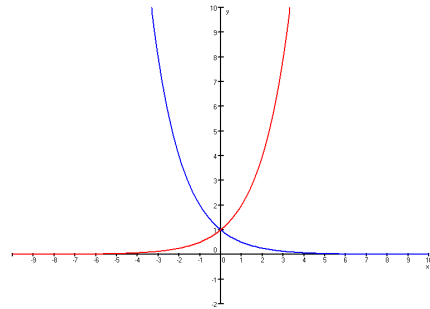


Hoofdstuk 5: Exponenten en logaritmen.

5.1 De standaardfunctie $f(x) = g^x$

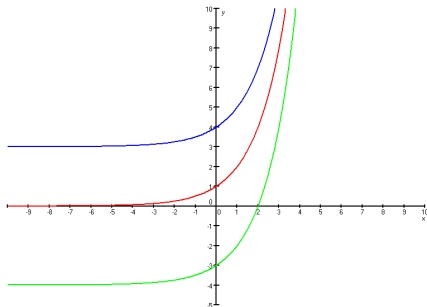
Opgave 1:

- De grafiek van g ontstaat door de grafiek van f te spiegelen in de x -as.
- $y = 0$
- $B_f = \langle 0, \rightarrow \rangle$
 $B_g = \langle 0, \rightarrow \rangle$



Opgave 2:

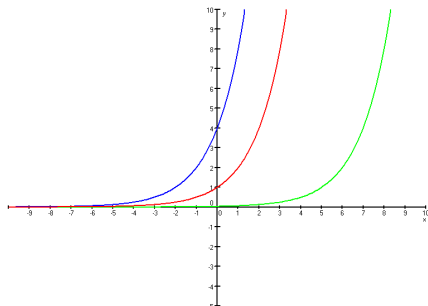
a.



y_2 ontstaat door translatie over $(0,3)$

y_3 ontstaat door translatie over $(0,-4)$

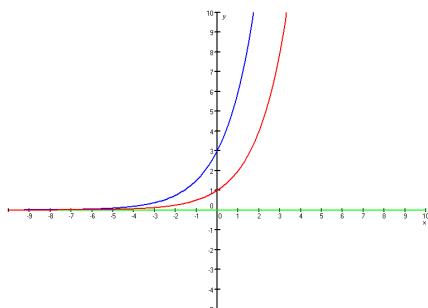
b.



y_4 ontstaat door translatie over $(-2,0)$

y_5 ontstaat door translatie over $(5,0)$

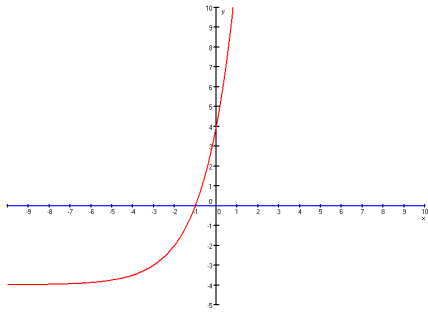
c.



y_6 ontstaat door vermenigvuldiging ten opzichte van de x -as met 3

Opgave 3:a. translatie over $(-3, -4)$

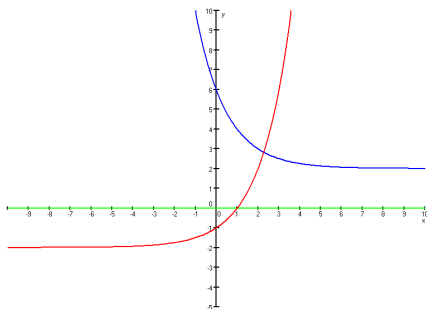
b.



$$B_f = \langle -4, \rightarrow \rangle$$

c. $y_1 = 2^{x+3} - 4$ en $y_2 = 2$
intersect geeft $x = -0,42$ dus $x \leq -0,42$ d. $f(3) = 60$ dus $-4 < x \leq 60$ **Opgave 4:**a. translatie over $(1, 5)$ H.A.: $y = 5$ b. translatie over $(-1, 0)$ daarna vermenigvuldiging t.o.v. de x -as met 5H.A.: $y = 0$ c. translatie over $(0, -7)$ daarna vermenigvuldiging t.o.v. de x -as met 4H.A.: $y = -7$ d. vermenigvuldiging t.o.v. de x -as met -2 daarna translatie over $(0, 3)$ H.A.: $y = 3$ **Opgave 5:**a. $N = -6$ b. $N = 5$ c. $N = 1000$ d. $N = 100$ **Opgave 6:**a. f : translatie over $(0, -2)$ g: translatie over $(2, 2)$

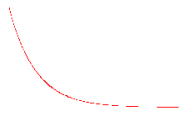
b.



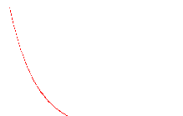
- c. $B_f = \langle -2, \rightarrow \rangle$
 $B_g = \langle 2, \rightarrow \rangle$
- d. $g(4) = 2\frac{1}{4}$
 $2 < g(x) \leq 2\frac{1}{4}$
- e. $y_1 = 2^x - 2$ en $y_2 = (\frac{1}{2})^{x-2} + 2$
intersect geeft: $x = 2,27$
dus $x \leq 2,27$
- f. $p \leq -2$
- g. $f(3) = 6$ en $g(3) = 2,5$
 $AB = 6 - 2,5 = 3,5$
- h. $y_3 = 7$
 y_1 en y_3 intersect geeft $x = 3,17$
 y_2 en y_3 intersect geeft $x = -0,32$
dus $CD = 3,17 - -0,32 = 3,49$

Opgave 7:

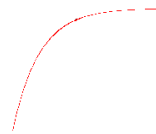
$$0 < g < 1$$



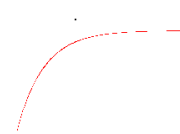
$$a > 0, b > 0$$



$$a > 0, b < 0$$

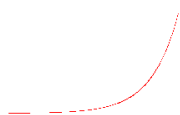


$$a < 0, b > 0$$

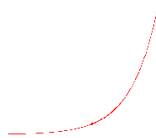


$$a < 0, b < 0$$

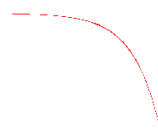
$$g > 1$$



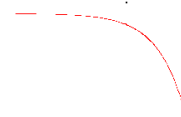
$$a > 0, b > 0$$



$$a > 0, b < 0$$



$$a < 0, b > 0$$



$$a < 0, b < 0$$

Opgave 8:

- a. $2^{x+1} = 64$
 $2^{x+1} = 2^6$
 $x + 1 = 6$
 $x = 5$
- b. $2^{x-3} = \frac{1}{8}$
 $2^{x-3} = \frac{1}{2^3}$

$$2^{x-3} = 2^{-3}$$

$$x - 3 = -3$$

$$x = 0$$

c. $3^{4x-1} = \frac{1}{27}\sqrt{3}$

$$3^{4x-1} = \frac{1}{3^3} \cdot 3^{\frac{1}{2}}$$

$$3^{4x-1} = 3^{-2\frac{1}{2}}$$

$$4x - 1 = -2\frac{1}{2}$$

$$4x = -1\frac{1}{2}$$

$$x = -\frac{3}{8}$$

d. $5^{-x+6} = 625$

$$5^{-x+6} = 5^4$$

$$-x + 6 = 4$$

$$-x = -2$$

$$x = 2$$

e. $(\frac{1}{3})^x - 2 = 25$

$$(3^{-1})^x = 27$$

$$3^{-x} = 3^3$$

$$-x = 3$$

$$x = -3$$

f. $5 \cdot (\frac{1}{2})^x + 11 = 91$

$$5 \cdot (\frac{1}{2})^x = 80$$

$$(\frac{1}{2})^x = 16$$

$$(2^{-1})^x = 2^4$$

$$2^{-x} = 2^4$$

$$-x = 4$$

$$x = -4$$

g. $2^{x+3} = \sqrt{2}$

$$2^{x+3} = 2^{\frac{1}{2}}$$

$$x + 3 = \frac{1}{2}$$

$$x = -2\frac{1}{2}$$

h. $3^{x+2} = 9\sqrt{3}$

$$3^{x+2} = 3^2 \cdot 3^{\frac{1}{2}}$$

$$3^{x+2} = 3^{2\frac{1}{2}}$$

$$x + 2 = 2\frac{1}{2}$$

$$x = \frac{1}{2}$$

i. $4^{2x-1} = 64$

$$4^{2x-1} = 4^3$$

$$2x - 1 = 3$$

$$2x = 4$$

$$x = 2$$

Opgave 9:

a. $2^{3x+5} = 16\sqrt{2}$

$2^{3x+5} = 2^4 \cdot 2^{\frac{1}{2}}$

$2^{3x+5} = 2^{4\frac{1}{2}}$

$3x + 5 = 4\frac{1}{2}$

$3x = -\frac{1}{2}$

$x = -\frac{1}{6}$

b. $3^{4x} = \frac{1}{81} \cdot \sqrt[4]{9}$

$3^{4x} = \frac{1}{3^4} \cdot \sqrt[4]{3^2}$

$3^{4x} = 3^{-4} \cdot 3^{\frac{2}{4}}$

$3^{4x} = 3^{-3\frac{1}{2}}$

$4x = -3\frac{1}{2}$

$x = -\frac{7}{8}$

c. $3 \cdot 5^{2x-1} = 0,6$

$5^{2x-1} = 0,2$

$5^{2x-1} = \frac{1}{5}$

$5^{2x-1} = 5^{-1}$

$2x - 1 = -1$

$2x = 0$

$x = 0$

d. $3^{3x-3} = \frac{1}{3} \cdot \sqrt[4]{3}$

$3^{3x-3} = 3^{-1} \cdot 3^{\frac{1}{4}}$

$3^{3x-3} = 3^{-\frac{3}{4}}$

$3x - 3 = -\frac{3}{4}$

$3x = 2\frac{1}{4}$

$x = \frac{3}{4}$

e. $3 \cdot \left(\frac{1}{2}\right)^{x-1} - 1 = -0,25$

$3 \cdot \left(\frac{1}{2}\right)^{x-1} = 0,75$

$\left(\frac{1}{2}\right)^{x-1} = 0,25$

$\left(\frac{1}{2}\right)^{x-1} = \frac{1}{4}$

$\left(\frac{1}{2}\right)^{x-1} = \left(\frac{1}{2}\right)^2$

$x - 1 = 2$

$x = 3$

f. $3 \cdot 5^{2x+1} = 75\sqrt{5}$

$5^{2x+1} = 25\sqrt{5}$

$5^{2x+1} = 5^2 \cdot 5^{\frac{1}{2}}$

$5^{2x+1} = 5^{2\frac{1}{2}}$

$2x + 1 = 2\frac{1}{2}$

$$2x = 1\frac{1}{2}$$

$$x = \frac{3}{4}$$

g. $2^{4x-1} = 2^{2x-3}$
 $4x-1 = 2x-3$
 $2x = -2$
 $x = -1$

h. $3^{x^2} = 3^{x+6}$
 $x^2 = x+6$
 $x^2 - x - 6 = 0$
 $(x-3)(x+2) = 0$
 $x = 3 \quad \vee \quad x = -2$

i. $4^{|2x+1|} = 16$
 $4^{|2x+1|} = 4^2$
 $|2x+1| = 2$
 $2x+1 = 2 \quad \vee \quad 2x+1 = -2$
 $2x = 1 \quad \vee \quad 2x = -3$
 $x = \frac{1}{2} \quad \vee \quad x = -1\frac{1}{2}$

Opgave 10:

a. $3^{x-4} < 3\sqrt{3}$
 $3^{x-4} < 3 \cdot 3^{\frac{1}{2}}$
 $3^{x-4} < 3^{1\frac{1}{2}}$
 $x-4 < 1\frac{1}{2}$
 $x < 5\frac{1}{2}$

b. $0,2^x + 5 \geq 6$
 $0,2^x \geq 1$
 $0,2^x \geq 0,2^0$
 $x \leq 0$

c. $5 - 2^{x+1} > 4\frac{1}{2}$
 $-2^{x+1} > -\frac{1}{2}$
 $2^{x+1} < \frac{1}{2}$
 $2^{x+1} < 2^{-1}$
 $x+1 < -1$
 $x < -2$

Opgave 11:

$$2^{4x-1} = 4^{x-3}$$
$$2^{4x-1} = (2^2)^{x-3}$$
$$2^{4x-1} = 2^{2x-6}$$
$$4x-1 = 2x-6$$
$$2x = -5$$
$$x = -2\frac{1}{2}$$

Opgave 12:

- a. $2^{x+1} + 2^x = 48$
 $2 \cdot 2^x + 2^x = 48$ want de regel $a^p \cdot a^q = a^{p+q}$ gebruik je nu van rechts naar links
- b. $2 \cdot 2^x + 1 \cdot 2^x = 48$
 $3 \cdot 2^x = 48$
 $2^x = 16$
 $2^x = 2^4$
 $x = 4$

Opgave 13:

- a. $2^{x+1} = 4^{3x+1}$
 $2^{x+1} = (2^2)^{3x+1}$
 $2^{x+1} = 2^{6x+2}$
 $x+1 = 6x+2$
 $-5x = 1$
 $x = -\frac{1}{5}$
- b. $4^{x-1} = 8^{3x-3}$
 $(2^2)^{x-1} = (2^3)^{3x-3}$
 $2^{2x-2} = 2^{9x-9}$
 $2x-2 = 9x-9$
 $-7x = -7$
 $x = 1$
- c. $2^{x^2} = \left(\frac{1}{4}\right)^x$
 $2^{x^2} = \left(\frac{1}{2^2}\right)^x$
 $2^{x^2} = (2^{-2})^x$
 $2^{x^2} = 2^{-2x}$
 $x^2 = -2x$
 $x^2 + 2x = 0$
 $x(x+2) = 0$
 $x = 0 \vee x = -2$
- d. $25^{x-3} = 5 \cdot 5^{2-x}$
 $(5^2)^{x-3} = 5^{3-x}$
 $5^{2x-6} = 5^{3-x}$
 $2x-6 = 3-x$
 $3x = 9$
 $x = 3$
- e. $27^x = 3 \cdot 9^{2x}$
 $(3^3)^x = 3 \cdot (3^2)^{2x}$
 $3^{3x} = 3 \cdot 3^{4x}$
 $3^{3x} = 3^{4x+1}$
 $3x = 4x+1$
 $-x = 1$
 $x = -1$

$$\begin{aligned}
 \text{f. } 0,5^x &= 0,25 \cdot 2^x \\
 \left(\frac{1}{2}\right)^x &= \frac{1}{4} \cdot 2^x \\
 (2^{-1})^x &= \frac{1}{2^2} \cdot 2^x \\
 2^{-x} &= 2^{-2} \cdot 2^x \\
 2^{-x} &= 2^{x-2} \\
 -x &= x-2 \\
 -2x &= -2 \\
 x &= 1
 \end{aligned}$$

Opgave 14:

$$\begin{aligned}
 \text{a. } 3^{x+2} + 3^x &= 810 \\
 3^2 \cdot 3^x + 3^x &= 810 \\
 9 \cdot 3^x + 3^x &= 810 \\
 10 \cdot 3^x &= 810 \\
 3^x &= 81 \\
 3^x &= 3^4 \\
 x &= 4 \\
 \text{b. } 2^{x-1} + 2^{x+1} &= 10 \\
 2^{-1} \cdot 2^x + 2^1 \cdot 2^x &= 10 \\
 \frac{1}{2} \cdot 2^x + 2 \cdot 2^x &= 10 \\
 2\frac{1}{2} \cdot 2^x &= 10 \\
 2^x &= 4 \\
 2^x &= 2^2 \\
 x &= 2 \\
 \text{c. } 2^{x+3} - 2^x &= \frac{7}{8} \\
 2^3 \cdot 2^x - 2^x &= \frac{7}{8} \\
 8 \cdot 2^x - 2^x &= \frac{7}{8} \\
 7 \cdot 2^x &= \frac{7}{8} \\
 2^x &= \frac{1}{8} \\
 2^x &= \frac{1}{2^3} \\
 2^x &= 2^{-3} \\
 x &= -3 \\
 \text{d. } 3^{x+2} &= 24 + 3^x \\
 3^2 \cdot 3^x - 3^x &= 24 \\
 9 \cdot 3^x - 3^x &= 24 \\
 8 \cdot 3^x &= 24 \\
 3^x &= 3 \\
 x &= 1 \\
 \text{e. } 3^x - 3^{x-1} &= 2\sqrt{3} \\
 3^x - 3^{-1} \cdot 3^x &= 2\sqrt{3} \\
 3^x - \frac{1}{3} \cdot 3^x &= 2\sqrt{3}
 \end{aligned}$$

$$\frac{2}{3} \cdot 3^x = 2\sqrt{3}$$

$$3^x = 3\sqrt{3}$$

$$3^x = 3^1 \cdot 3^{\frac{1}{2}}$$

$$3^x = 3^{1\frac{1}{2}}$$

$$x = 1\frac{1}{2}$$

f. $5^{x-1} + 5^{x-2} = 6\sqrt{5}$

$$5^{-1} \cdot 5^x + 5^{-2} \cdot 5^x = 6\sqrt{5}$$

$$\frac{1}{5} \cdot 5^x + \frac{1}{25} \cdot 5^x = 6\sqrt{5}$$

$$\frac{6}{25} \cdot 5^x = 6\sqrt{5}$$

$$5^x = 25\sqrt{5}$$

$$5^x = 5^2 \cdot 5^{\frac{1}{2}}$$

$$5^x = 5^{2\frac{1}{2}}$$

$$x = 2\frac{1}{2}$$

Opgave 15:

a. $3^{x+1} = 9^{x+2}$

$$3^{x+1} = (3^2)^{x+2}$$

$$3^{x+1} = 3^{2x+4}$$

$$x+1 = 2x+4$$

$$-x = 3$$

$$x = -3$$

b. $3^{x+1} - 3^{x-1} = 8\sqrt{3}$

$$3^1 \cdot 3^x - 3^{-1} \cdot 3^x = 8\sqrt{3}$$

$$3 \cdot 3^x - \frac{1}{3} \cdot 3^x = 8\sqrt{3}$$

$$\frac{8}{3} \cdot 3^x = 8\sqrt{3}$$

$$3^x = 3\sqrt{3}$$

$$3^x = 3^1 \cdot 3^{\frac{1}{2}}$$

$$3^x = 3^{1\frac{1}{2}}$$

$$x = 1\frac{1}{2}$$

c. $3^{x^2} = \left(\frac{1}{3}\right)^{x-6}$

$$3^{x^2} = (3^{-1})^{x-6}$$

$$3^{x^2} = 3^{-x+6}$$

$$x^2 = -x+6$$

$$x^2 + x - 6 = 0$$

$$(x+3)(x-2) = 0$$

$$x = -3 \quad \vee \quad x = 2$$

d. $5^x + 5^{x+1} = \frac{6}{25}$

$$5^x + 5^1 \cdot 5^x = \frac{6}{25}$$

$$6 \cdot 5^x = \frac{6}{25}$$

$$5^x = \frac{1}{25}$$

$$5^x = \frac{1}{5^2}$$

$$5^x = 5^{-2}$$

$$x = -2$$

e. $5^{x^2+5} = 125^{x+1}$

$$5^{x^2+5} = (5^3)^{x+1}$$

$$5^{x^2+5} = 5^{3x+3}$$

$$x^2 + 5 = 3x + 3$$

$$x^2 - 3x + 2 = 0$$

$$(x-1)(x-2) = 0$$

$$x = 1 \quad \vee \quad x = 2$$

f. $2^{x+2} - \left(\frac{1}{2}\right)^{-x+1} = 28$

$$2^2 \cdot 2^x - (2^{-1})^{-x+1} = 28$$

$$4 \cdot 2^x - 2^{x-1} = 28$$

$$4 \cdot 2^x - 2^{-1} \cdot 2^x = 28$$

$$4 \cdot 2^x - \frac{1}{2} \cdot 2^x = 28$$

$$3\frac{1}{2} \cdot 2^x = 28$$

$$2^x = 8$$

$$2^x = 2^3$$

$$x = 3$$

g. $4^{x^2+1} = 8^{x^2-1}$

$$(2^2)^{x^2+1} = (2^3)^{x^2-1}$$

$$2^{2x^2+2} = 2^{3x^2-3}$$

$$2x^2 + 2 = 3x^2 - 3$$

$$-x^2 = -5$$

$$x^2 = 5$$

$$x = \sqrt{5} \quad \vee \quad x = -\sqrt{5}$$

h. $2^{x+3} - 4^{\frac{1}{2}x-1} = 3\frac{7}{8}$

$$2^3 \cdot 2^x - (2^2)^{\frac{1}{2}x-1} = \frac{31}{8}$$

$$8 \cdot 2^x - 2^{x-2} = \frac{31}{8}$$

$$8 \cdot 2^x - 2^{-2} \cdot 2^x = \frac{31}{8}$$

$$8 \cdot 2^x - \frac{1}{4} \cdot 2^x = \frac{31}{8}$$

$$7\frac{3}{4} \cdot 2^x = \frac{31}{8}$$

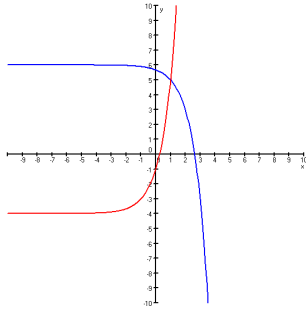
$$2^x = \frac{1}{2}$$

$$2^x = 2^{-1}$$

$$x = -1$$

Opgave 16:a. f : translatie over $(-1, -4)$ g: vermenigvuldiging t.o.v. de x -as met -1 , daarna translatie over $(1, 6)$.

b.



c. $B_f = \langle -4, \rightarrow \rangle$

$B_g = \langle \leftarrow, 6 \rangle$

d. $3^{x+1} - 4 \leq 6 - 3^{x-1}$

$3^{x+1} + 3^{x-1} \leq 10$

$3^1 \cdot 3^x + 3^{-1} \cdot 3^x \leq 10$

$3 \cdot 3^x + \frac{1}{3} \cdot 3^x \leq 10$

$3\frac{1}{3} \cdot 3^x \leq 10$

$3^x \leq 3$

$x \leq 1$

e. $f(2\frac{1}{2}) = 3^{3\frac{1}{2}} - 4 = 3^3 \cdot 3^{\frac{1}{2}} - 4 = 27\sqrt{3} - 4$

$g(2\frac{1}{2}) = 6 - 3^{1\frac{1}{2}} = 6 - 3^1 \cdot 3^{\frac{1}{2}} = 6 - 3\sqrt{3}$

$AB = f(2\frac{1}{2}) - g(2\frac{1}{2}) = 27\sqrt{3} - 4 - (6 - 3\sqrt{3}) = 27\sqrt{3} - 4 - 6 + 3\sqrt{3} = 30\sqrt{3} - 10$

f. $f(x) - g(x) = 80$

$3^{x+1} - 4 - (6 - 3^{x-1}) = 80$

$3^{x+1} - 4 - 6 + 3^{x-1} = 80$

$3^1 \cdot 3^x + 3^{-1} \cdot 3^x = 90$

$3 \cdot 3^x + \frac{1}{3} \cdot 3^x = 90$

$\frac{10}{3} \cdot 3^x = 90$

$3^x = 27$

$3^x = 3^3$

$x = 3$

g. $g(x) - f(x) = p$

$6 - 3^{x-1} - (3^{x+1} - 4) = p$

$6 - 3^{x-1} - 3^{x+1} + 4 = p$

$-3^{x-1} - 3^{x+1} = p - 10$

$3^{x-1} + 3^{x+1} = -p + 10$

$3^{-1} \cdot 3^x + 3^1 \cdot 3^x = -p + 10$

$\frac{1}{3} \cdot 3^x + 3 \cdot 3^x = -p + 10$

$3\frac{1}{3} \cdot 3^x = -p + 10$

$$3^x = -\frac{3}{10}p + 3$$

$3^x > 0$ voor iedere x dus er zijn geen oplossingen als $-\frac{3}{10}p + 3 \leq 0$

$$-\frac{3}{10}p \leq -3$$

$$p \geq 10$$