrt{15} + \frac{1}{2}\sqrt{21}$
- c.  $(a - \sqrt{3})(a + \sqrt{3}) = a^2 - 3$
- d.  $\frac{20}{\sqrt{6} - 1} = \frac{20}{\sqrt{6} - 1} \cdot \frac{\sqrt{6} + 1}{\sqrt{6} + 1} = \frac{20\sqrt{6} + 20}{6 - 1} = \frac{20\sqrt{6} + 20}{5} = 4\sqrt{6} + 4$
- e.  $(2a - \sqrt{7})^2 = 4a^2 - 4a\sqrt{7} + 7$
- f.  $\left(\frac{2}{\sqrt{5} - 1}\right)^2 = \frac{4}{(\sqrt{5} - 1)^2} = \frac{4}{5 - 2\sqrt{5} + 1} = \frac{4}{6 - 2\sqrt{5}} = \frac{4}{6 - 2\sqrt{5}} \cdot \frac{6 + 2\sqrt{5}}{6 + 2\sqrt{5}} = \frac{24 + 8\sqrt{5}}{36 - 20} = \frac{24 + 8\sqrt{5}}{16} = 1\frac{1}{2} + \frac{1}{2}\sqrt{5}$

#### Opgave 3:

- a.  $\frac{1}{3a} - \frac{1}{4a} = \frac{4}{12a} - \frac{3}{12a} = \frac{1}{12a}$
- b.  $\frac{1}{5x} + \frac{1}{10x} = \frac{2}{10x} + \frac{1}{10x} = \frac{3}{10x}$
- c.  $\frac{1}{x-2} - \frac{2}{x+1} = \frac{x+1}{(x-2)(x+1)} - \frac{2(x-2)}{(x-2)(x+1)} = \frac{x+1}{(x-2)(x+1)} - \frac{2x-4}{(x-2)(x+1)} = \frac{-x+5}{(x-2)(x+1)}$
- d.  $\frac{2x}{x+1} + \frac{5}{x-3} = \frac{2x(x-3)}{(x+1)(x-3)} + \frac{5(x+1)}{(x+1)(x-3)} = \frac{2x^2 - 6x}{(x+1)(x-3)} + \frac{5x+5}{(x+1)(x-3)} = \frac{2x^2 - x + 5}{(x+1)(x-3)}$
- e.  $x + \frac{3}{x+1} = \frac{x(x+1)}{x+1} + \frac{3}{x+1} = \frac{x^2 + x}{x+1} + \frac{3}{x+1} = \frac{x^2 + x + 3}{x+1}$

$$f. \quad \frac{2a}{b} + \frac{a}{a+b} = \frac{2a(a+b)}{b(a+b)} + \frac{ab}{b(a+b)} = \frac{2a^2 + 2ab}{b(a+b)} + \frac{ab}{b(a+b)} = \frac{2a^2 + 3ab}{b(a+b)}$$

**Opgave 4:**

$$a. \quad \frac{x^2 - 6x + 5}{x^2 - 25} = \frac{(x-5)(x-1)}{(x-5)(x+5)} = \frac{x-1}{x+5}$$

$$b. \quad \frac{6x^2 + 6x}{x^2 + 3x + 2} = \frac{6x(x+1)}{(x+2)(x+1)} = \frac{6x}{x+2}$$

$$c. \quad \frac{x^2 + 6x + 8}{x+2} + \frac{x^2 + 8}{x} = \frac{(x+2)(x+4)}{x+2} + \frac{x^2 + 8}{x} = x + 4 + x + \frac{8}{x} = 2x + 4 + \frac{8}{x}$$

**Opgave 5:**

$$a. \quad \frac{6}{x} - \frac{4}{x+2} = 2$$

$$\frac{6(x+2)}{x(x+2)} - \frac{4x}{x(x+2)} = 2$$

$$\frac{6x+12}{x(x+2)} - \frac{4x}{x(x+2)} = 2$$

$$\frac{2x+12}{x(x+2)} = 2$$

$$2x(x+2) = 2x+12$$

$$2x^2 + 4x = 2x + 12$$

$$2x^2 + 2x - 12 = 0$$

$$x^2 + x - 6 = 0$$

$$(x+3)(x-2) = 0$$

$$x = -3 \quad \vee \quad x = 2$$

$$b. \quad \frac{x}{16} = \frac{x^2 - 4}{x^2 + 6x + 8}$$

$$\frac{x}{16} = \frac{(x+2)(x-2)}{(x+2)(x+4)}$$

$$\frac{x}{16} = \frac{x-2}{x+4}$$

$$x(x+4) = 16(x-2)$$

$$x^2 + 4x = 16x - 32$$

$$x^2 - 12x + 32 = 0$$

$$(x-4)(x-8) = 0$$

$$x = 4 \quad \vee \quad x = 8$$

$$c. \quad \frac{x^2 - 9}{x^2 + 4x + 3} = \frac{5}{3x}$$

$$\frac{(x+3)(x-3)}{(x+3)(x+1)} = \frac{5}{3x}$$

$$\frac{x-3}{x+1} = \frac{5}{3x}$$

$$3x(x-3) = 5(x+1)$$

$$3x^2 - 9x = 5x + 5$$

$$3x^2 - 14x - 5 = 0$$

$$x = \frac{14 \pm \sqrt{196 + 60}}{6} = \frac{14 \pm \sqrt{256}}{6} = \frac{14 \pm 16}{6}$$

$$x = \frac{14+16}{6} = 5 \quad \vee \quad x = \frac{14-16}{6} = -\frac{1}{3}$$

### **Opgave 6:**

a.  $2a^3 \cdot 3a^6 = 6a^9$

b.  $a^{12} \cdot \frac{1}{a^4} = \frac{a^{12}}{a^4} = a^8$

c.  $(2a)^3 - a \cdot 7a^2 = 8a^3 - 7a^3 = a^3$

d.  $\frac{14a^8}{2a^5} = 7a^3$

e.  $(3a^2)^4 + 5(a^4)^2 = 81a^8 + 5a^8 = 86a^8$

f.  $\frac{1}{a^6} \cdot (a^2)^3 = \frac{1}{a^6} \cdot a^6 = \frac{a^6}{a^6} = 1$

### **Opgave 7:**

a.  $\frac{1}{a^3} = a^{-3}$

b.  $a^4 \cdot \frac{1}{a^7} = \frac{a^4}{a^7} = a^{-3}$

c.  $\sqrt[5]{a^3} = a^{\frac{3}{5}}$

d.  $\frac{\sqrt{a}}{a^2} = \frac{a^{\frac{1}{2}}}{a^2} = a^{-1\frac{1}{2}}$

e.  $a^2 \cdot \sqrt[3]{a} = a^2 \cdot a^{\frac{1}{3}} = a^{2\frac{1}{3}}$

f.  $\frac{1}{\sqrt[3]{a^2}} = \frac{1}{a^{\frac{2}{3}}} = a^{-\frac{2}{3}}$

### **Opgave 8:**

a.  $\left(a^{-\frac{1}{4}}\right)^3 = a^{-\frac{3}{4}} = \frac{1}{a^{\frac{3}{4}}} = \frac{1}{\sqrt[4]{a^3}}$

b.  $a^{-2} \cdot b^{\frac{1}{5}} = \frac{1}{a^2} \cdot \sqrt[5]{b} = \frac{\sqrt[5]{b}}{a^2}$

c.  $7a^{-\frac{1}{3}} \cdot b^{\frac{3}{5}} = \frac{7 \cdot \sqrt[5]{b^3}}{a^{\frac{1}{3}}} = \frac{7 \cdot \sqrt[5]{b^3}}{\sqrt[3]{a}}$

### **Opgave 9:**

a.  $3x^{1,6} + 2 = 7$

$$3x^{1,6} = 5$$

$$x^{1,6} = \frac{5}{3}$$

$$x = \left(\frac{5}{3}\right)^{\frac{1}{1,6}} = 1,376$$

b.  $\frac{1}{4}x^{-3,7} = 160$

$$x^{-3,7} = 640$$

$$x = 640^{\frac{1}{-3,7}} = 0,174$$

c.  $7 \cdot \sqrt[5]{x^3} = 48$

$$x^{\frac{3}{5}} = \frac{48}{7}$$

$$x = \left(\frac{48}{7}\right)^{\frac{5}{3}} = 24,750$$

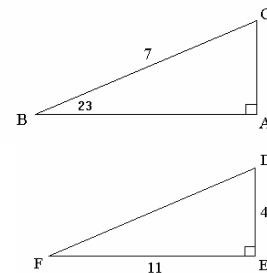
### Opgave 10:

a.  $\cos 23^\circ = \frac{AB}{7}$

$$AB = 7 \cos 23^\circ = 6,44$$

b.  $\tan \angle D = \frac{11}{4}$

$$\angle D = 70^\circ$$



### Opgave 11:

$$\sin 45^\circ = \frac{CD}{6}$$

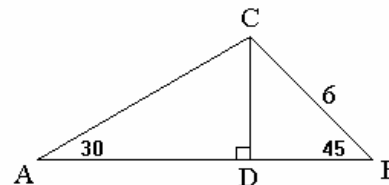
$$CD = 6 \cdot \sin 45^\circ = 6 \cdot \frac{1}{2} \sqrt{2} = 3\sqrt{2}$$

$$BD = 3\sqrt{2}$$

$$\tan 30^\circ = \frac{CD}{AD} = \frac{3\sqrt{2}}{AD}$$

$$AD = \frac{3\sqrt{2}}{\tan 30^\circ} = \frac{3\sqrt{2}}{\frac{1}{\sqrt{3}}} = \frac{3\sqrt{2}}{\frac{1}{\sqrt{3}}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{6}}{1} = 3\sqrt{6}$$

$$Opp(\triangle ABC) = \frac{1}{2} \cdot (3\sqrt{6} + 3\sqrt{2}) \cdot 3\sqrt{2} = 4\frac{1}{2} \sqrt{12} + 9 = 4\frac{1}{2} \cdot 2\sqrt{3} + 9 = 9\sqrt{3} + 9$$



### Opgave 12:

a.  $\frac{\sin \alpha}{a} = \frac{\sin \beta}{b}$

$$\frac{\sin 35^\circ}{3} = \frac{\sin \beta}{5}$$

$$\sin \beta = \frac{5 \cdot \sin 35^\circ}{3} = 0,956$$

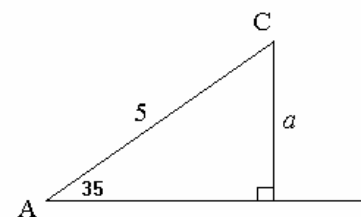
$$\beta = 73^\circ \quad \vee \quad \beta = 107^\circ$$

$$\gamma = 180^\circ - 35^\circ - 73^\circ = 72^\circ \quad \vee \quad \gamma = 180^\circ - 35^\circ - 107^\circ = 38^\circ$$

b.  $\sin 35^\circ = \frac{a}{5}$

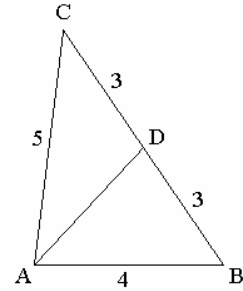
$$a = 5 \cdot \sin 35^\circ = 2,87$$

$$a = 2,87 \quad \vee \quad a \geq 5$$



**Opgave 13:**

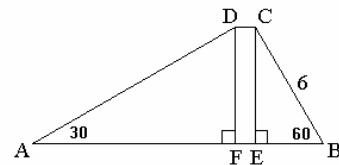
- a.  $AC^2 = AB^2 + BC^2 - 2 \cdot AB \cdot BC \cdot \cos \angle B$   
 $5^2 = 4^2 + 6^2 - 2 \cdot 4 \cdot 6 \cdot \cos \angle B$   
 $25 = 16 + 36 - 48 \cos \angle B$   
 $48 \cos \angle B = 27$   
 $\cos \angle B = \frac{27}{48}$   
 $\angle B = 56^\circ$
- b.  $AD^2 = AB^2 + BD^2 - 2 \cdot AB \cdot BD \cdot \cos \angle B$   
 $AD^2 = 4^2 + 3^2 - 2 \cdot 4 \cdot 3 \cdot \frac{27}{48} = 11,5$   
 $AD = 3,39$

**Opgave 14:**

- a.  $AM = \sqrt{AD^2 + DM^2} = \sqrt{a^2 + (\frac{1}{2}a)^2} = \sqrt{a^2 + 2\frac{1}{4}a^2} = \sqrt{3\frac{1}{4}a^2} = \sqrt{\frac{13}{4}a^2} = \frac{1}{2}a\sqrt{13}$
- b.  $AM = BM = \frac{1}{2}a\sqrt{13}$   
 $FM = \sqrt{BM^2 + BF^2} = \sqrt{(\frac{1}{2}a\sqrt{13})^2 + (2a)^2} = \sqrt{3\frac{1}{4}a^2 + 4a^2} = \sqrt{7\frac{1}{4}a^2} = \sqrt{\frac{29}{4}a^2} = \frac{1}{2}a\sqrt{29}$
- c.  $FH = \sqrt{EF^2 + EH^2} = \sqrt{(3a)^2 + a^2} = \sqrt{9a^2 + a^2} = \sqrt{10a^2} = a\sqrt{10}$   
 $FS = \frac{1}{2}FH = \frac{1}{2}a\sqrt{10}$   
 $BS = \sqrt{BF^2 + FS^2} = \sqrt{(2a)^2 + (\frac{1}{2}a\sqrt{10})^2} = \sqrt{4a^2 + 2\frac{1}{2}a^2} = \sqrt{6\frac{1}{2}a^2} = \sqrt{\frac{26}{4}a^2} = \frac{1}{2}a\sqrt{26}$

**Opgave 15:**

- $\sin 60^\circ = \frac{CE}{6}$   
 $CE = 6 \cdot \sin 60^\circ = 6 \cdot \frac{1}{2}\sqrt{3} = 3\sqrt{3}$
- $\cos 60^\circ = \frac{BE}{6}$   
 $BE = 6 \cdot \cos 60^\circ = 6 \cdot \frac{1}{2} = 3$
- $DF = CE = 3\sqrt{3}$
- $\tan 30^\circ = \frac{DF}{AF} = \frac{3\sqrt{3}}{AF}$   
 $AF = \frac{3\sqrt{3}}{\tan 30^\circ} = \frac{3\sqrt{3}}{\frac{1}{3}\sqrt{3}} = 9$
- $Opp(ABCD) = \frac{1}{2} \cdot CE \cdot (AB + CD) = \frac{1}{2} \cdot 3\sqrt{3} \cdot (9 + EF + 3 + CD) = 1\frac{1}{2}\sqrt{3} \cdot (12 + 2 \cdot CD) = 36$
- $12 + 2 \cdot CD = \frac{36}{1\frac{1}{2}\sqrt{3}} = \frac{24}{\sqrt{3}} = \frac{24}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{24\sqrt{3}}{3} = 8\sqrt{3}$
- $2 \cdot CD = 8\sqrt{3} - 12$   
 $CD = 4\sqrt{3} - 6$
- $AB = AF + EF + BE = 9 + 4\sqrt{3} - 6 + 3 = 6 + 4\sqrt{3}$



**Opgave 16:**

$$CP = \sqrt{AC^2 - AD^2} = \sqrt{25^2 - 7^2} = \sqrt{625 - 49} = \sqrt{576} = 24$$

$\triangle AKN \sim \triangle APC$  (snavelfiguur)

$$\frac{AK}{AP} = \frac{KN}{PC}$$

$$\frac{x}{7} = \frac{KN}{24}$$

$$KN = \frac{24 \cdot x}{7} = \frac{24}{7}x$$

$$Opp(KLMN) = KL \cdot KN = 2 \cdot (7 - x) \cdot \frac{24}{7}x = 48x - \frac{48}{7}x^2$$

$$Opp' = 48 - \frac{96}{7}x$$

$$-\frac{96}{7}x = -48$$

$$x = 3\frac{1}{2}$$

$$Opp(KLMN) = 48 \cdot 3\frac{1}{2} - \frac{48}{7} \cdot (3\frac{1}{2})^2 = 84$$

