

## HOOFDSTUK 4: Algebra en meetkunde.

### 4.2 Gebroken vormen

#### Opgave 10:

I: niet waar,  $\frac{1}{x} + \frac{1}{y} = \frac{y}{xy} + \frac{x}{xy} = \frac{x+y}{xy}$

II: waar,  $\frac{1}{x} - \frac{1}{y} = \frac{y}{xy} - \frac{x}{xy} = \frac{y-x}{xy}$

III: niet waar,  $\frac{3}{2x} + \frac{2}{3x} = \frac{9}{6x} + \frac{4}{6x} = \frac{13}{6x}$

IV: waar,  $\frac{3}{2x} - \frac{2}{3x} = \frac{9}{6x} - \frac{4}{6x} = \frac{5}{6x}$

#### Opgave 11:

a.  $\frac{1}{2x} + \frac{2}{x} = \frac{1}{2x} + \frac{4}{2x} = \frac{5}{2x}$

b.  $\frac{3}{2a} - \frac{2}{3a} = \frac{9}{6a} - \frac{4}{6a} = \frac{5}{6a}$

c.  $\frac{1}{ab} - \frac{1}{b} = \frac{1}{ab} - \frac{a}{ab} = \frac{1-a}{ab}$

d.  $\frac{a}{b} - \frac{1}{a} = \frac{a^2}{ab} - \frac{b}{ab} = \frac{a^2-b}{ab}$

e.  $2 + \frac{1}{x} = \frac{2x}{x} + \frac{1}{x} = \frac{2x+1}{x}$

f.  $3a - \frac{2}{a} = \frac{3a^2}{a} - \frac{2}{a} = \frac{3a^2-2}{a}$

#### Opgave 12:

a.  $\frac{1}{x} + \frac{1}{x+2} = \frac{x+2}{x(x+2)} + \frac{x}{x(x+2)} = \frac{2x+2}{x(x+2)}$

b.  $\frac{1}{x+3} + \frac{1}{x+4} = \frac{x+4}{(x+3)(x+4)} + \frac{x+3}{(x+3)(x+4)} = \frac{2x+7}{(x+3)(x+4)}$

c.  $\frac{x}{x-2} - \frac{1}{x+2} = \frac{x(x+2)}{(x-2)(x+2)} - \frac{x-2}{(x-2)(x+2)} = \frac{x^2+2x}{(x-2)(x+2)} - \frac{x-2}{(x-2)(x+2)} = \frac{x^2+x+2}{(x-2)(x+2)}$

d.  $\frac{x+2}{x+3} - \frac{x}{x-2} = \frac{(x+2)(x-2)}{(x+3)(x-2)} - \frac{x(x+3)}{(x+3)(x-2)} = \frac{x^2-4}{(x+3)(x-2)} - \frac{x^2+3x}{(x+3)(x-2)} = \frac{-3x-4}{(x+3)(x-2)}$

e.  $\frac{2x}{x+2} - \frac{3x}{x+3} = \frac{2x(x+3)}{(x+2)(x+3)} - \frac{3x(x+2)}{(x+2)(x+3)} = \frac{2x^2+6x}{(x+2)(x+3)} - \frac{3x^2+6x}{(x+2)(x+3)} =$

$$f. \frac{-x^2}{(x+2)(x+3)} - \frac{x+2}{x+3} - \frac{x+3}{x+2} = \frac{(x+2)(x+2)}{(x+2)(x+3)} - \frac{(x+3)(x+3)}{(x+2)(x+3)} = \frac{x^2+4x+4}{(x+2)(x+3)} - \frac{x^2+6x+9}{(x+2)(x+3)} = \frac{-2x-5}{(x+2)(x+3)}$$

### **Opgave 13:**

$$a. \frac{1}{a} = b + \frac{1}{c} = \frac{bc}{c} + \frac{1}{c} = \frac{bc+1}{c}$$

$$a(bc+1) = 1 \cdot c$$

$$a = \frac{c}{bc+1}$$

$$b. \frac{1}{p} = 2q - \frac{1}{q} = \frac{2q^2}{q} - \frac{1}{q} = \frac{2q^2-1}{q}$$

$$p = \frac{q}{2q^2-1}$$

$$c. \frac{3}{y} = x - \frac{x}{x-1} = \frac{x(x-1)}{x-1} - \frac{x}{x-1} = \frac{x^2-x}{x-1} - \frac{x}{x-1} = \frac{x^2-2x}{x-1}$$

$$\frac{y}{3} = \frac{x-1}{x^2-2x}$$

$$y = \frac{3x-3}{x^2-2x}$$

### **Opgave 14:**

$$a. \frac{3}{x^2y} - \frac{2}{xy^2} = \frac{3y}{x^2y^2} - \frac{2x}{x^2y^2} = \frac{3y-2x}{x^2y^2}$$

$$b. 2x - \frac{x^2}{x+1} = \frac{2x(x+1)}{x+1} - \frac{x^2}{x+1} = \frac{2x^2+2x}{x+1} - \frac{x^2}{x+1} = \frac{x^2+2x}{x+1}$$

$$c. \frac{5a}{3b} - \frac{a}{b+1} = \frac{5a(b+1)}{3b(b+1)} - \frac{3ab}{3b(b+1)} = \frac{5ab+5a}{3b(b+1)} - \frac{3ab}{3b(b+1)} = \frac{2ab+5a}{3b(b+1)}$$

$$d. \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} = \frac{x^2}{x^3} + \frac{x}{x^3} + \frac{1}{x^3} = \frac{x^2+x+1}{x^3}$$

$$e. \frac{2x+1}{x+1} - \frac{x-1}{x+2} = \frac{(2x+1)(x+2)}{(x+1)(x+2)} - \frac{(x-1)(x+1)}{(x+1)(x+2)} = \frac{2x^2+5x+2}{(x+1)(x+2)} - \frac{x^2-1}{(x+1)(x+2)} =$$

$$\frac{x^2+5x+3}{(x+1)(x+2)}$$

$$f. \frac{p^2}{p+1} - \frac{p^3}{p+2} = \frac{p^2(p+2)}{(p+1)(p+2)} - \frac{p^3(p+1)}{(p+1)(p+2)} = \frac{p^3+2p^2}{(p+1)(p+2)} - \frac{p^4+p^3}{(p+1)(p+2)} =$$

$$\frac{-p^4+2p^2}{(p+1)(p+2)}$$

**Opgave 15:**

a.  $\frac{1}{2} + \frac{1}{x+1} = \frac{x+1}{x+4}$   
 $\frac{x+1}{2(x+1)} + \frac{2}{2(x+1)} = \frac{x+1}{x+4}$   
 $\frac{x+3}{2(x+1)} = \frac{x+1}{x+4}$   
 $2(x+1)(x+1) = (x+3)(x+4)$   
 $2(x^2 + 2x + 1) = x^2 + 7x + 12$   
 $2x^2 + 4x + 2 = x^2 + 7x + 12$   
 $x^2 - 3x - 10 = 0$   
 $(x+2)(x-5) = 0$   
 $x = -2 \quad \vee \quad x = 5$

b.  $x+1 + \frac{1}{x-1} = \frac{x}{x+3}$   
 $\frac{(x+1)(x-1)}{x-1} + \frac{1}{x-1} = \frac{x}{x+3}$   
 $\frac{x^2-1}{x-1} + \frac{1}{x-1} = \frac{x}{x+3}$   
 $\frac{x^2}{x-1} = \frac{x}{x+3}$   
 $x^2(x+3) = x(x-1)$   
 $x^3 + 3x^2 = x^2 - x$   
 $x^3 + 2x^2 + x = 0$   
 $x(x^2 + 2x + 1) = 0$   
 $x(x+1)(x+1) = 0$   
 $x = 0 \quad \vee \quad x = -1$

c.  $\frac{1}{x} + \frac{1}{x-3} = \frac{3}{x+1}$   
 $\frac{x-3}{x(x-3)} + \frac{x}{x(x-3)} = \frac{3}{x+1}$   
 $\frac{2x-3}{x(x-3)} = \frac{3}{x+1}$   
 $3x(x-3) = (2x-3)(x+1)$   
 $3x^2 - 9x = 2x^2 - x - 3$   
 $x^2 - 8x + 3 = 0$   
 $x = \frac{8 \pm \sqrt{64-12}}{2} = \frac{8 \pm \sqrt{52}}{2} = \frac{8 \pm 2\sqrt{13}}{2}$   
 $x = 4 + \sqrt{13} \quad \vee \quad x = 4 - \sqrt{13}$

**Opgave 16:**

I: waar,  $\frac{2x^2+1}{x} = \frac{2x^2}{x} + \frac{1}{x} = 2x + \frac{1}{x}$

II: niet waar,  $\frac{x^2-1}{x} = \frac{x^2}{x} - \frac{1}{x} = x - \frac{1}{x}$

III: waar,  $\frac{x^2-1}{x^2+2x+1} = \frac{(x-1)(x+1)}{(x+1)(x+1)} = \frac{x-1}{x+1}$

**Opgave 17:**

a.  $\frac{x^2-9}{x^2+6x+9} = \frac{(x-3)(x+3)}{(x+3)(x+3)} = \frac{x-3}{x+3}$

b.  $\frac{x^2-5x}{x^2-x-20} = \frac{x(x-5)}{(x+4)(x-5)} = \frac{x}{x+4}$

c.  $\frac{a^2-4a}{a^2+a} = \frac{a(a-4)}{a(a+1)} = \frac{a-4}{a+1}$

d.  $\frac{a^2-4a-5}{a^3+a^2} = \frac{(a-5)(a+1)}{a^2(a+1)} = \frac{a-5}{a^2}$

e.  $\frac{x^3-11x^2+30x}{x^2-10x+25} = \frac{x(x^2-11x+30)}{(x-5)(x-5)} = \frac{x(x-5)(x-6)}{(x-5)(x-5)} = \frac{x(x-6)}{x-5}$

f.  $\frac{x^2+6x+5}{2x+2} = \frac{(x+1)(x+5)}{2(x+1)} = \frac{x+5}{2} = \frac{1}{2}x + 2\frac{1}{2}$

**Opgave 18:**

a.  $A = \frac{p^2+p}{p^2-1} = \frac{p(p+1)}{(p-1)(p+1)} = \frac{p}{p-1}$

b.  $T = \frac{t^3+4t^2}{t^2-16} = \frac{t^2(t+4)}{(t-4)(t+4)} = \frac{t^2}{t-4}$

c.  $N = \frac{a^4+a^2-2}{a^4+3a^2+2} = \frac{(a^2+2)(a^2-1)}{(a^2+2)(a^2+1)} = \frac{a^2-1}{a^2+1}$

**Opgave 19:**

a.  $\frac{4x^2+7}{x} = \frac{4x^2}{x} + \frac{7}{x} = 4x + \frac{7}{x}$

b.  $\frac{a^2-2a+6}{2a} = \frac{a^2}{2a} - \frac{2a}{2a} + \frac{6}{2a} = \frac{1}{2}a - 1 + \frac{3}{a}$

c.  $\frac{p^3-3p^2+2}{2p} = \frac{p^3}{2p} - \frac{3p^2}{2p} + \frac{2}{2p} = \frac{1}{2}p^2 - 1\frac{1}{2}p + \frac{1}{p}$

**Opgave 20:**

a.  $F = \frac{a^2+2a-3}{a-1} + \frac{a^2+1}{a} = \frac{(a+3)(a-1)}{a-1} + \frac{a^2}{a} + \frac{1}{a} = a+3+a+\frac{1}{a} = 2a+3+\frac{1}{a}$

$$\text{b. } R = \frac{m^4 - 4}{m^4 + 2m^2} + \frac{m^2 + 6}{2m^2} = \frac{(m^2 + 2)(m^2 - 2)}{m^2(m^2 + 2)} + \frac{m^2}{2m^2} + \frac{6}{2m^2} = \frac{m^2 - 2}{m^2} + \frac{1}{2} + \frac{3}{m^2} =$$

$$\frac{m^2}{m^2} - \frac{2}{m^2} + \frac{1}{2} + \frac{3}{m^2} = 1 + \frac{1}{m^2} + \frac{1}{2} = 1\frac{1}{2} + \frac{1}{m^2}$$

$$\text{c. } H = \frac{c^3 + 4c^2 + 1}{2c^2} - \frac{c^2 - 5c + 6}{2c - 6} = \frac{c^3}{2c^2} + \frac{4c^2}{2c^2} + \frac{1}{2c^2} - \frac{(c-2)(c-3)}{2(c-3)} = \frac{1}{2}c + 2 + \frac{1}{2c^2} - \frac{c-2}{2}$$

$$= \frac{1}{2}c + 2 + \frac{1}{2c^2} - \frac{c}{2} + \frac{2}{2} = \frac{1}{2}c + 2 + \frac{1}{2c^2} - \frac{1}{2}c + 1 = 3 + \frac{1}{2c^2}$$

### **Opgave 21:**

$$\text{a. } \frac{x^2 + 4x + 4}{x^2 - 4} = \frac{10}{x - 2}$$

$$\frac{(x+2)(x+2)}{(x-2)(x+2)} = \frac{10}{x-2}$$

$$\frac{x+2}{x-2} = \frac{10}{x-2}$$

$$x+2 = 10$$

$$x = 8$$

$$\text{b. } \frac{x^2 - 9x + 14}{x^2 + x - 6} = \frac{3 - x}{2x - 6}$$

$$\frac{(x-7)(x-2)}{(x+3)(x-2)} = \frac{-(x-3)}{2(x-3)}$$

$$\frac{x-7}{x+3} = -\frac{1}{2}$$

$$2(x-7) = -(x+3)$$

$$2x - 14 = -x - 3$$

$$3x = 11$$

$$x = 3\frac{2}{3}$$

$$\text{c. } \frac{x^2 - 6}{x - 3} = \frac{x^2 - 4}{x^2 - x - 2}$$

$$\frac{x^2 - 6}{x - 3} = \frac{(x-2)(x+2)}{(x-2)(x+1)}$$

$$\frac{x^2 - 6}{x - 3} = \frac{x+2}{x+1}$$

$$(x^2 - 6)(x+1) = (x+2)(x-3)$$

$$x^3 + x^2 - 6x - 6 = x^2 - x - 6$$

$$x^3 - 5x = 0$$

$$x(x^2 - 5) = 0$$

$$x = 0 \quad \vee \quad x^2 = 5$$

$$x = 0 \quad \vee \quad x = -\sqrt{5} \quad \vee \quad x = \sqrt{5}$$