

HOOFDSTUK 4: Algebra en meetkunde.

4.5 Lengte en oppervlakte

Opgave 64:

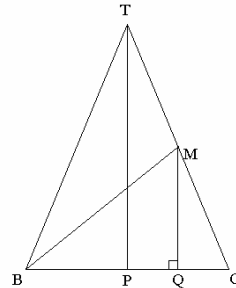
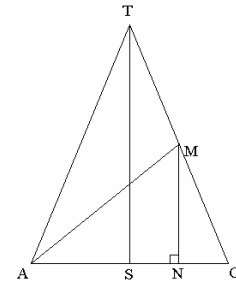
- a. $BE = \sqrt{AB^2 + AE^2} = \sqrt{6^2 + 6^2} = \sqrt{72} = 6\sqrt{2}$
 b. $BH = \sqrt{BE^2 + EH^2} = \sqrt{(6\sqrt{2})^2 + 6^2} = \sqrt{72 + 36} = \sqrt{108} = 6\sqrt{3}$

Opgave 65:

- a. $AF = \sqrt{AB^2 + BF^2} = \sqrt{(2a)^2 + a^2} = \sqrt{4a^2 + a^2} = \sqrt{5a^2} = a\sqrt{5}$
 b. $AG = \sqrt{AF^2 + FG^2} = \sqrt{(a\sqrt{5})^2 + a^2} = \sqrt{5a^2 + a^2} = \sqrt{6a^2} = a\sqrt{6}$
 c. $AC = \sqrt{AB^2 + BC^2} = \sqrt{(2a)^2 + a^2} = \sqrt{4a^2 + a^2} = \sqrt{5a^2} = a\sqrt{5}$
 $AM = \sqrt{AC^2 + CM^2} = \sqrt{(a\sqrt{5})^2 + (\frac{1}{2}a)^2} = \sqrt{5a^2 + \frac{1}{4}a^2} = \sqrt{5\frac{1}{4}a^2} = \frac{1}{2}a\sqrt{21}$

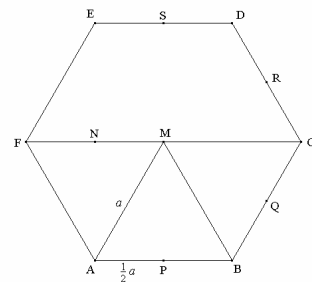
Opgave 66:

- a. $AC = a\sqrt{2}$ dus $AS = \frac{1}{2}a\sqrt{2}$
 $AT = \sqrt{AS^2 + TS^2} = \sqrt{(\frac{1}{2}a\sqrt{2})^2 + (2a)^2} = \sqrt{\frac{1}{2}a^2 + 4a^2} = \sqrt{4\frac{1}{2}a^2} = \sqrt{\frac{18}{4}a^2} = \frac{3}{2}a\sqrt{2}$
 b. $AN = \frac{3}{4}AC = \frac{3}{4}a\sqrt{2}$
 $MN = \frac{1}{2}TS = a$
 $AM = \sqrt{AN^2 + MN^2} = \sqrt{(\frac{3}{4}a\sqrt{2})^2 + a^2} = \sqrt{\frac{9}{8}a^2 + a^2} = \sqrt{\frac{17}{8}a^2} = \sqrt{\frac{34}{16}a^2} = \frac{1}{4}a\sqrt{34}$
 c. $TP = \sqrt{TS^2 + PS^2} = \sqrt{(2a)^2 + (\frac{1}{2}a)^2} = \sqrt{4a^2 + \frac{1}{4}a^2} = \sqrt{4\frac{1}{4}a^2} = \sqrt{\frac{17}{4}a^2} = \frac{1}{2}a\sqrt{17}$
 $QM = \frac{1}{2}TP = \frac{1}{4}a\sqrt{17}$
 $BQ = \frac{3}{4}BC = \frac{3}{4}a$
 $BM = \sqrt{BQ^2 + QM^2} = \sqrt{(\frac{3}{4}a)^2 + (\frac{1}{4}a\sqrt{17})^2} = \sqrt{\frac{9}{16}a^2 + \frac{17}{16}a^2} = \sqrt{\frac{26}{16}a^2} = \frac{1}{4}a\sqrt{26}$

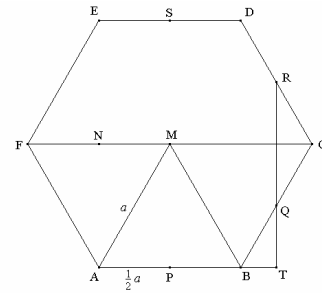


Opgave 67:

- a. $PM = \sqrt{AM^2 - AP^2} = \sqrt{a^2 - (\frac{1}{2}a)^2} = \sqrt{a^2 - \frac{1}{4}a^2} = \sqrt{\frac{3}{4}a^2} = \frac{1}{2}a\sqrt{3}$
 $PS = 2 \cdot PM = 2 \cdot \frac{1}{2}a\sqrt{3} = a\sqrt{3}$
 b. $AC = \sqrt{AN^2 + CN^2} = \sqrt{(\frac{1}{2}a\sqrt{3})^2 + (1\frac{1}{2}a)^2} = \sqrt{\frac{3}{4}a^2 + 2\frac{1}{4}a^2} = \sqrt{3a^2} = a\sqrt{3}$



$$\begin{aligned}
 \text{c. } AQ &= \sqrt{AT^2 + QT^2} = \sqrt{\left(1\frac{1}{4}a\right)^2 + \left(\frac{1}{4}a\sqrt{3}\right)^2} = \\
 &= \sqrt{\frac{25}{16}a^2 + \frac{3}{16}a^2} = \sqrt{\frac{28}{16}a^2} = \sqrt{\frac{7}{4}a^2} = \frac{1}{2}a\sqrt{7} \\
 AR &= \sqrt{AT^2 + RT^2} = \sqrt{\left(1\frac{1}{4}a\right)^2 + \left(\frac{3}{4}a\sqrt{3}\right)^2} = \\
 &= \sqrt{\frac{25}{16}a^2 + \frac{27}{16}a^2} = \sqrt{\frac{52}{16}a^2} = \sqrt{\frac{13}{4}a^2} = \frac{1}{2}a\sqrt{13} \\
 AS &= \sqrt{AP^2 + PS^2} = \sqrt{\left(\frac{1}{2}a\right)^2 + (a\sqrt{3})^2} = \sqrt{\frac{1}{4}a^2 + 3a^2} = \\
 &= \sqrt{3\frac{1}{4}a^2} = \sqrt{\frac{13}{4}a^2} = \frac{1}{2}a\sqrt{13}
 \end{aligned}$$



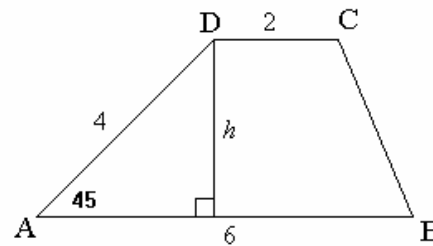
Opgave 68:

$$\sin 45^\circ = \frac{h}{4}$$

$$h = 4 \cdot \sin 45^\circ = 4 \cdot \frac{1}{2}\sqrt{2} = 2\sqrt{2}$$

$$\text{Opp}(\text{trapezium}) = \frac{1}{2} \cdot h \cdot (\text{basis}_1 + \text{basis}_2)$$

$$\text{Opp}(ABCD) = \frac{1}{2} \cdot 2\sqrt{2} \cdot (6 + 2) = 8\sqrt{2}$$



Opgave 69:

$$\text{a. } \tan 60^\circ = \frac{h}{AE} = \frac{4}{AE}$$

$$AE = \frac{4}{\tan 60^\circ} = \frac{4}{\sqrt{3}} = \frac{4}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{3} = \frac{4}{3}\sqrt{3}$$

$$BF = CF = h = 4$$

$$CD = EF = 10 - AE - BF = 10 - \frac{4}{3}\sqrt{3} - 4 = 6 - \frac{4}{3}\sqrt{3}$$

$$\text{Opp}(ABCD) = \frac{1}{2} \cdot 4 \cdot (10 + 6 - \frac{4}{3}\sqrt{3}) = 32 - \frac{8}{3}\sqrt{3}$$

$$\text{b. } AE = \frac{h}{\tan 60^\circ} = \frac{h}{\sqrt{3}} = \frac{h}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{h \cdot \sqrt{3}}{3} = \frac{1}{3}h \cdot \sqrt{3}$$

$$BF = h$$

$$AE + EF + FB = 10$$

$$\frac{1}{3}h \cdot \sqrt{3} + 2 + h = 10$$

$$h \cdot \left(\frac{1}{3}\sqrt{3} + 1\right) = 8$$

$$h = \frac{8}{\frac{1}{3}\sqrt{3} + 1} = \frac{8}{\frac{1}{3}\sqrt{3} + 1} \cdot \frac{\frac{1}{3}\sqrt{3} - 1}{\frac{1}{3}\sqrt{3} - 1} = \frac{\frac{8}{3}\sqrt{3} - 8}{\frac{1}{3} - 1} = \frac{\frac{8}{3}\sqrt{3} - 8}{-\frac{2}{3}} = -4\sqrt{3} + 12 = 12 - 4\sqrt{3}$$

$$\text{Opp}(ABCD) = \frac{1}{2} \cdot (12 - 4\sqrt{3}) \cdot (10 + 2) = 72 - 24\sqrt{3}$$

$$\text{c. } AE = \frac{1}{3}h \cdot \sqrt{3} \text{ en } BF = h$$

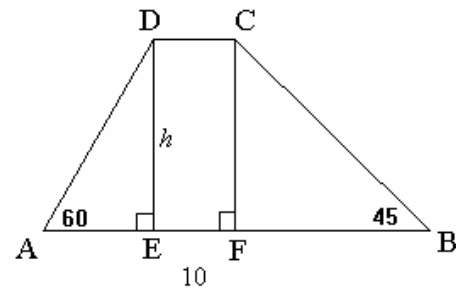
$$CD = 10 - \frac{1}{3}h \cdot \sqrt{3} - h$$

$$\text{Opp}(ABCD) = \frac{1}{2} \cdot h \cdot (10 + 10 - \frac{1}{3}h\sqrt{3} - h) = \frac{1}{2}h \cdot (20 - \frac{1}{3}h\sqrt{3} - h) = 25$$

$$y_1 = \frac{1}{2}x \cdot (20 - \frac{1}{3}x\sqrt{3} - x) \text{ en } y_2 = 25$$

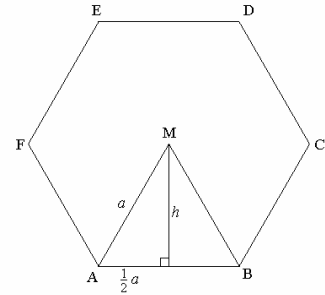
$$\text{calc-menu optie intersection geeft } x = 3,43 \quad \vee \quad x = 9,25$$

$$CD = 10 - \frac{1}{3} \cdot 3,43 \cdot \sqrt{3} - 3,43 = 4,60 \quad \vee \quad x = 10 - \frac{1}{3} \cdot 9,25 \cdot \sqrt{3} - 9,25 = -4,59 \text{ (vervalt)}$$



Opgave 70:

- a. $h = \sqrt{a^2 - (\frac{1}{2}a)^2} = \sqrt{a^2 - \frac{1}{4}a^2} = \sqrt{\frac{3}{4}a^2} = \frac{1}{2}a\sqrt{3}$
 $Opp(\triangle ABM) = \frac{1}{2} \cdot a \cdot \frac{1}{2}a\sqrt{3} = \frac{1}{4}a^2\sqrt{3}$
 $Opp(ABCDEF) = 6 \cdot Opp(\triangle ABM) = 6 \cdot \frac{1}{4}a^2\sqrt{3} = 1\frac{1}{2}a^2\sqrt{3}$
- b. $Opp(cirkel) = \pi \cdot r^2 = \pi \cdot (\frac{1}{2}a\sqrt{3})^2 = \pi \cdot \frac{3}{4}a^2 = \frac{3}{4}\pi \cdot a^2$
- c. $Opp = 1\frac{1}{2}a^2\sqrt{3} - \frac{3}{4}\pi \cdot a^2 = 10$
 $y_1 = 1\frac{1}{2}x^2\sqrt{3} - \frac{3}{4}\pi \cdot x^2$ en $y_2 = 10$
 calc-menu optie intersection geeft $x = 6,43$
 dus $a = 6,43$

**Opgave 71:**

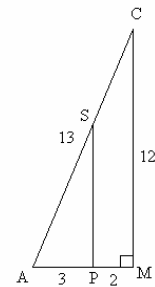
$$CM = \sqrt{13^2 - 5^2} = \sqrt{169 - 25} = \sqrt{144} = 12$$

snavefiguur dus $\frac{AP}{AM} = \frac{PS}{CM}$

$$\frac{3}{5} = \frac{PS}{12}$$

$$PS = \frac{3 \cdot 12}{5} = 7,2$$

$$Opp(PQRS) = 4 \cdot 7,2 = 28,8$$

**Opgave 72:**

a. $BC = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

snavefiguur dus $\frac{PQ}{AB} = \frac{CP}{AC} = \frac{CQ}{BC}$

$$\frac{PQ}{3} = \frac{x}{4} = \frac{CQ}{5}$$

$$PQ = \frac{3x}{4} = \frac{3}{4}x$$

$$CQ = \frac{5x}{4} = \frac{5}{4}x$$

$$AP = 4 - CP = 4 - x$$

$$Opp(\triangle BPQ) = Opp(\triangle ABC) - Opp(\triangle CPQ) - Opp(\triangle ABP) =$$

$$\frac{1}{2} \cdot 3 \cdot 4 - \frac{1}{2} \cdot x \cdot \frac{3}{4}x - \frac{1}{2} \cdot 3 \cdot (4 - x) = 6 - \frac{3}{8}x^2 - 6 + 1\frac{1}{2}x = 1\frac{1}{2}x - \frac{3}{8}x^2$$

b. $Opp' = 1\frac{1}{2} - \frac{3}{4}x = 0$

$$-\frac{3}{4}x = -1\frac{1}{2}$$

$$x = 2$$

$$Opp = 1\frac{1}{2} \cdot 2 - \frac{3}{8} \cdot 2^2 = 3 - 1\frac{1}{2} = 1\frac{1}{2}$$

Opgave 73:

a. $\tan 30^\circ = \frac{x}{AP}$

$$AP = \frac{x}{\tan 30^\circ} = \frac{x}{\frac{1}{3}\sqrt{3}} = \frac{x}{\frac{1}{3}\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} =$$

$$\frac{x\sqrt{3}}{1} = x\sqrt{3}$$

$$\cos 30^\circ = \frac{AK}{6}$$

$$AK = 6 \cdot \cos 30^\circ = 6 \cdot \frac{1}{2}\sqrt{3} = 3\sqrt{3}$$

$$AB = 2 \cdot 3\sqrt{3} + 6 = 6\sqrt{3} + 6$$

$$PQ = AB - 2 \cdot AP = 6\sqrt{3} + 6 - 2 \cdot x\sqrt{3}$$

$$Opp(PQRS) = PQ \cdot PS = (6\sqrt{3} + 6 - 2x\sqrt{3}) \cdot x$$

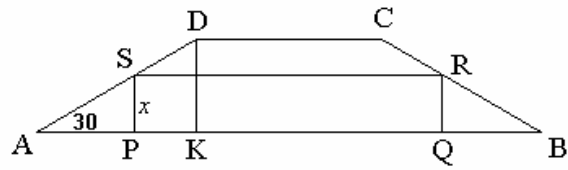
b. $Opp = (6\sqrt{3} + 6 - 2x\sqrt{3}) \cdot x = 6x\sqrt{3} + 6x - 2x^2\sqrt{3}$

$$Opp' = 6\sqrt{3} + 6 - 4x\sqrt{3} = 0$$

$$-4x\sqrt{3} = -6\sqrt{3} - 6$$

$$x\sqrt{3} = 1\frac{1}{2}\sqrt{3} + 1\frac{1}{2}$$

$$x = 1\frac{1}{2} + \frac{1\frac{1}{2}}{\sqrt{3}} = 1\frac{1}{2} + \frac{1\frac{1}{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = 1\frac{1}{2} + \frac{1\frac{1}{2}\sqrt{3}}{3} = 1\frac{1}{2} + \frac{1}{2}\sqrt{3} = PS$$

**Opgave 74:**

a. $AE = DE = CF = BF = h$

$$\cos 45^\circ = \frac{h}{AD}$$

$$AD = \frac{h}{\cos 45^\circ} = \frac{h}{\frac{1}{2}\sqrt{2}} = \frac{h}{\frac{1}{2}\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{h\sqrt{2}}{1} = h\sqrt{2}$$

$$Omtrek = 2h + 2h\sqrt{2} + 2x = 60$$

$$2x = 60 - 2h - 2h\sqrt{2}$$

$$x = 30 - h - h\sqrt{2}$$

b. $Opp(ABCD) = \frac{1}{2}h \cdot (2h + x + x) = \frac{1}{2}h \cdot (2h + 2x) = \frac{1}{2}h \cdot (2h + 60 - 2h - 2h\sqrt{2}) =$
 $\frac{1}{2}h \cdot (60 - 2h\sqrt{2}) = 30h - h^2\sqrt{2} = 0$

$$Opp' = 30 - 2h\sqrt{2} = 0$$

$$-2h\sqrt{2} = -30$$

$$h = \frac{-30}{-2\sqrt{2}} = \frac{-30}{-2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{-30\sqrt{2}}{-4} = 7\frac{1}{2}\sqrt{2}$$

$$Opp(ABCD) = 30 \cdot 7\frac{1}{2}\sqrt{2} - (7\frac{1}{2}\sqrt{2})^2 \cdot \sqrt{2} = 225\sqrt{2} - 112\frac{1}{2}\sqrt{2} = 112\frac{1}{2}\sqrt{2}$$

Opgave 75:

$$MR = 6$$

$$RS = \sqrt{MR^2 - MS^2} = \sqrt{6^2 - x^2} = \sqrt{36 - x^2}$$

$$Opp(\Delta PQR) = \frac{1}{2} \cdot PQ \cdot PS = \frac{1}{2} \cdot 2\sqrt{36 - x^2} \cdot (6 + x)$$

$$y_1 = (6 + x) \cdot \sqrt{36 - x^2}$$

calc-menu optie maximum geeft $Opp = 46,77$