

HOOFDSTUK 4: Algebra en meetkunde.

4.1 Rekenen met wortels

Opgave 1:

II, III, IV, VI, VII, VIII

Toelichting:

$$\text{IV: } \frac{\sqrt{18}}{\sqrt{2}} = \sqrt{\frac{18}{2}} = \sqrt{9} = 3$$

$$\text{VI: } \sqrt{8} - \sqrt{2} = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$$

$$\text{VII: } \frac{2}{\sqrt{5}} = \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5} = \frac{2}{5}\sqrt{5}$$

$$\text{VIII: } \sqrt{4\frac{1}{2}} = \sqrt{2\frac{1}{4} \cdot 2} = \sqrt{2\frac{1}{4}} \cdot \sqrt{2} = 1\frac{1}{2}\sqrt{2}$$

Opgave 2:

$$\text{a. } 2\sqrt{3} \cdot 3\sqrt{5} = 6\sqrt{15}$$

$$\text{b. } \frac{5\sqrt{10}}{\sqrt{5}} = 5\sqrt{2}$$

$$\text{c. } 3a\sqrt{2} \cdot a\sqrt{7} = 3a^2\sqrt{14}$$

$$\text{d. } \frac{2\sqrt{14}}{3\sqrt{7}} = \frac{2}{3}\sqrt{2}$$

$$\text{e. } \frac{1}{2}a\sqrt{2} \cdot \frac{1}{2}a\sqrt{3} = \frac{1}{4}a^2\sqrt{6}$$

$$\text{f. } \frac{6}{5\sqrt{2}} = \frac{6}{5\sqrt{2}} \cdot \frac{5\sqrt{2}}{5\sqrt{2}} = \frac{30\sqrt{2}}{50} = \frac{3}{5}\sqrt{2}$$

Opgave 3:

$$\text{a. } \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3} = \frac{1}{3}\sqrt{3}$$

$$\text{b. } \sqrt{\frac{1}{2}} = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{1}{2}\sqrt{2}$$

$$\text{c. } \sqrt{4\frac{1}{2}} = \sqrt{2\frac{1}{4} \cdot 2} = \sqrt{2\frac{1}{4}} \cdot \sqrt{2} = 1\frac{1}{2}\sqrt{2}$$

$$\text{d. } \left(\frac{1}{2}\sqrt{5}\right)^2 = \frac{1}{2}\sqrt{5} \cdot \frac{1}{2}\sqrt{5} = \frac{1}{4} \cdot 5 = 1\frac{1}{4}$$

$$\text{e. } \left(\frac{1}{2}a\sqrt{2}\right)^2 = \frac{1}{2}a\sqrt{2} \cdot \frac{1}{2}a\sqrt{2} = \frac{1}{4}a^2 \cdot 2 = \frac{1}{2}a^2$$

$$\text{f. } \left(\frac{2}{3}a\sqrt{3}\right)^2 = \frac{2}{3}a\sqrt{3} \cdot \frac{2}{3}a\sqrt{3} = \frac{4}{9}a^2 \cdot 3 = 1\frac{1}{3}a^2$$

Opgave 4:

$$\text{a. } \sqrt{24} + \sqrt{6} = \sqrt{4 \cdot 6} + \sqrt{6} = 2\sqrt{6} + \sqrt{6} = 3\sqrt{6}$$

$$\text{b. } \sqrt{80} - \frac{10}{\sqrt{5}} = \sqrt{16 \cdot 5} - \frac{10}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = 4\sqrt{5} - \frac{10\sqrt{5}}{5} = 4\sqrt{5} - 2\sqrt{5} = 2\sqrt{5}$$

$$\text{c. } \sqrt{18a} - \sqrt{8a} = \sqrt{9 \cdot 2a} - \sqrt{4 \cdot 2a} = 3\sqrt{2a} - 2\sqrt{2a} = \sqrt{2a}$$

- d. $\sqrt{3a^2} + \sqrt{12a^2} = |a| \cdot \sqrt{3} + \sqrt{4a^2 \cdot 3} = |a| \cdot \sqrt{3} + 2 \cdot |a| \cdot \sqrt{3} = 3 \cdot |a| \cdot \sqrt{3}$
 e. $\sqrt{\frac{3}{4}a^2} = \sqrt{\frac{1}{4}a^2 \cdot 3} = \frac{1}{2} \cdot |a| \cdot \sqrt{3}$
 f. $\sqrt{\frac{7}{9}a^2} = \sqrt{\frac{1}{9}a^2 \cdot 7} = \frac{1}{3} \cdot |a| \cdot \sqrt{7}$

Opgave 5:

- a. $a\sqrt{8} - a\sqrt{2} = 2a\sqrt{2} - a\sqrt{2} = a\sqrt{2}$
 b. $\sqrt{2a^2} + \sqrt{\frac{1}{2}a^2} = |a| \cdot \sqrt{2} + |a| \cdot \sqrt{\frac{1}{2}} = |a| \cdot \sqrt{2} + |a| \cdot \sqrt{\frac{2}{4}} = |a| \cdot \sqrt{2} + \frac{1}{2} \cdot |a| \cdot \sqrt{2} = 1\frac{1}{2} \cdot |a| \cdot \sqrt{2}$
 c. $\sqrt{24\frac{1}{2}a^2} - \sqrt{2a^2} = \sqrt{12\frac{1}{4}a^2 \cdot 2} - \sqrt{2a^2} = 3\frac{1}{2} \cdot |a| \cdot \sqrt{2} - |a| \cdot \sqrt{2} = 2\frac{1}{2} \cdot |a| \cdot \sqrt{2}$
 d. $a^2 \cdot \sqrt{50} - a^2 \cdot \sqrt{32} = a^2 \cdot \sqrt{25 \cdot 2} - a^2 \cdot \sqrt{16 \cdot 2} = 5a^2 \sqrt{2} - 4a^2 \sqrt{2} = a^2 \sqrt{2}$
 e. $(\frac{1}{4}a\sqrt{2})^2 + (\frac{3}{4}a\sqrt{2})^2 = \frac{1}{16}a^2 \cdot 2 + \frac{9}{16}a^2 \cdot 2 = \frac{1}{8}a^2 + \frac{9}{8}a^2 = 1\frac{1}{4}a^2$
 f. $\frac{a}{2\sqrt{3}} + \frac{a}{\sqrt{3}} = \frac{a}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} + \frac{a}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{a\sqrt{3}}{6} + \frac{a\sqrt{3}}{3} = \frac{1}{6}a\sqrt{3} + \frac{2}{6}a\sqrt{3} = \frac{1}{2}a\sqrt{3}$

Opgave 6:

- I: niet waar, $(\sqrt{2} + \sqrt{3})^2 = 2 + 2\sqrt{6} + 3 = 5 + 2\sqrt{6}$
 II: waar
 III: waar
 IV: niet waar, $(10 - \sqrt{3})^2 = 100 - 20\sqrt{3} + 3 = 103 - 20\sqrt{3}$

Opgave 7:

- a. $(3\sqrt{2} - \sqrt{5})^2 = 18 - 6\sqrt{10} + 5 = 23 - 6\sqrt{10}$
 b. $(2\sqrt{2} + 3\sqrt{3})^2 = 8 + 12\sqrt{6} + 27 = 35 + 12\sqrt{6}$
 c. $(5\sqrt{3} + 2)(5\sqrt{3} - 2) = 75 - 4 = 71$
 d. $(a - \sqrt{3})^2 = a^2 - 2a\sqrt{3} + 3$
 e. $(a - a\sqrt{2})^2 = a^2 - 2a^2 \cdot \sqrt{2} + 2a^2 = 3a^2 - 2a^2 \cdot \sqrt{2}$
 f. $(4 - \frac{1}{2}a\sqrt{2})^2 = 16 - 4a\sqrt{2} + \frac{1}{4}a^2 \cdot 2 = 16 - 4a\sqrt{2} + \frac{1}{2}a^2$

Opgave 8:

- a. $\frac{2}{\sqrt{5}-1} = \frac{2}{\sqrt{5}-1} \cdot \frac{\sqrt{5}+1}{\sqrt{5}+1} = \frac{2\sqrt{5}+2}{5-1} = \frac{2\sqrt{5}+2}{4} = \frac{1}{2}\sqrt{5} + \frac{1}{2}$
 b. $\frac{10}{\sqrt{2}+\sqrt{3}} = \frac{10}{\sqrt{2}+\sqrt{3}} \cdot \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}} = \frac{10\sqrt{2}-10\sqrt{3}}{2-3} = \frac{10\sqrt{2}-10\sqrt{3}}{-1} = -10\sqrt{2} + 10\sqrt{3}$
 c. $\frac{12\sqrt{2}}{\sqrt{10}-\sqrt{2}} = \frac{12\sqrt{2}}{\sqrt{10}-\sqrt{2}} \cdot \frac{\sqrt{10}+\sqrt{2}}{\sqrt{10}+\sqrt{2}} = \frac{12\sqrt{20}+24}{10-2} = \frac{24\sqrt{5}+24}{8} = 3\sqrt{5} + 3$

Opgave 9:

- a. $(2a\sqrt{2} - a\sqrt{3})^2 = 8a^2 - 4a^2 \cdot \sqrt{6} + 3a^2 = 11a^2 - 4a^2 \cdot \sqrt{6}$

$$\text{b. } \frac{\sqrt{5} + \sqrt{2}}{\sqrt{5} - \sqrt{2}} = \frac{\sqrt{5} + \sqrt{2}}{\sqrt{5} - \sqrt{2}} \cdot \frac{\sqrt{5} + \sqrt{2}}{\sqrt{5} + \sqrt{2}} = \frac{5 + 2\sqrt{10} + 2}{5 - 2} = \frac{7 + 2\sqrt{10}}{3} = 2\frac{1}{3} + \frac{2}{3}\sqrt{10}$$

$$\text{c. } \left(\frac{1}{2}\sqrt{2} + \frac{3}{4}\sqrt{3}\right)^2 = \frac{1}{2} + \frac{3}{4}\sqrt{6} + \frac{27}{16} = 2\frac{3}{16} + \frac{3}{4}\sqrt{6}$$

$$\text{d. } \frac{\sqrt{72}}{3 - \sqrt{3}} = \frac{6\sqrt{2}}{3 - \sqrt{3}} \cdot \frac{3 + \sqrt{3}}{3 + \sqrt{3}} = \frac{18\sqrt{2} + 6\sqrt{6}}{9 - 3} = \frac{18\sqrt{2} + 6\sqrt{6}}{6} = 3\sqrt{2} + \sqrt{6}$$

$$\text{e. } \left(\frac{1}{\sqrt{2} - 1}\right)^2 = \frac{1}{(\sqrt{2} - 1)^2} = \frac{1}{2 - 2\sqrt{2} + 1} = \frac{1}{3 - 2\sqrt{2}} = \frac{1}{3 - 2\sqrt{2}} \cdot \frac{3 + 2\sqrt{2}}{3 + 2\sqrt{2}} = \frac{3 + 2\sqrt{2}}{9 - 8} =$$

$$\frac{3 + 2\sqrt{2}}{1} = 3 + 2\sqrt{2}$$

$$\text{f. } \left(\frac{a}{2\sqrt{5}} + \frac{a}{\sqrt{5}}\right)^2 = \left(\frac{a}{2\sqrt{5}} + \frac{2a}{2\sqrt{5}}\right)^2 = \left(\frac{3a}{2\sqrt{5}}\right)^2 = \frac{9a^2}{20} = \frac{9}{20}a^2$$