

Gemengde opgaven hoofdstuk 5 Exponenten en logaritmen

Opgave 1:

a. $y = 2^x \xrightarrow{V_{x-0,3}} y = 3 \cdot 2^x \xrightarrow{T(0,-2)} y = 3 \cdot 2^x - 2$

$y = 2^x \xrightarrow{T(3,1)} y = 2^{x-3} + 1$

b. $f: \text{H.A.: } y = -2$ $g: \text{H.A.: } y = 1$

$B_f = \langle -2, \rightarrow \rangle$ $B_g = \langle 1, \rightarrow \rangle$

c. $y_1 = 3 \cdot 2^x - 2$ en $y_2 = 2^{x-3} + 1$
intersect geeft $(0,06;1,13)$

d. $3 \cdot 2^x - 2 = -\frac{1}{2}$

$3 \cdot 2^x = 1\frac{1}{2}$

$2^x = \frac{1}{2}$

$2^x = 2^{-1}$

$x = -1$

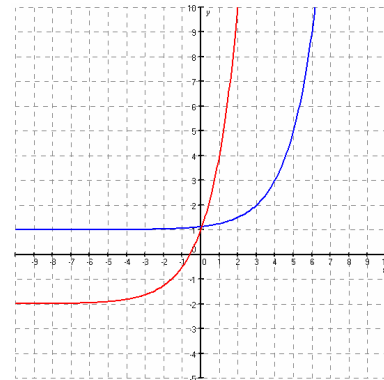
e. $g(7) = 2^4 + 1 = 17$

dus $1 < g(x) \leq 17$

f. $y_1 = 3 \cdot 2^x - 2$ en $y_3 = 9$ intersect geeft $x = 1,874$

$y_2 = 2^{x-3} + 1$ en $y_3 = 9$ intersect geeft $x = 6$

dus $AB = 6 - 1,873 = 4,13$



Opgave 2:

a. $(\frac{1}{2})^{x-5} - 1 = -\frac{3}{4}$

$(\frac{1}{2})^{x-1} = \frac{1}{4}$

$(\frac{1}{2})^{x-5} = (\frac{1}{2})^2$

$x - 5 = 2$

$x = 7$

b. $f(2) = 3 \cdot 2^0 - 3 = 0$

$B_f = \langle -3, \rightarrow \rangle$

dus $-3 < f(x) \leq 0$

c. $f(-1) = -2\frac{5}{8}$

$g(-1) = 63$

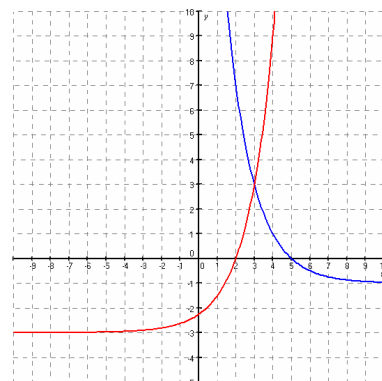
$AB = 63 - -2\frac{5}{8} = 65\frac{5}{8}$

d. $y_1 = 3 \cdot 2^{x-2} - 3$ en $y_3 = 4$ intersect geeft $x = 3,222$

$y_2 = 0,5^{x-5} - 1$ en $y_3 = 4$ intersect geeft $x = 2,678$

$PQ = 3,222 - 2,678 = 0,54$

e. $-3 < p \leq -1$



Opgave 3:

a. $30 - 3^{3x+1} = 3$

$$-3^{3x+1} = -27$$

$$3^{3x+1} = 27$$

$$3^{3x+1} = 3^3$$

$$3x + 1 = 3$$

$$3x = 2$$

$$x = \frac{2}{3}$$

b. $5 \cdot 3^{2x} = 15 \cdot \sqrt[4]{3}$

$$3^{2x} = 3 \cdot \sqrt[4]{3}$$

$$3^{2x} = 3^1 \cdot 3^{\frac{1}{4}}$$

$$3^{2x} = 3^{1\frac{1}{4}}$$

$$x = \frac{5}{8}$$

c. $4 \cdot {}^3\log(3x - 5) = 20$

$${}^3\log(3x - 5) = 5$$

$$3x - 5 = 3^5$$

$$3x - 5 = 243$$

$$3x = 248$$

$$x = 82\frac{2}{3}$$

d. $6^{-0,5}\log 3x = 8$

$$-0,5\log 3x = 2$$

$$0,5\log 3x = -2$$

$$3x = 0,5^{-2}$$

$$3x = 4$$

$$x = 1\frac{1}{3}$$

e. $2^{x^2-2} = 32$

$$2^{x^2-2} = 2^5$$

$$x^2 - 2 = 5$$

$$x^2 = 7$$

$$x = \sqrt{7} \quad \vee \quad x = -\sqrt{7}$$

f. $2 + 3^{\frac{1}{2}}\log(6x + 1) = -4$

$$3^{\frac{1}{2}}\log(6x + 1) = -6$$

$$\frac{1}{2}\log(6x + 1) = -2$$

$$6x + 1 = \left(\frac{1}{2}\right)^{-2}$$

$$6x + 1 = 4$$

$$6x = 3$$

$$x = \frac{1}{2}$$

g. $2 \cdot \left(\frac{1}{3}\right)^{x-1} + 5 = 59$

$$2 \cdot \left(\frac{1}{3}\right)^{x-1} = 54$$

$$\left(\frac{1}{3}\right)^{x-1} = 27$$

$$(3^{-1})^{x-1} = 3^3$$

$$3^{-x+1} = 3^3$$

$$-x + 1 = 3$$

$$-x = 2$$

$$x = -2$$

h. $4^{3x+1} = \frac{1}{8}\sqrt{2}$

$$(2^2)^{3x+1} = \frac{1}{2^3} \cdot 2^{\frac{1}{2}}$$

$$2^{6x+2} = 2^{-3} \cdot 2^{\frac{1}{2}}$$

$$2^{6x+2} = 2^{-2\frac{1}{2}}$$

$$6x + 2 = -2\frac{1}{2}$$

$$6x = -4\frac{1}{2}$$

$$x = -\frac{3}{4}$$

Opgave 4:

a. $5^{1-3x} = \frac{1}{5} \cdot \sqrt[3]{25}$

$$5^{1-3x} = 5^{-1} \cdot \sqrt[3]{5^2}$$

$$5^{1-3x} = 5^{-1} \cdot 5^{\frac{2}{3}}$$

$$5^{1-3x} = 5^{-\frac{1}{3}}$$

$$1 - 3x = -\frac{1}{3}$$

$$-3x = -1\frac{1}{3}$$

$$x = \frac{4}{9}$$

b. $4^{3x-x^2} = (\frac{1}{2})^{3-x}$

$$(2^2)^{3x-x^2} = (2^{-1})^{3-x}$$

$$2^{6x-2x^2} = 2^{-3+x}$$

$$6x - 2x^2 = -3 + x$$

$$-2x^2 + 5x + 3 = 0$$

$$x = \frac{-5 \pm \sqrt{25+24}}{-4} = \frac{-5 \pm 7}{-4}$$

$$x = \frac{-5-7}{-4} = 3 \quad \vee \quad x = \frac{-5+7}{-4} = -\frac{1}{2}$$

c. $3^{x-3} + 3^{x-4} = \frac{4}{3}\sqrt{3}$

$$3^{-3} \cdot 3^x + 3^{-4} \cdot 3^x = \frac{4}{3}\sqrt{3}$$

$$\frac{1}{27} \cdot 3^x + \frac{1}{81} \cdot 3^x = \frac{4}{3}\sqrt{3}$$

$$\frac{4}{81} \cdot 3^x = \frac{4}{3}\sqrt{3}$$

$$3^x = 27\sqrt{3}$$

$$3^x = 3^3 \cdot 3^{\frac{1}{2}}$$

$$3^x = 3^{3\frac{1}{2}}$$

$$x = 3\frac{1}{2}$$

d. $3^{-2}\log(x-5) = 1$

$$-2\log(x-5) = -2$$

$${}^2\log(x-5) = 2$$

$$x-5 = 2^2$$

$$x-5 = 4$$

$$x = 9$$

e. $(\frac{1}{3})^{x+2} = 9^{2x-5}$

$$(3^{-1})^{x+2} = (3^2)^{2x-5}$$

$$3^{-x-2} = 3^{4x-10}$$

$$-x-2 = 4x-10$$

$$-5x = -8$$

$$x = 1\frac{3}{5}$$

f. $2^{x+2} - 2^{x-1} = 14\sqrt{2}$

$$2^2 \cdot 2^x - 2^{-1} \cdot 2^x = 14\sqrt{2}$$

$$4 \cdot 2^x - \frac{1}{2} \cdot 2^x = 14\sqrt{2}$$

$$3\frac{1}{2} \cdot 2^x = 14\sqrt{2}$$

$$2^x = 4\sqrt{2}$$

$$2^x = 2^2 \cdot 2^{\frac{1}{2}}$$

$$2^x = 2^{2\frac{1}{2}}$$

$$x = 2\frac{1}{2}$$

g. $(\frac{1}{2})^{-x+2} + 2^{x+3} = 4\frac{1}{8}$

$$(2^{-1})^{-x+2} + 2^{x+3} = 4\frac{1}{8}$$

$$2^{x-2} + 2^{x+3} = 4\frac{1}{8}$$

$$2^{-2} \cdot 2^x + 2^3 \cdot 2^x = 4\frac{1}{8}$$

$$\frac{1}{4} \cdot 2^x + 8 \cdot 2^x = 4\frac{1}{8}$$

$$8\frac{1}{4} \cdot 2^x = 4\frac{1}{8}$$

$$2^x = \frac{1}{2}$$

$$2^x = 2^{-1}$$

$$x = -1$$

h. $5 - 3^{\frac{1}{3}} \log x^2 = -1$

$$-3^{\frac{1}{3}} \log x^2 = -6$$

$$\frac{1}{3} \log x^2 = 2$$

$$x^2 = (\frac{1}{3})^2$$

$$x^2 = \frac{1}{9}$$

$$x = \frac{1}{3} \quad \vee \quad x = -\frac{1}{3}$$

Opgave 5:

a. $g_{\text{jaar}} = 1,096$

$$g^{10} = 1,096^{10} = 2,50 \text{ dus een toename van } 150\%$$

b. $g_{\text{maand}} = 1,096^{\frac{1}{12}} = 1,008 \text{ dus een toename van } 0,8\%$

c. $1,096^t = 2$

$$t = \frac{\log 2}{\log 1,096} = 7,56 \text{ jaar , dus 7 jaar en 7 maanden}$$

d. $1,096^t = 10$

$$t = \frac{\log 10}{\log 1,096} = 25,1 \text{ dus 25 jaar}$$

Opgave 6:

a. $g_{dag} = 0,83$

$$g_{week} = 0,83^7 = 0,271 \text{ dus een afname van } 72,9\%$$

b. $g_{uur} = 0,83^{\frac{1}{24}} = 0,992 \text{ dus een afname van } 0,8\%$

c. $0,83^t = 0,5$

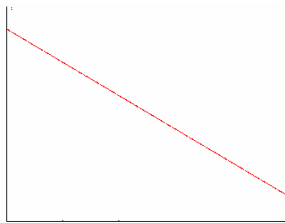
$$t = \frac{\log 0,5}{\log 0,83} = 3,72 \text{ dagen , dus 3 dagen en 17 uur}$$

d. $0,83^t = 0,25$

$$t = \frac{\log 0,25}{\log 0,83} = 7,44 \text{ dagen , dus 7 dagen en 11 uur}$$

Opgave 7:

a.



b. $g^{4,750} = \frac{567}{1013} = 0,56$

$$g = 0,885$$

$$P = 1013 \cdot 0,885^h$$

c. $0,885^{0,2} = 0,976 \text{ dus een afname van } 2,4\%$

d. $P = 1013 \cdot 0,885^{7,5} = 405$

Opgave 8:

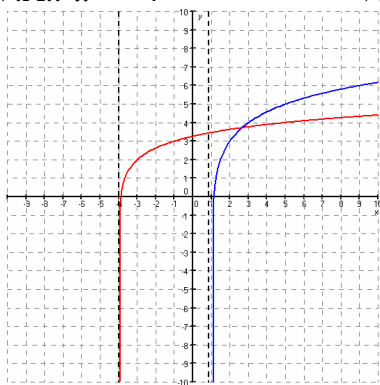
a. $D_f = \langle -4, \rightarrow \rangle$

$$D_g = \langle 1, \rightarrow \rangle$$

V.A.: $x = -4$

V.A.: $x = 1$

b.



c. $y_1 = 2 + \log(x+4)/\log(3)$ en $y_2 = 3 + \log(x-1)/\log(2)$

intersect geeft $x = 2,65$

dus $1 < x \leq 2,65$

- d. $2+^3\log(x+4) \leq 5$
 $^3\log(x+4) = 3$
 $x+4 = 3^3$
 $x+4 = 27$
 $x = 23$
dus $-4 < x \leq 23$
- e. $f(6) = 4,096$
 $g(6) = 5,322$
 $AB = 5,322 - 4,096 = 1,23$
- f. $2+^3\log(x+4) = 2$ $3+^2\log(x-1) = 2$
 $^3\log(x+4) = 0$ $^2\log(x-1) = -1$
 $x+4 = 3^0$ $x-1 = 2^{-1}$
 $x+4 = 1$ $x-1 = \frac{1}{2}$
 $x = -3$ $x = 1\frac{1}{2}$
 $PQ = 1\frac{1}{2} - (-3) = 4\frac{1}{2}$

Opgave 9:

- a. $N = 1 \cdot 1,05^{20} \cdot 0,92^{11} = 1,06$ miljoen
- b. $1,05^{20} = 2,653$
 $2,653 \cdot g^{11} = 1$
 $g^{11} = 0,377$
 $g = 0,915$ dus een afname van 8,5%
- c. $1,05^t \cdot 0,9^{31-t} = 1$
 $y_1 = 1,05^x \cdot 0,9^{31-x}$ en $y_2 = 1$
intersect geeft $x = 21,2$ dus op 22 mei

Opgave 10:

- a. $0,7^5 = 0,168$ dus 16,8%
- b. $0,7^{10} = 0,028$ dus 97,2% wordt geabsorbeerd
- c. $0,7^d = 0,01$
 $d = \frac{\log 0,01}{\log 0,7} = 12,9$ dus minstens 13 mm

Opgave 11:

- a. $g^7 = 0,3$
 $g = \sqrt[7]{0,3} = 0,842$
- b. $0,842^t = 0,6$
 $t = \frac{\log 0,6}{\log 0,842} = 2,97$ dagen, dus 71 uur
- c. $M = 500 \cdot 0,842^t$
 $\left[\frac{dM}{dt}\right]_{t=2} = -61 \frac{\text{mg}}{\text{dag}} = -2,5 \frac{\text{mg}}{\text{uur}}$
- d. $M(7) = 500 \cdot 0,842^7 = 150$
 $M(10) = 650 \cdot 0,842^3 = 388$ mg

e. $M(14) = 650 \cdot 0,842^7 = 195$
 $M = 195 \cdot 0,842^{t-14}$

Opgave 12:

a. $a \cdot \log 19 = 100$

$$a = \frac{100}{\log 19} = 78,201$$

b. $78 \cdot \log(x+1) = 75$

$$\log(x+1) = 0,962$$

$$x+1 = 10^{0,962} = 9,2$$

$$x = 8,2$$

c. bij de stand $-1,3$ hoort $x = \frac{1,7}{6} \cdot 18 = 5,1$

$$P = 78 \cdot \log 6,1 = 61$$