

HOOFDSTUK 4: Algebra en meetkunde.

4.3 Machten met gehele en gebroken exponenten

Opgave 22:

I: niet waar, $x^2 \cdot x^3 = x^5$

II: waar

III: niet waar, $(2x)^3 = 2^3 \cdot x^3 = 8x^3$

IV: waar

Opgave 23:

a. $2a^3 \cdot 4a^7 = 8a^{10}$

b. $2a^3 + 4a^7 - a^3 = a^3 + 4a^7$

c. $a^6 \cdot \frac{1}{a^4} = \frac{a^6}{a^4} = a^2$

d. $(3ab^2)^4 = 3^4 a^4 (b^2)^4 = 81a^4 b^8$

e. $(5a^3)^3 \cdot 2b^7 = 5^3 (a^3)^3 \cdot 2b^7 = 125a^9 \cdot 2b^7 = 250a^9 b^7$

f. $\frac{15a^{18}}{3a^6} = 5a^{12}$

g. $(-2a)^3 \cdot 3a^3 = (-2)^3 a^3 \cdot 3a^3 = -8a^3 \cdot 3a^3 = -24a^6$

h. $(-2a)^2 + 3a^2 = (-2)^2 a^2 + 3a^2 = 4a^2 + 3a^2 = 7a^2$

i. $\frac{1}{a^8} \cdot (a^3)^4 = \frac{1}{a^8} \cdot a^{12} = \frac{a^{12}}{a^8} = a^4$

Opgave 24:

a. $7a^3 + 5a^3 = 12a^3$

b. $7a^3 - a^3 = 6a^3$

c. $7a^5 : a^3 = 7a^2$

d. $7a^3 \cdot 5a^3 = 35a^6$

e. $(7a^3)^5 = 7^5 (a^3)^5 = 16807a^{15}$

f. $(7a)^3 + 5a^3 = 7^3 a^3 + 5a^3 = 343a^3 + 5a^3 = 348a^3$

g. $(2a)^2 + (\frac{1}{2}a)^2 = 2^2 a^2 + (\frac{1}{2})^2 a^2 = 4a^2 + \frac{1}{4}a^2 = 4\frac{1}{4}a^2$

h. $(3a)^2 - 8a^2 = 3^2 a^2 - 8a^2 = 9a^2 - 8a^2 = a^2$

i. $(\frac{1}{3}a)^3 - a^3 = (\frac{1}{3})^3 a^3 - a^3 = \frac{1}{27}a^3 - a^3 = -\frac{26}{27}a^3$

Opgave 25:

a. de grafiek van y_1 is hetzelfde als de grafiek van y_2 .

b. $y = \frac{1}{x}$

c. $x^0 = 1$

Opgave 26:

- a. $a^2 : \frac{1}{a^4} = a^2 : a^{-4} = a^6$
b. $a^8 : a^0 = a^8$
c. $(a^3)^{-2} = a^{-6}$
d. $\frac{a}{a^{12}} = a^{-11}$
e. $\frac{1}{a^5} : a = a^{-5} : a = a^{-6}$
f. $1 = a^0$

Opgave 27:

- a. $7^{-2} = \frac{1}{7^2} = \frac{1}{49}$
b. $(\frac{1}{3})^{-2} = (3^{-1})^{-2} = 3^2 = 9$
c. $3 \cdot 5^{-2} = \frac{3}{5^2} = \frac{3}{25}$
d. $(\frac{2}{5})^{-1} = \frac{2^{-1}}{5^{-1}} = \frac{5}{2} = 2\frac{1}{2}$
e. $4 \cdot 10^{-3} = 4 \cdot \frac{1}{10^3} = \frac{4}{1000} = \frac{1}{250}$
f. $\frac{1}{2} : 6^{-2} = \frac{2^{-1}}{6^{-2}} = \frac{6^2}{2^1} = \frac{36}{2} = 18$

Opgave 28:

- a. $6a^{-5} \cdot b^3 = \frac{6}{a^5} \cdot b^3 = \frac{6b^3}{a^5}$
b. $\frac{1}{3}a^{-3} = \frac{1}{3a^3}$
c. $3a^{-4} = \frac{3}{a^4}$
d. $(\frac{1}{2}a)^{-3} = (\frac{1}{2})^{-3} a^{-3} = (2^{-1})^{-3} a^{-3} = 2^3 a^{-3} = \frac{8}{a^3}$
e. $-4 \cdot (\frac{2}{3}a)^{-2} = -4 \cdot (\frac{2}{3})^{-2} a^{-2} = -4 \cdot \frac{2^{-2}}{3^{-2}} a^{-2} = -4 \cdot \frac{3^2}{2^2} a^{-2} = -4 \cdot \frac{9}{4} a^{-2} = -9a^{-2} = -\frac{9}{a^2}$
f. $(3a)^{-2} \cdot b^{-3} = 3^{-2} a^{-2} b^{-3} = \frac{1}{3^2 a^2 b^3} = \frac{1}{9a^2 b^3}$

Opgave 29:

- a. $y_1 = y_3$

Opgave 30:

- a. $5a^{\frac{1}{3}} = 5 \cdot \sqrt[3]{a}$
- b. $2a^{-\frac{1}{4}} \cdot b = \frac{2}{a^{\frac{1}{4}}} \cdot b = \frac{2b}{\sqrt[4]{a}}$
- c. $3a^{-\frac{2}{3}} = \frac{3}{a^{\frac{2}{3}}} = \frac{3}{\sqrt[3]{a^2}}$
- d. $a^{-3} \cdot b^{\frac{1}{3}} = \frac{1}{a^3} \cdot \sqrt[3]{b} = \frac{\sqrt[3]{b}}{a^3}$
- e. $\frac{1}{5}a^{-\frac{1}{2}} \cdot b^{\frac{1}{5}} = \frac{1}{5a^{\frac{1}{2}}} \cdot \sqrt[5]{b} = \frac{\sqrt[5]{b}}{5\sqrt{a}}$
- f. $(5a)^{-\frac{1}{2}} = \frac{1}{(5a)^{\frac{1}{2}}} = \frac{1}{\sqrt{5a}}$

Opgave 31:

- a. $a \cdot \sqrt[3]{a} = a \cdot a^{\frac{1}{3}} = a^{\frac{4}{3}}$
- b. $\frac{1}{\sqrt{a}} = \frac{1}{a^{\frac{1}{2}}} = a^{-\frac{1}{2}}$
- c. $\frac{1}{a\sqrt{a}} = \frac{1}{a \cdot a^{\frac{1}{2}}} = \frac{1}{a^{\frac{3}{2}}} = a^{-\frac{3}{2}}$
- d. $\frac{1}{\sqrt[4]{a^3}} = \frac{1}{a^{\frac{3}{4}}} = a^{-\frac{3}{4}}$
- e. $a^2 \cdot \sqrt{a} = a^2 \cdot a^{\frac{1}{2}} = a^{\frac{5}{2}}$
- f. $\sqrt[3]{\frac{1}{a^2}} = \sqrt[3]{a^{-2}} = a^{-\frac{2}{3}}$
- g. $\sqrt[3]{a^{12}} = a^{\frac{12}{3}} = a^4$
- h. $a^4 \cdot \sqrt[3]{a} = a^4 \cdot a^{\frac{1}{3}} = a^{\frac{13}{3}}$
- i. $\frac{a^3}{\sqrt[3]{a}} = \frac{a^3}{a^{\frac{1}{3}}} = a^{\frac{8}{3}}$

Opgave 32:

- a. $\frac{x^6}{x^2 \cdot \sqrt{x}} = \frac{x^6}{x^2 \cdot x^{\frac{1}{2}}} = \frac{x^6}{x^{\frac{5}{2}}} = x^{\frac{7}{2}}$
- b. $x \cdot \sqrt[7]{x^3} = x \cdot x^{\frac{3}{7}} = x^{\frac{10}{7}}$
- c. $\frac{x}{\sqrt[5]{x}} = \frac{x}{x^{\frac{1}{5}}} = x^{\frac{4}{5}}$
- d. $x^4 \cdot \sqrt{x} = x^4 \cdot x^{\frac{1}{2}} = x^{\frac{9}{2}}$
- e. $\frac{\sqrt[3]{x}}{\sqrt{x}} = \frac{x^{\frac{1}{3}}}{x^{\frac{1}{2}}} = x^{-\frac{1}{6}}$
- f. $\frac{1}{x^2} : \sqrt{x} = x^{-2} : x^{\frac{1}{2}} = x^{-\frac{5}{2}}$

g. $\sqrt[3]{x^2} \cdot \frac{1}{x^3} = x^{\frac{2}{3}} \cdot x^{-3} = x^{-2\frac{1}{3}}$

h. $x^5 \cdot \sqrt[3]{x^6} = x^5 \cdot x^{\frac{6}{3}} = x^5 \cdot x^2 = x^7$

i. $\frac{x^4 \cdot \sqrt[5]{x}}{x^5 \cdot \sqrt[4]{x}} = \frac{x^4 \cdot x^{\frac{1}{5}}}{x^5 \cdot x^{\frac{1}{4}}} = \frac{x^{\frac{4\frac{1}{5}}{5}}}{x^{\frac{5\frac{1}{4}}{4}}} = x^{-1\frac{1}{20}}$

Opgave 33:

a. $x^{1,6} = 50$
 $x = 50^{\frac{1}{1,6}} = 11,531$

b. $x^{-4,1} = 5$
 $x = 5^{\frac{1}{-4,1}} = 0,675$

c. $x^{-1,3} = 11$
 $x = 11^{\frac{1}{-1,3}} = 0,158$

d. $x^{-1} = 21$
 $x = 21^{\frac{1}{-1}} = 0,048$

e. $x^{0,55} = 18$
 $x = 18^{\frac{1}{0,55}} = 191,564$

f. $\sqrt[3]{x^2} = 28$
 $x^{\frac{2}{3}} = 28$
 $x = 28^{\frac{3}{2}} = 148,162$

Opgave 34:

a. $3x^{2,25} + 1 = 27$
 $3x^{2,25} = 26$
 $x^{2,25} = \frac{26}{3}$
 $x = \left(\frac{26}{3}\right)^{\frac{1}{2,25}} = 2,611$

b. $5x^{-1,3} + 8 = 21$
 $5x^{-1,3} = 13$
 $x^{-1,3} = 2,6$
 $x = 2,6^{\frac{1}{-1,3}} = 0,480$

c. $4x^{-1,8} + 16 = 5000$
 $4x^{-1,8} = 4984$
 $x^{-1,8} = 1246$
 $x = 1246^{\frac{1}{-1,8}} = 0,019$

d. $8 - 3x^{1,16} = 1$
 $-3x^{1,16} = -7$
 $x^{1,16} = \frac{7}{3}$
 $x = \left(\frac{7}{3}\right)^{\frac{1}{1,16}} = 2,076$

$$\begin{aligned} \text{e. } 5 \cdot \sqrt[3]{x} &= 8 \\ \sqrt[3]{x} &= 1,6 \\ x &= 1,6^3 = 4,096 \end{aligned}$$

$$\begin{aligned} \text{f. } 3 \cdot \sqrt[4]{x^3} - 1 &= 36 \\ 3 \cdot \sqrt[4]{x^3} &= 37 \\ \sqrt[4]{x^3} &= \frac{37}{3} \\ x^{\frac{3}{4}} &= \frac{37}{3} \\ x &= \left(\frac{37}{3}\right)^{\frac{4}{3}} = 28,495 \end{aligned}$$

Opgave 35:

$$\text{a. } P = 800 \cdot l^{-2,25} = \frac{800}{l^{2,25}}$$

Als l groter wordt, dan wordt $l^{2,25}$ groter, dus je deelt door een groter getal, dus P wordt kleiner.

$$\text{b. } P = 800 \cdot 0,9^{-2,25} = 1014 \text{ per km}^2$$

$$\begin{aligned} \text{c. } 800 \cdot l^{-2,25} &= 1350 \\ l^{-2,25} &= 1,6875 \end{aligned}$$

$$l = 1,6875^{-\frac{1}{2,25}} = 0,79 \text{ m}$$

$$\begin{aligned} \text{d. } P &= 800 \cdot 2,15^{-2,25} = 142,9 \\ 142,9 \cdot 250 &= 35731 \end{aligned}$$

$$\begin{aligned} \text{e. } \frac{160000}{5} &= 32000 \\ 800 \cdot l^{-2,25} &= 32000 \\ l^{-2,25} &= 40 \\ l &= 40^{-\frac{1}{2,25}} = 0,19 \text{ m} \end{aligned}$$

Opgave 36:

$$\begin{aligned} \text{a. } T &= a \cdot R^{1,5} \\ a &= \frac{T}{R^{1,5}} = \frac{1,9}{2,95^{1,5}} = 0,37 \end{aligned}$$

$$\text{b. } T = 0,37 \cdot 35,6^{1,5} = 79 \text{ dagen}$$

$$\begin{aligned} \text{c. } 0,37 \cdot R^{1,5} &= \frac{15}{24} \\ R^{1,5} &= 1,69 \\ R &= 1,69^{\frac{1}{1,5}} = 1,42 \text{ dus } 1,42 \cdot 10^5 \text{ km} \end{aligned}$$

$$\text{d. } \left(\frac{25}{11}\right)^{1,5} = 3,4 \text{ dus } 3,4 \text{ keer zo groot}$$

Opgave 37:

$$\begin{aligned} \text{a. } W &= a \cdot m^{0,75} \\ a &= \frac{W}{m^{0,75}} = \frac{6700}{40^{0,75}} = 421 \end{aligned}$$

b. $W = 421 \cdot 4^{0,75} = 1191 \text{ kJ}$

c. $421 \cdot m^{0,75} = 50000$

$$m^{0,75} = 118,8$$

$$m = 118,8^{\frac{1}{0,75}} = 584$$