

5.3 Logaritmen

Opgave 37:

- a. $2^3 = 8$
- b. $2^{-2} = \frac{1}{4}$
- c. $2^{\frac{1}{2}} = \sqrt{2}$
- d. $3^2 = 9$
- e. $3^{-3} = \frac{1}{27}$
- f. $3^{\frac{1}{5}} = \sqrt[5]{3}$

Opgave 38:

- a. ${}^5\log 125 = {}^5\log 5^3 = 3$
- b. ${}^{10}\log 0,1 = {}^{10}\log \frac{1}{10} = {}^{10}\log 10^{-1} = -1$
- c. ${}^2\log 4 = {}^2\log 2^2 = 2$
- d. ${}^7\log 49 = {}^7\log 7^2 = 2$
- e. ${}^2\log \sqrt{2} = {}^2\log 2^{\frac{1}{2}} = \frac{1}{2}$
- f. ${}^2\log 0,5 = {}^2\log \frac{1}{2} = {}^2\log 2^{-1} = -1$
- g. ${}^4\log 0,25 = {}^4\log \frac{1}{4} = {}^4\log 4^{-1} = -1$
- h. ${}^4\log 4 = {}^4\log 4^1 = 1$
- i. ${}^4\log 1 = {}^4\log 4^0 = 0$

Opgave 39:

- a. ${}^2\log 64\sqrt{2} = {}^2\log(2^6 \cdot 2^{\frac{1}{2}}) = {}^2\log 2^{6\frac{1}{2}} = 6\frac{1}{2}$
- b. ${}^3\log \frac{1}{9}\sqrt{3} = {}^3\log(\frac{1}{3^2} \cdot 3^{\frac{1}{2}}) = {}^3\log(3^{-2} \cdot 3^{\frac{1}{2}}) = {}^3\log 3^{-1\frac{1}{2}} = -1\frac{1}{2}$
- c. ${}^3\log 3^{21,5} = 21,5$
- d. ${}^5\log \frac{1}{125} = {}^5\log \frac{1}{5^3} = {}^5\log 5^{-3} = -3$
- e. $\frac{1}{3}\log \frac{1}{27} = \frac{1}{3}\log(\frac{1}{3})^3 = 3$
- f. $\frac{1}{2}\log \frac{1}{4} = \frac{1}{2}\log(\frac{1}{2})^2 = 2$
- g. ${}^2\log(\frac{1}{32} \cdot \sqrt[3]{2}) = {}^2\log(\frac{1}{2^5} \cdot 2^{\frac{1}{3}}) = {}^2\log(2^{-5} \cdot 2^{\frac{1}{3}}) = {}^2\log 2^{-4\frac{2}{3}} = -4\frac{2}{3}$
- h. ${}^5\log 1 = {}^5\log 5^0 = 0$
- i. ${}^3\log(81 \cdot \sqrt[5]{27}) = {}^3\log(3^4 \cdot \sqrt[5]{3^3}) = {}^3\log(3^4 \cdot 3^{\frac{3}{5}}) = {}^3\log 3^{4\frac{3}{5}} = 4\frac{3}{5}$

Opgave 40:

- a. ${}^2\log 2^8 = 8$ dus ${}^2\log 256 = 8$
- b. ${}^3\log 3^{-3} = -3$ dus ${}^3\log \frac{1}{3^3} = -3$ dus ${}^3\log \frac{1}{27} = -3$
- c. ${}^5\log 5^{\frac{1}{2}} = \frac{1}{2}$ dus ${}^5\log \sqrt{5} = \frac{1}{2}$

Opgave 41:

a. ${}^3\log(x+2) = 2$

$x+2 = 3^2$

$x+2 = 9$

$x = 7$

b. $1+\frac{1}{2}\log x = 4$

$\frac{1}{2}\log x = 3$

$x = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

c. ${}^3\log(2x+1) = 4$

$2x+1 = 3^4$

$2x+1 = 81$

$2x = 80$

$x = 40$

d. $5+{}^4\log x = 3$

${}^4\log x = -2$

$x = 4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

e. $\frac{1}{2}\log(x-1) = 3$

$x-1 = \left(\frac{1}{2}\right)^3$

$x-1 = \frac{1}{8}$

$x = 1\frac{1}{8}$

f. ${}^2\log(x^2-4) = 5$

$x^2-4 = 2^5$

$x^2-4 = 32$

$x^2 = 36$

$x = 6 \quad \vee \quad x = -6$

Opgave 42:

a. $4 \cdot {}^3\log x = 2$

${}^3\log x = \frac{1}{2}$

$x = 3^{\frac{1}{2}} = \sqrt{3}$

b. ${}^3\log(4x-1) = -2$

$4x-1 = 3^{-2}$

$4x-1 = \frac{1}{3^2}$

$4x-1 = \frac{1}{9}$

$4x = 1\frac{1}{9}$

$x = \frac{10}{36} = \frac{5}{18}$

c. $3+{}^2\log x = -1$

${}^2\log x = -4$

$x = 2^{-4} = \frac{1}{2^4} = \frac{1}{16}$

d. ${}^5\log(3x + 2) = 1$

$$3x + 2 = 5^1$$

$$3x + 2 = 5$$

$$3x = 3$$

$$x = 1$$

e. ${}^3\log(0,4x - 5) = 2$

$$0,4x - 5 = 3^2$$

$$0,4x - 5 = 9$$

$$0,4x = 14$$

$$x = 35$$

f. $4 + 2 \cdot {}^2\log x = 7$

$$2 \cdot {}^2\log x = 3$$

$${}^2\log x = 1\frac{1}{2}$$

$$x = 2^{1\frac{1}{2}} = 2^1 \cdot 2^{\frac{1}{2}} = 2\sqrt{2}$$

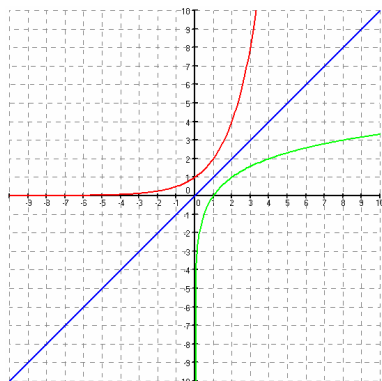
Opgave 43:

a.

x	-3	-2	-1	0	1	2	3
$y = 2^x$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

x	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
$y = {}^2\log x$	-3	-2	-1	0	1	2	3

b.



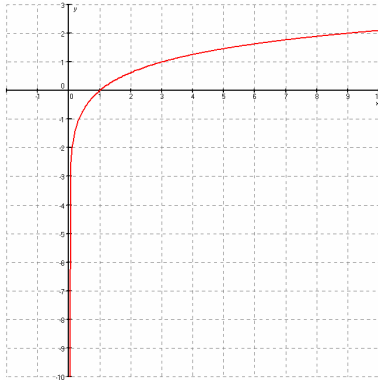
c. spiegeling in de lijn $y = x$

Opgave 44:

a.

x	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9
$f(x)$	-2	-1	0	1	2

b.



c. ${}^3\log x = 1\frac{1}{2}$

$$x = 3^{1\frac{1}{2}} = 3\sqrt{3}$$

dus $0 < x \leq 1\frac{1}{2}$

d. $f(\sqrt{3}) = {}^3\log \sqrt{3} = \frac{1}{2}$

$$f(27) = {}^3\log 27 = 3$$

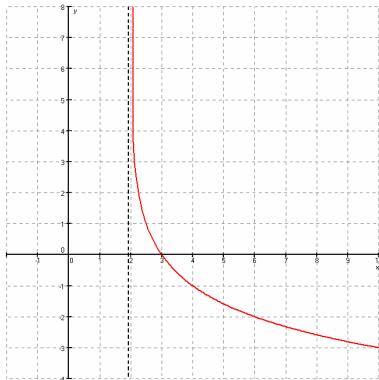
dus $\frac{1}{2} \leq f(x) \leq 3$

Opgave 45:

a.

x	6	4	3	$2\frac{1}{2}$	$2\frac{1}{4}$
$f(x)$	-2	-1	0	1	2

b.



c. $f(2\frac{1}{8}) = \frac{1}{2}\log \frac{1}{8} = 3$ dus $f(x) \leq 3$

d. $\frac{1}{2}\log(x-2) = -3$

$$x-2 = (\frac{1}{2})^{-3}$$

$$x-2 = 8$$

$$x = 10$$

dus $2 < x \leq 10$

Opgave 46:

$$2^{2\log 8} = 2^3 = 8$$

$$3^{3\log 9} = 3^2 = 9$$

$$2^{2 \log_2 \frac{1}{2}} = 2^{-1} = \frac{1}{2}$$

Opgave 47:

- a. $\log 100 = 2$
 $\log 1000 = 3$
- b. $g = 10$

Opgave 48:

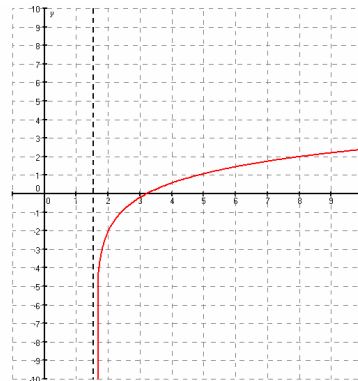
- a. ${}^3 \log 5 = \frac{\log 5}{\log 3} = 1,46$
- b. ${}^{\frac{1}{7}} \log 18 = \frac{\log 18}{\log \frac{1}{7}} = -1,49$
- c. ${}^2 \log 20 - {}^2 \log 6 = \frac{\log 20}{\log 2} - \frac{\log 6}{\log 2} = 1,74$
- d. ${}^{\frac{1}{3}} \log 10 + \log \frac{1}{3} = \frac{\log 10}{\log \frac{1}{3}} + \frac{\log \frac{1}{3}}{\log 10} = -2,57$
- e. $3 \cdot {}^2 \log 7 = 3 \cdot \frac{\log 7}{\log 2} = 8,42$
- f. ${}^4 \log 12 = \frac{5}{\frac{\log 12}{\log 4}} = 2,79$

Opgave 49:

- a. $T(0,3)$
- b. $T(-3,0)$
- c. $V_{x-as,3}$
- d. $D_f = \langle 0, \rightarrow \rangle$ V.A.: $x = 0$
 $D_g = \langle 0, \rightarrow \rangle$ V.A.: $x = 0$
 $D_h = \langle -3, \rightarrow \rangle$ V.A.: $x = -3$
 $D_k = \langle 0, \rightarrow \rangle$ V.A.: $x = 0$

Opgave 50:

- a. $5x - 8 > 0$
 $5x > 8$
 $x > 1,6$
 $D_f = \langle 1,6; \rightarrow \rangle$
V.A.: $x = 1,6$
- b. $-3 + {}^2 \log(5x - 8) = 0$
 ${}^2 \log(5x - 8) = 3$
 $5x - 8 = 2^3$
 $5x - 8 = 8$
 $5x = 16$
 $x = 3,2$



$$1,6 < x \leq 3,2$$

c. $f(8) = -3 + {}^2\log 32 = -3 + 5 = 2$
 $f(x) \leq 2$

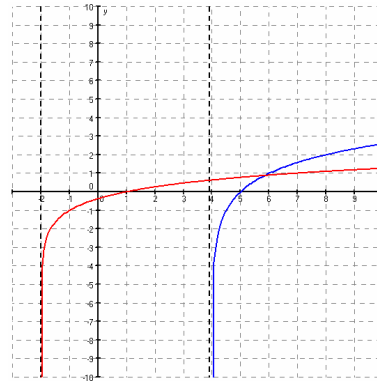
Opgave 51:

a. $x + 2 > 0$
 $x > -2$
 $D_f = \langle -2, \rightarrow \rangle$
V.A.: $x = -2$

$x - 4 > 0$
 $x > 4$
 $D_g = \langle 4, \rightarrow \rangle$
V.A.: $x = 4$

b. $y_1 = -1 + \log(x + 2) / \log(3)$
 $y_2 = \log(x - 4) / \log(2)$
Intersect geeft: $x = 5,83$ en $y = 0,87$

c. $4 < x \leq 5,83$



Opgave 52:

- a. $T(5, -3)$ dus $D_f = \langle 5, \rightarrow \rangle$ en V.A.: $x = 5$
b. $T(-1, 3)$ dus $D_g = \langle -1, \rightarrow \rangle$ en V.A.: $x = -1$
c. $V_{x-as,5}$ dan $T(2, 0)$ dus $D_h = \langle 2, \rightarrow \rangle$ en V.A.: $x = 2$
d. $V_{x-as,-2}$ dan $T(0, -4)$ dus $D_k = \langle 0, \rightarrow \rangle$ en V.A.: $x = 0$

Opgave 53:

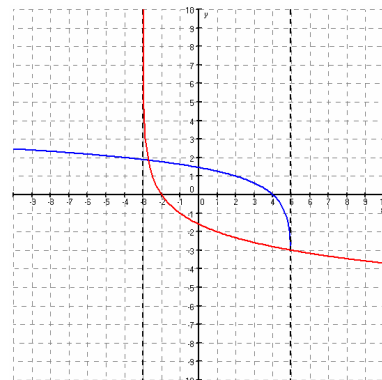
a. $D_f = \langle -3, \rightarrow \rangle$ en V.A.: $x = -3$

$-x + 5 > 0$
 $-x > -5$
 $x < 5$
 $D_g = \langle \leftarrow, 5 \rangle$
V.A.: $x = 5$

b. $\frac{1}{2} \log(x + 3) = 5$
 $x + 3 = \left(\frac{1}{2}\right)^5$
 $x + 3 = \frac{1}{32}$
 $x = -2 \frac{31}{32}$

c. $g(-4) = {}^3\log 9 = 2$
 $g(x) \leq 2$

d. $\frac{1}{2} \log(x + 3) = 1$
 $x + 3 = \left(\frac{1}{2}\right)^1$
 $x + 3 = \frac{1}{2}$



$$x = -2\frac{1}{2}$$

$$x \leq -2\frac{1}{2}$$

e. $y_1 = \log(x+3)/\log(0.5)$

$$y_2 = \log(-x+5)/\log(3)$$

Intersect geeft $x = -2,72 \vee x = 4,96$

$$-2,72 \leq x \leq 4,96$$

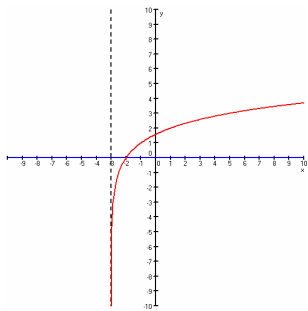
f. $y_3 = 2,5$

y_1 en y_3 intersect geeft: $x = -2,823$

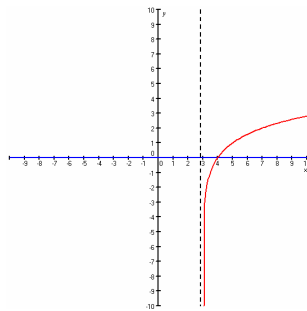
y_2 en y_3 intersect geeft: $x = -10,588$

$$AB = -2,823 - -10,588 = 7,77$$

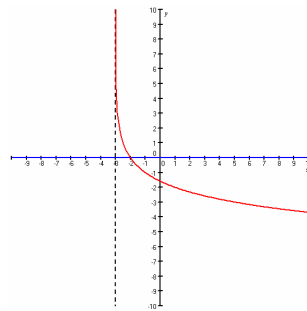
Opgave 54:



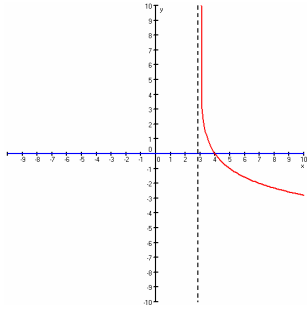
$$a > 0; b > 0; g > 1$$



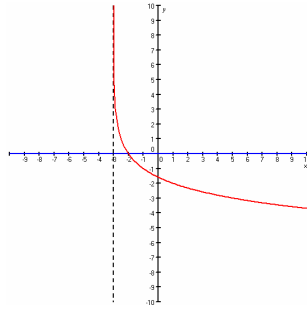
$$a > 0; b < 0; g > 1$$



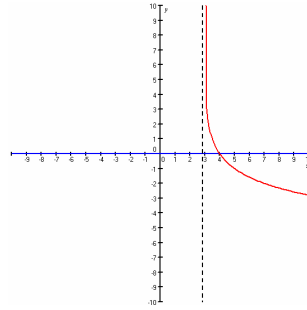
$$a > 0; b > 0; g < 1$$



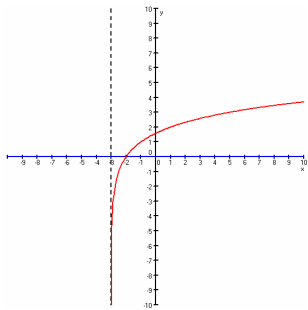
$$a > 0; b < 0; g < 1$$



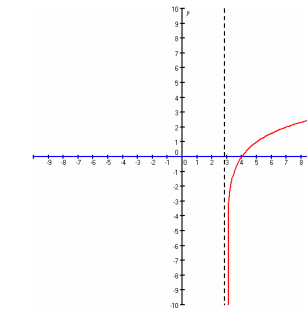
$$a < 0; b > 0; g > 1$$



$$a < 0; b < 0; g > 1$$



$$a < 0; b > 0; g < 1$$



$$a < 0; b < 0; g < 1$$

Opgave 55:

- a. $21 = 1 + k \cdot \log 100$
 $21 = 1 + k \cdot 2$
 $20 = 2k$
 $k = 10$
- b. $DIN = 1 + 10 \cdot \log 400 = 27$
- c. $1 + 10 \cdot \log ASA = 24$
 $10 \cdot \log ASA = 23$
 $\log ASA = 2,3$
 $ASA = 200$

Opgave 56:

- a. $R = 0,67 \cdot \log(1,5 \cdot 10^7) - 0,9 = 3,9$
- b. $0,67 \cdot \log E - 0,9 = 9,3$
 $0,67 \cdot \log E = 10,2$
 $\log E = 15,22$
 $E = 10^{15,22} = 1,7 \cdot 10^{15} \text{ kJ}$
- c. $\frac{1,7 \cdot 10^{15}}{1,5 \cdot 10^7} = 1,1 \cdot 10^8$

Opgave 57:

Zie de tabel, als de geluidsintensiteit $10\times$ zo groot is, dan is het aantal decibel 10 hoger.
 Dus het aantal decibel van 10 leerlingen is 65.

Opgave 58:

Vrachtwagen: $10 \cdot \log \frac{I}{10^{-12}} = 65$

$$\log \frac{I}{10^{-12}} = 6,5$$

$$\frac{I}{10^{-12}} = 3,16 \cdot 10^6$$

$$I = 3,16 \cdot 10^{-6}$$

Trein: $10 \cdot \log \frac{I}{10^{-12}} = 72$

$$\log \frac{I}{10^{-12}} = 7,2$$

$$\frac{I}{10^{-12}} = 1,58 \cdot 10^7$$

$$I = 1,58 \cdot 10^{-5}$$

$$I_{\text{total}} = I_v + I_t = 3,16 \cdot 10^{-6} + 1,58 \cdot 10^{-5} = 1,9 \cdot 10^{-5}$$

$$L = 10 \cdot \log \frac{1,9 \cdot 10^{-5}}{10^{-12}} = 72,8$$

Dus het geluidsniveau stijgt met $72,8 - 65 = 7,8 \text{ dB}$

Opgave 59:

a. stel $I_1 = 10^{-4}$

dan is $I_2 = \frac{I_1}{4} = \frac{10^{-4}}{4} = 2,5 \cdot 10^{-5}$

$$L_1 = 10 \cdot \log \frac{10^{-4}}{10^{-12}} = 80$$

$$L_2 = 10 \cdot \log \frac{2,5 \cdot 10^{-5}}{10^{-12}} = 74$$

Dus het geluidsniveau daalt inderdaad 6 dB.

b. De afstand wordt $2\times$ verdubbeld, dus het geluidsniveau daalt met $2 \cdot 6 = 12$ dB.

Dus $85 - 12 = 73$ dB.

Opgave 60:

$$10 \cdot \log \frac{I_1}{10^{-12}} = 80$$

$$10 \cdot \log \frac{I_2}{10^{-12}} = 90$$

$$\log \frac{I_1}{10^{-12}} = 8$$

$$\log \frac{I_2}{10^{-12}} = 9$$

$$\frac{I_1}{10^{-12}} = 10^8$$

$$\frac{I_2}{10^{-12}} = 10^9$$

$$I_1 = 10^{-4}$$

$$I_2 = 10^{-3}$$

$$\frac{I_2}{I_1} = \frac{10^{-3}}{10^{-4}} = 10 \text{ dus } I_2 \text{ geldt voor } 10 \text{ boxen}$$

Dus men mag er nog 45 bij plaatsen.