

Gemengde opgaven H6: Goniometrische formules

Opgave 13:

$$x_A = \cos 40^\circ = 0,766 \quad y_A = \sin 40^\circ = 0,643 \quad \text{dus } A = (0,766; 0,643)$$

$$x_B = \cos 160^\circ = -0,940 \quad y_B = \sin 160^\circ = 0,342 \quad \text{dus } B = (-0,940; 0,342)$$

$$x_C = \cos 280^\circ = 0,174 \quad y_C = \sin 280^\circ = -0,985 \quad \text{dus } C = (0,174; -0,985)$$

Opgave 14:

a. $x_A = \cos \frac{2}{3}\pi = -\frac{1}{2} \quad y_A = \sin \frac{2}{3}\pi = \frac{1}{2}\sqrt{3} \quad \text{dus } A = (-\frac{1}{2}, \frac{1}{2}\sqrt{3})$

b. $x_C = \cos(-\frac{1}{6}\pi) = \frac{1}{2}\sqrt{3} \quad y_C = \sin(-\frac{1}{6}\pi) = -\frac{1}{2} \quad \text{dus } C = (\frac{1}{2}\sqrt{3}, -\frac{1}{2})$

c. $x_B = -\frac{1}{2}\sqrt{2}$ en B ligt in het derde kwadrant, dus $\beta = -\frac{3}{4}\pi$

d. cirkelboog $BC = \frac{1}{6}\pi + 1\frac{1}{4}\pi = 1\frac{5}{12}\pi$

Opgave 15:

a. bij $t = 2$ hoort $\alpha = \frac{2}{12} \cdot 2\pi = \frac{1}{3}\pi$

$$x_P = \cos \frac{1}{3}\pi = \frac{1}{2} \quad y_P = \sin \frac{1}{3}\pi = \frac{1}{2}\sqrt{3} \quad \text{dus } P_{t=2} = (\frac{1}{2}, \frac{1}{2}\sqrt{3})$$

bij $t = 7\frac{1}{2}$ hoort $\alpha = \frac{7\frac{1}{2}}{12} \cdot 2\pi = 1\frac{1}{4}\pi$

$$x_P = \cos 1\frac{1}{4}\pi = -\cos \frac{1}{4}\pi = -\frac{1}{2}\sqrt{2} \quad y_P = \sin 1\frac{1}{4}\pi = -\sin \frac{1}{4}\pi = -\frac{1}{2}\sqrt{2}$$

$$\text{dus } P_{t=7\frac{1}{2}} = (-\frac{1}{2}\sqrt{2}, -\frac{1}{2}\sqrt{2})$$

bij $t = 11$ hoort $\alpha = \frac{11}{12} \cdot 2\pi = 1\frac{5}{6}\pi$

$$x_P = \cos 1\frac{5}{6}\pi = \cos \frac{1}{6}\pi = \frac{1}{2}\sqrt{3} \quad y_P = \sin 1\frac{5}{6}\pi = -\sin \frac{1}{6}\pi = -\frac{1}{2} \quad \text{dus } P_{t=11} = (\frac{1}{2}\sqrt{3}, -\frac{1}{2})$$

b. $\cos \alpha = -\frac{1}{2}$

$$\alpha = \frac{2}{3}\pi \quad \vee \quad \alpha = 1\frac{1}{3}\pi$$

$$t = 4 \quad \vee \quad t = 8$$

Opgave 16:

a. $\cos(3x - \frac{1}{2}\pi) = \frac{1}{2}\sqrt{2}$

$$3x - \frac{1}{2}\pi = \frac{1}{4}\pi + k \cdot 2\pi \quad \vee \quad 3x - \frac{1}{2}\pi = -\frac{1}{4}\pi + k \cdot 2\pi$$

$$3x = \frac{3}{4}\pi + k \cdot 2\pi \quad \vee \quad 3x = \frac{1}{4}\pi + k \cdot 2\pi$$

$$x = \frac{1}{4}\pi + k \cdot \frac{2}{3}\pi \quad \vee \quad x = \frac{1}{12}\pi + k \cdot \frac{2}{3}\pi$$

b. $\sin(\frac{1}{3}x + \frac{1}{4}\pi) = -\frac{1}{2}$

$$\frac{1}{3}x + \frac{1}{4}\pi = 1\frac{1}{6}\pi + k \cdot 2\pi \quad \vee \quad \frac{1}{3}x + \frac{1}{4}\pi = -\frac{1}{6}\pi + k \cdot 2\pi$$

$$\frac{1}{3}x = \frac{11}{12}\pi + k \cdot 2\pi \quad \vee \quad \frac{1}{3}x = -\frac{5}{12}\pi + k \cdot 2\pi$$

$$x = 2\frac{3}{4}\pi + k \cdot 6\pi \quad \vee \quad x = -1\frac{1}{4}\pi + k \cdot 6\pi$$

c. $\sin(\frac{1}{2}x - \frac{1}{3}\pi) \cdot \cos 2x = 0$

$$\sin(\frac{1}{2}x - \frac{1}{3}\pi) = 0 \quad \vee \quad \cos 2x = 0$$

$$\frac{1}{2}x - \frac{1}{3}\pi = 0 + k \cdot \pi \quad \vee \quad 2x = \frac{1}{2}\pi + k \cdot \pi$$

$$\frac{1}{2}x = \frac{1}{3}\pi + k \cdot \pi \quad \vee \quad x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$$

$$x = \frac{2}{3}\pi + k \cdot 2\pi \quad \vee \quad x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$$

d. $4 \cos^2(2\pi x - \frac{1}{2}\pi) = 3$
 $\cos^2(2\pi x - \frac{1}{2}\pi) = \frac{3}{4}$
 $\cos(2\pi x - \frac{1}{2}\pi) = \sqrt{\frac{3}{4}} = \frac{1}{2}\sqrt{3} \quad \vee \quad \cos(2\pi x - \frac{1}{2}\pi) = -\frac{1}{2}\sqrt{3}$
 $2\pi x - \frac{1}{2}\pi = \frac{1}{6}\pi + k \cdot 2\pi \quad \vee \quad 2\pi x - \frac{1}{2}\pi = -\frac{1}{6}\pi + k \cdot 2\pi$
 $\vee \quad 2\pi x - \frac{1}{2}\pi = \frac{5}{6}\pi + k \cdot 2\pi \quad \vee \quad 2\pi x - \frac{1}{2}\pi = -\frac{5}{6}\pi + k \cdot 2\pi$
 $2\pi x = \frac{2}{3}\pi + k \cdot 2\pi \quad \vee \quad 2\pi x = \frac{1}{3}\pi + k \cdot 2\pi \quad \vee \quad 2\pi x = 1\frac{1}{3}\pi + k \cdot 2\pi$
 $\vee \quad 2\pi x = -\frac{1}{3}\pi + k \cdot 2\pi$
 $x = \frac{1}{3} + k \cdot 1 \quad \vee \quad x = \frac{1}{6} + k \cdot 1 \quad \vee \quad x = \frac{2}{3} + k \cdot 1 \quad \vee \quad x = -\frac{1}{6} + k \cdot 1$

Opgave 17:

a. $\cos(2x - \frac{1}{2}\pi) = \cos(\pi - x)$
 $2x - \frac{1}{2}\pi = \pi - x + k \cdot 2\pi \quad \vee \quad 2x - \frac{1}{2}\pi = -(\pi - x) + k \cdot 2\pi$
 $3x = 1\frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad 2x - \frac{1}{2}\pi = -\pi + x + k \cdot 2\pi$
 $x = \frac{1}{2}\pi + k \cdot \frac{2}{3}\pi \quad \vee \quad x = -\frac{1}{2}\pi + k \cdot 2\pi$

b. $\sin(2x + \frac{1}{3}\pi) = \sin(x - \frac{1}{2}\pi)$
 $2x + \frac{1}{3}\pi = x - \frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad 2x + \frac{1}{3}\pi = \pi - (x - \frac{1}{2}\pi) + k \cdot 2\pi$
 $x = -\frac{5}{6}\pi + k \cdot 2\pi \quad \vee \quad 2x + \frac{1}{3}\pi = \pi - x + \frac{1}{2}\pi + k \cdot 2\pi$
 $x = -\frac{5}{6}\pi + k \cdot 2\pi \quad \vee \quad 3x = 1\frac{1}{6}\pi + k \cdot 2\pi$
 $x = -\frac{5}{6}\pi + k \cdot 2\pi \quad \vee \quad x = \frac{7}{18}\pi + k \cdot \frac{2}{3}\pi$

c. $\sin(\pi x) = \sin(2\pi x)$
 $\pi x = 2\pi x + k \cdot 2\pi \quad \vee \quad \pi x = \pi - 2\pi x + k \cdot 2\pi$
 $-\pi x = 0 + k \cdot 2\pi \quad \vee \quad 3\pi x = \pi + k \cdot 2\pi$
 $x = 0 + k \cdot 2 \quad \vee \quad x = \frac{1}{3} + k \cdot \frac{2}{3}$

d. $\cos(10\pi x) = \cos(5\pi x - 6\pi)$
 $10\pi x = 5\pi x - 6\pi + k \cdot 2\pi \quad \vee \quad 10\pi x = -5\pi x + 6\pi + k \cdot 2\pi$
 $5\pi x = -6\pi + k \cdot 2\pi \quad \vee \quad 15\pi x = 6\pi + k \cdot 2\pi$
 $x = -1\frac{1}{5} + k \cdot \frac{2}{5} \quad \vee \quad x = \frac{2}{5} + k \cdot \frac{2}{15}$

Opgave 18:

a. $\sin(1\frac{1}{2}x - \frac{1}{6}\pi) = \frac{1}{2}\sqrt{3}$
 $1\frac{1}{2}x - \frac{1}{6}\pi = \frac{1}{3}\pi + k \cdot 2\pi \quad \vee \quad 1\frac{1}{2}x - \frac{1}{6}\pi = \frac{2}{3}\pi + k \cdot 2\pi$
 $1\frac{1}{2}x = \frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad 1\frac{1}{2}x = \frac{5}{6}\pi + k \cdot 2\pi$
 $x = \frac{1}{3}\pi + k \cdot \frac{4}{3}\pi \quad \vee \quad x = \frac{5}{9}\pi + k \cdot \frac{4}{3}\pi$
 $x = \frac{1}{3}\pi \quad \vee \quad x = \frac{5}{9}\pi \quad \vee \quad x = 1\frac{2}{3}\pi \quad \vee \quad x = 1\frac{8}{9}\pi$

b. $\cos^3(2\frac{1}{2}x) + \cos(2\frac{1}{2}x) = 0$
 $\cos(2\frac{1}{2}x) \cdot (\cos^2(2\frac{1}{2}x) + 1) = 0$
 $\cos(2\frac{1}{2}x) = 0 \quad \vee \quad \cos^2(2\frac{1}{2}x) = -1$
 $2\frac{1}{2}x = \frac{1}{2}\pi + k \cdot \pi$
 $x = \frac{1}{5}\pi + k \cdot \frac{2}{5}\pi$

$$x = \frac{1}{5}\pi \quad \vee \quad x = \frac{3}{5}\pi \quad \vee \quad x = \pi \quad \vee \quad x = 1\frac{2}{5}\pi \quad \vee \quad x = 1\frac{4}{5}\pi$$

c. $\sin^2(1,5x) = \sin(1,5x) + 2$

$$\sin^2(1,5x) - \sin(1,5x) - 2 = 0$$

$$(\sin(1,5x) - 2)(\sin(1,5x) + 1) = 0$$

$$\sin(1,5x) = 2 \quad \vee \quad \sin(1,5x) = -1$$

kan niet $1,5x = 1\frac{1}{2}\pi + k \cdot 2\pi$

$$x = \pi + k \cdot \frac{4}{3}\pi$$

$$x = \pi$$

d. $\cos(2x + \frac{1}{3}\pi) = \cos(3x - \frac{1}{6}\pi)$

$$2x + \frac{1}{3}\pi = 3x - \frac{1}{6}\pi + k \cdot 2\pi \quad \vee \quad 2x + \frac{1}{3}\pi = -3x + \frac{1}{6}\pi + k \cdot 2\pi$$

$$-x = -\frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad 5x = -\frac{1}{6}\pi + k \cdot 2\pi$$

$$x = \frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad x = -\frac{1}{30}\pi + k \cdot \frac{2}{5}\pi$$

$$x = \frac{11}{30}\pi \quad \vee \quad x = \frac{1}{2}\pi \quad \vee \quad x = \frac{23}{30}\pi \quad \vee \quad x = 1\frac{1}{6}\pi \quad \vee \quad x = 1\frac{17}{30}\pi \quad \vee \quad x = 1\frac{29}{30}\pi$$

Opgave 19:

a. 1. $T(0, -2)$ geeft $y = -2 + \cos x$

2. $V_{x-as,3}$ geeft $y = -6 + 3\cos x$

b. 1. $T(\frac{1}{3}\pi, 0)$ geeft $y = \cos(x - \frac{1}{3}\pi)$

2. $V_{y-as,\frac{1}{2}}$ geeft $y = \cos(2x - \frac{1}{3}\pi)$

c. 1. $V_{x-as,3}$ geeft $y = 3\cos x$

2. $T(0, -2)$ geeft $y = -2 + 3\cos x$

3. $V_{y-as,\frac{1}{2}}$ geeft $y = -2 + 3\cos 2x$

d. 1. $V_{y-as,\frac{1}{2}}$ geeft $y = \cos 2x$

2. $V_{x-as,3}$ geeft $y = 3\cos 2x$

3. $T(0, -2)$ geeft $y = -2 + 3\cos 2x$

e. 1. $V_{x-as,3}$ geeft $y = 3\cos x$

2. $V_{y-as,\frac{1}{2}}$ geeft $y = 3\cos 2x$

3. $T(0, -2)$ geeft $y = -2 + 3\cos 2x$

4. $V_{y-as,\frac{1}{2}}$ geeft $y = -2 + 3\cos 4x$

f. 1. $T(\frac{1}{3}\pi, 0)$ geeft $y = \cos(x - \frac{1}{3}\pi)$

2. $V_{x-as,3}$ geeft $y = 3\cos(x - \frac{1}{3}\pi)$

3. $V_{y-as,\frac{1}{2}}$ geeft $y = 3\cos(2x - \frac{1}{3}\pi)$

4. $T(0, -2)$ geeft $y = -2 + 3\cos(2x - \frac{1}{3}\pi)$

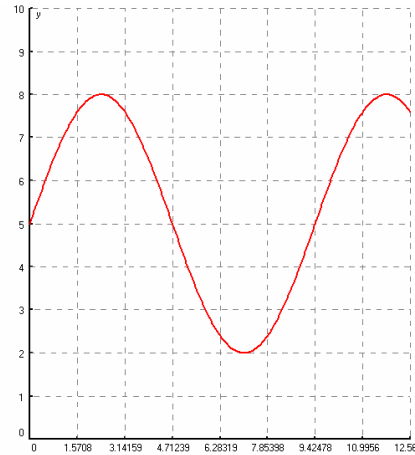
Opgave 20:

a. 1. $V_{y-as,1\frac{1}{2}}$

2. $V_{x-as,3}$

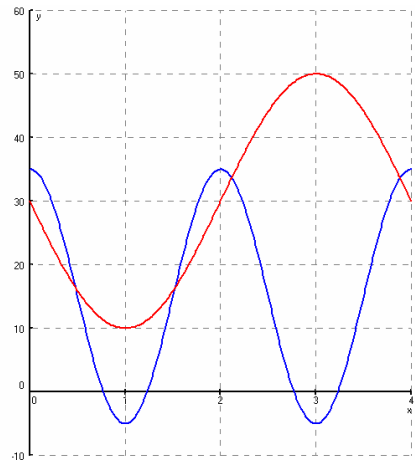
3. $T(0, 5)$

- b. evenwichts as: 5
 amplitude: 3
 periode: $\frac{2\pi}{3} = 3\pi$
 beginpunt: (0,5)
- c. $B_f = [2,8]$
- d. $y_1 = 5 + 3\sin(\frac{2}{3}x)$ en $y_2 = 3$
 intersect geeft: $x = 5,81 \vee x = 8,33$
 $0 \leq x < 5,81 \vee 8,33 < x \leq 4\pi$
- e. de kleinste helling is in het punt waar de grafiek dalend door de evenwichts-as gaat.
 Dat is voor $x = 1\frac{1}{2}\pi$.
- $$\left[\frac{dy}{dx}\right]_{1\frac{1}{2}\pi} = -2$$



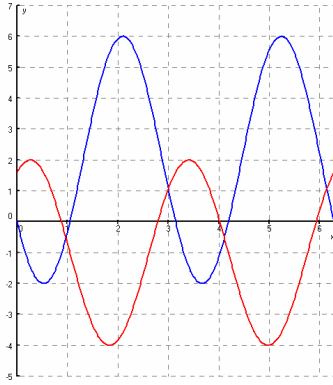
Opgave 21:

- a. f : 1. $V_{y-as, \frac{2}{\pi}}$
 2. $V_{x-as, -20}$
 3. $T(0,30)$
 g : 1. $V_{y-as, \frac{1}{\pi}}$
 2. $V_{x-as, 20}$
 3. $T(0,15)$
- b. f : evenwichts as: 30
 amplitude: 20
 periode: $\frac{2\pi}{\frac{1}{2}\pi} = 4$
 beginpunt: (0,30) dalend
 g : evenwichts as: 15
 amplitude: 20
 periode: $\frac{2\pi}{\pi} = 2$
 beginpunt: (0,35)
- c. $f(2,25) = 37,654$
 $g(2,25) = 29,142$
 $AB = 37,654 - 29,142 = 8,51$
- d. $-5 \leq p < 10$



Opgave 22:

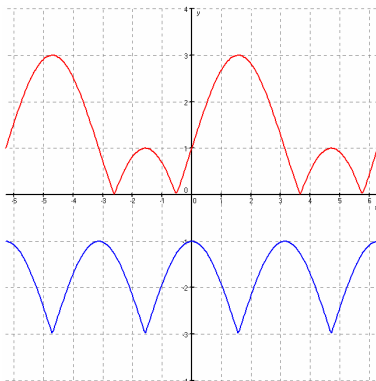
- a. f : 1. $V_{y-as, \frac{1}{2}}$ 2. $V_{x-as, 3}$ 3. $T(-\frac{1}{6}\pi, -1)$
 $g(x) = 2 - 4\cos 2(x - \frac{1}{6}\pi)$
 1. $V_{y-as, \frac{1}{2}}$ 2. $V_{x-as, -4}$ 3. $T(\frac{1}{6}\pi, 2)$
- b. f : evenwichts as: -1
 amplitude: 3
 periode: π
 beginpunt: $(\frac{1}{6}\pi, -1)$
- g : evenwichts as: 2
 amplitude: 4
 periode: π
 beginpunt: $(\frac{1}{6}\pi, 2)$ laagste punt



- c. $y_1 = -1 + 3 \sin 2(x + \frac{1}{6}\pi)$ en $y_2 = 2 - 4 \cos 2(x - \frac{1}{6}\pi)$
 neem $y_3 = y_2 - y_1$ de optie maximum geeft: $x = 1,98 \vee x = 5,12$
 dus $a = 1,98 \vee a = 5,12$
- d. neem $y_3 = y_1 + y_2$
 minimum: $-1,05$
 maximum: $3,05$
 evenwichts as: 1
 amplitude: $2,05$
 periode: π
 beginpunt: $x = 1,72$
 $h(x) = 1 + 2,05 \sin 2(x - 1,72)$

Opgave 23:

a.



- b. $|1 + 2 \sin x| = 1$
 $1 + 2 \sin x = 1 \vee 1 + 2 \sin x = -1$
 $2 \sin x = 0 \vee 2 \sin x = -2$
 $\sin x = 0 \vee \sin x = -1$
 $x = 0 + k \cdot \pi \vee x = 1\frac{1}{2}\pi + k \cdot 2\pi$
 $x = -2\pi \vee x = -\pi \vee x = -\frac{1}{2}\pi \vee x = 0 \vee x = \pi \vee x = 1\frac{1}{2}\pi \vee x = 2\pi$
- c. $y_1 = \text{abs}(2 \cos x) - 3$ en $y_2 = -1,65$ intersect geeft:
 $x = -5,45 \vee x = -3,97 \vee x = -2,31 \vee x = -0,83 \vee x = 0,83 \vee x = 2,31 \vee$
 $x = 3,97 \vee x = 5,45$

$$-2\pi \leq x < -5,54 \vee -3,97 < x < -2,31 \vee -0,83 < x < 0,83 \vee 2,31 < x < 3,97 \vee 5,45 < x \leq 2\pi$$

d. $f(2,1) = 2,726$ $g(2,1) = -1,990$ dus $AB = 2,726 - (-1,990) = 4,72$

Opgave 24:

a. $6500 - 1500 = 5000 \text{ cm}^3$

b. per periode: $4 \frac{1}{2}$ seconde

de periode is 15 seconden

dus per minuut: $4 \cdot 4 \frac{1}{2} = 18 \text{ sec}$

c. G7: per periode: $4000 - 3500 = 500 \text{ cm}^3$ bij het inademen

de periode is 6 sec

dus per minuut: $10 \cdot 500 = 5000 \text{ cm}^3$

G8: $4 \cdot 5000 = 20000 \text{ cm}^3$

$G7 : G8 = 5000 : 20000 = 1 : 4$

d. evenwichts as: 3750

amplitude: 250

periode: 6 dus $c = \frac{2\pi}{6} = \frac{1}{3}\pi$

beginpunt: $t = 0$

$$V = 3750 + 250 \sin \frac{1}{3}\pi t$$

e. evenwichts as: 4000

amplitude: 2500

periode: 15 dus $c = \frac{2\pi}{15} = \frac{2}{15}\pi$

beginpunt: $t = \frac{15}{4} = 3,75$

$$V = 4000 + 2500 \cos \frac{2}{15}\pi(t - 3,75)$$

f. periode: $\frac{60}{40} = 1,5 \text{ sec}$ dus $c = \frac{2\pi}{1,5} = \frac{4}{3}\pi$

evenwichts as: 4200

amplitude: 2500

beginpunt: $t = 0$ laagste punt

$$V = 4200 - 2500 \cos \frac{4}{3}\pi t$$

Opgave 25:

a. evenwichts as: $\frac{3,5-1,5}{2} = 1$

amplitude: 2,5

periode: $\frac{4}{3}\pi$ dus $c = \frac{2\pi}{\frac{4}{3}\pi} = 1 \frac{1}{2}$

beginpunt: $x = \frac{1}{3}\pi$

$$y = 1 + 2,5 \sin 1 \frac{1}{2}(x - \frac{1}{3}\pi)$$

b. evenwichts as: 20

amplitude: 20

periode: 12 dus $c = \frac{2\pi}{12} = \frac{1}{6}\pi$

begin: $t = 1$

$$N = 20 + 20 \cos \frac{1}{6}\pi(t - 1)$$

Opgave 26:

a. $y_1 = 3 + 3\sin(0,469x)$ en $y_2 = 3,8$ intersect geeft: $x = 0,576 \vee x = 6,123$
dus de breedte is: $6,123 - 0,576 = 5,5$ cm

b. periode: $\frac{2\pi}{0,469} = 13,4$

$$\frac{67}{13,4} = 5 \text{ golven}$$

$$PR = \sqrt{67^2 + 55^2} = 86,68 \text{ cm}$$

$$\text{periode: } \frac{86,68}{5} = 17,34$$

$$\text{dus } c = \frac{2\pi}{17,34} = 0,362$$

$$\text{dus } y = 3 + 3\sin(0,362x)$$