6.5 Diagnostische toets

Opgave 1:

a. $\sin(-270°) = \sin 90° = 1$

b. $\sin 135° = \sin 45° = \frac{1}{2}\sqrt{2}$

c. $\cos 225° = -\cos 45° = -\frac{1}{2}\sqrt{2}$

d. $\cos(-120°) = -\cos 60° = -\frac{1}{2}$

e. $\sin 240° = -\sin 60° = -\frac{1}{2}\sqrt{3}$

f. $\cos 330° = \cos 30° = \frac{1}{2}\sqrt{3}$

Opgave 2:

a. $x_P = \cos 205° = -0{,}91 \quad y_P = \sin 205° = -0{,}42$ dus $P = (-0{,}91; -0{,}42)$

b. $x_Q = \cos(-37°) = 0{,}80 \quad y_Q = \sin(-37°) = -0{,}60$ dus $Q = (0{,}80; -0{,}60)$

Opgave 3:

a. $\frac{1}{5}\pi \ rad = 36°$

b. $10\pi \ rad = 1800°$

c. $-4\pi \ rad = -720°$

d. $-4 \ rad = \frac{-4}{2\pi} \cdot 360 = -229{,}2°$

e. $\frac{2}{3}\pi \ rad = 120°$

f. $\frac{2}{3} \ rad = \frac{\frac{2}{3}}{2\pi} \cdot 360 = 38{,}2°$

Opgave 4:

a. $270° = 1\frac{1}{2}\pi \ rad$

b. $60° = \frac{1}{3}\pi \ rad$

c. $-150° = -\frac{5}{6}\pi \ rad$

d. $-135° = -\frac{3}{4}\pi \ rad$

e. $540° = 3\pi \ rad$

f. $390° = 2\frac{1}{6}\pi \ rad$

Opgave 5:

a. $26° = \frac{26}{360} \cdot 2\pi = 0{,}45 \ rad$

b. $-73° = \frac{-73}{360} \cdot 2\pi = -1{,}27 \ rad$

c. $1010° = \frac{1010}{360} \cdot 2\pi = 17{,}63 \ rad$

Opgave 6:

a. $\sin \frac{2}{7}\pi = 0{,}78$

b. $\sin \frac{2}{7} = 0{,}28$

c. $\cos 1\frac{3}{5}\pi = 0{,}31$

Opgave 7:

a. $\sin\frac{5}{6}\pi = \sin\frac{1}{6}\pi = \frac{1}{2}$

b. $\cos\frac{3}{4}\pi = -\cos\frac{1}{4}\pi = -\frac{1}{2}\sqrt{2}$

c. $\cos 1\frac{1}{3}\pi = -\cos\frac{1}{3}\pi = -\frac{1}{2}$

Opgave 8:

a. $\sin\alpha = \frac{1}{2}$

$\alpha = \frac{1}{6}\pi \quad \vee \quad \alpha = \frac{5}{6}\pi$

b. $\sin\alpha = -\frac{1}{2}\sqrt{2}$

$\alpha = 1\frac{1}{4}\pi \quad \vee \quad \alpha = 1\frac{3}{4}\pi$

c. $\cos\alpha = \frac{1}{2}\sqrt{3}$

$\alpha = \frac{1}{6}\pi \quad \vee \quad \alpha = 1\frac{5}{6}\pi$

Opgave 9:

a. $\sin(2x + \frac{1}{2}\pi) = 0$

$2x + \frac{1}{2}\pi = 0 + k \cdot \pi$

$2x = -\frac{1}{2}\pi + k \cdot \pi$

$x = -\frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$

b. $\cos(2x + \frac{1}{6}\pi) = 1$

$2x + \frac{1}{6}\pi = 0 + k \cdot 2\pi$

$2x = -\frac{1}{6}\pi + k \cdot 2\pi$

$x = -\frac{1}{12}\pi + k \cdot \pi$

c. $\sin^2(\frac{1}{2}x) - \sin(\frac{1}{2}x) = 0$

$\sin(\frac{1}{2}x) \cdot (\sin(\frac{1}{2}x) - 1) = 0$

$\sin(\frac{1}{2}x) = 0 \quad \vee \quad \sin(\frac{1}{2}x) = 1$

$\frac{1}{2}x = 0 + k \cdot \pi \quad \vee \quad \frac{1}{2}x = \frac{1}{2}\pi + k \cdot 2\pi$

$x = 0 + k \cdot 2\pi \quad \vee \quad x = \pi + k \cdot 4\pi$

Opgave 10:

a. $\sin(\frac{1}{2}x + \pi) = \frac{1}{2}\sqrt{2}$

$\frac{1}{2}x + \pi = \frac{1}{4}\pi + k \cdot 2\pi \quad \vee \quad \frac{1}{2}x + \pi = \frac{3}{4}\pi + k \cdot 2\pi$

$\frac{1}{2}x = -\frac{3}{4}\pi + k \cdot 2\pi \quad \vee \quad \frac{1}{2}x = -\frac{1}{4}\pi + k \cdot 2\pi$

$x = -1\frac{1}{2}\pi + k \cdot 4\pi \quad \vee \quad x = -\frac{1}{2}\pi + k \cdot 4\pi$

b. $\cos(-\frac{1}{3}x + \frac{1}{2}\pi) = -\frac{1}{2}$

$-\frac{1}{3}x + \frac{1}{2}\pi = \frac{2}{3}\pi + k \cdot 2\pi \quad \vee \quad -\frac{1}{3}x + \frac{1}{2}\pi = -\frac{2}{3}\pi + k \cdot 2\pi$

$-\frac{1}{3}x = \frac{1}{6}\pi + k \cdot 2\pi \quad \vee \quad -\frac{1}{3}x = -1\frac{1}{6}\pi + k \cdot 2\pi$

$x = -\frac{1}{2}\pi + k \cdot 6\pi \quad \vee \quad x = -3\frac{1}{2}\pi + k \cdot 6\pi$

c. $4\cos^2(\frac{1}{2}\pi x) = 3$

$\cos^2(\frac{1}{2}\pi x) = \frac{3}{4}$

$\cos(\frac{1}{2}\pi x) = \sqrt{\frac{3}{4}} = \frac{1}{2}\sqrt{3} \quad \vee \quad \cos(\frac{1}{2}\pi x) = -\frac{1}{2}\sqrt{3}$

$\frac{1}{2}\pi x = \frac{1}{6}\pi + k \cdot 2\pi \quad \vee \quad \frac{1}{2}\pi x = 1\frac{1}{6}\pi + k \cdot 2\pi \quad \vee \quad \frac{1}{2}\pi x = \frac{5}{6}\pi + k \cdot 2\pi$

$\vee \quad \frac{1}{2}\pi x = -\frac{5}{6}\pi + k \cdot 2\pi$

$x = \frac{1}{3} + k \cdot 4 \quad \vee \quad x = -\frac{1}{3} + k \cdot 4 \quad \vee \quad x = \frac{5}{3} + k \cdot 4 \quad \vee \quad x = -\frac{5}{3} + k \cdot 4$

Opgave 11:

a. $2\sin 2x = -\sqrt{3}$

$\sin 2x = -\frac{1}{2}\sqrt{3}$

$2x = 1\frac{1}{3}\pi + k \cdot 2\pi \quad \vee \quad 2x = 1\frac{2}{3}\pi + k \cdot 2\pi$

$x = \frac{2}{3}\pi + k \cdot \pi \quad \vee \quad x = \frac{5}{6}\pi + k \cdot \pi$

$x = \frac{2}{3}\pi \quad \vee \quad x = \frac{5}{6}\pi \quad \vee \quad x = 1\frac{2}{3}\pi \quad \vee \quad x = 1\frac{5}{6}\pi$

b. $2\cos(1\frac{1}{2}x - \frac{1}{6}\pi) = -\sqrt{2}$

$\cos(1\frac{1}{2}x - \frac{1}{6}\pi) = -\frac{1}{2}\sqrt{2}$

$1\frac{1}{2}x - \frac{1}{6}\pi = \frac{3}{4}\pi + k \cdot 2\pi \quad \vee \quad 1\frac{1}{2}x - \frac{1}{6}\pi = 1\frac{1}{4}\pi + k \cdot 2\pi$

$1\frac{1}{2}x = \frac{11}{12}\pi + k \cdot 2\pi \quad \vee \quad 1\frac{1}{2}x = 1\frac{5}{12}\pi + k \cdot 2\pi$

$x = \frac{11}{18}\pi + k \cdot \frac{4}{3}\pi \quad \vee \quad x = \frac{17}{18}\pi + k \cdot \frac{4}{3}\pi$

$x = \frac{11}{18}\pi \quad \vee \quad x = \frac{17}{18}\pi \quad \vee \quad x = 1\frac{17}{18}\pi$

c. $\sin^2 x - \frac{1}{2}\sin x - \frac{1}{2} = 0$

$(\sin x - 1)(\sin x + \frac{1}{2}) = 0$

$\sin x = 1 \quad \vee \quad \sin x = -\frac{1}{2}$

$x = \frac{1}{2}\pi \quad \vee \quad x = 1\frac{1}{6}\pi \quad \vee \quad x = 1\frac{5}{6}\pi$

Opgave 12:

a. $\sin(2x - 1) = \sin(x + 2)$

$2x - 1 = x + 2 + k \cdot 2\pi \quad \vee \quad 2x - 1 = \pi - (x + 2) + k \cdot 2\pi$

$x = 3 + k \cdot 2\pi \quad \vee \quad 2x - 1 = \pi - x - 2 + k \cdot 2\pi$

$\vee \quad 3x = \pi - 1 + k \cdot 2\pi$

$\vee \quad x = \frac{1}{3}\pi - \frac{1}{3} + k \cdot \frac{2}{3}\pi$

b. $\cos(x + \frac{1}{3}\pi) = \cos(2x - \frac{1}{2}\pi)$

$x + \frac{1}{3}\pi = 2x - \frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad x + \frac{1}{3}\pi = -2x + \frac{1}{2}\pi + k \cdot 2\pi$

$-x = -\frac{5}{6}\pi + k \cdot 2\pi \quad \vee \quad 3x = \frac{1}{6}\pi + k \cdot 2\pi$

$x = \frac{5}{6}\pi + k \cdot 2\pi \quad \vee \quad x = \frac{1}{18}\pi + k \cdot \frac{2}{3}\pi$

c. $\sin(\frac{1}{2}\pi x) = \sin(\pi(x + 1))$

$\frac{1}{2}\pi x = \pi(x + 1) + k \cdot 2\pi \quad \vee \quad \frac{1}{2}\pi x = \pi - \pi(x + 1) + k \cdot 2\pi$

$\frac{1}{2}\pi x = \pi x + \pi + k \cdot 2\pi \quad \vee \quad \frac{1}{2}\pi x = \pi - \pi x - \pi + k \cdot 2\pi$

$-\frac{1}{2}\pi x = \pi + k \cdot 2\pi \quad \vee \quad 1\frac{1}{2}\pi x = 0 + k \cdot 2\pi$

$x = -2 + k \cdot 4 \quad \vee \quad x = 0 + k \cdot \frac{4}{3}$

Opgave 13:

a. $f(x) = 2\sin(3x - \frac{1}{2}\pi) = 2\sin 3(x - \frac{1}{6}\pi)$

1. $V_{y-as,\frac{1}{3}}$ 2. $V_{x-as,2}$ 3. $T(\frac{1}{6}\pi,0)$

b. 1. $V_{y-as,3}$ 2. $T(-2,5)$

c. $h(x) = 1 + 2\sin(3x - \frac{1}{4}\pi) = 1 + 2\sin 3(x - \frac{1}{12}\pi)$

1. $V_{y-as,\frac{1}{3}}$ 2. $V_{x-as,2}$ 3. $T(\frac{1}{12}\pi,1)$

Opgave 14:

$y = \sin x \xrightarrow{T(\frac{1}{2}\pi,3)} y = 3 + \sin(x - \frac{1}{2}\pi) \xrightarrow{V_{y-as,\frac{1}{5}}} y = 3 + \sin(5x - \frac{1}{2}\pi)$

Opgave 15:

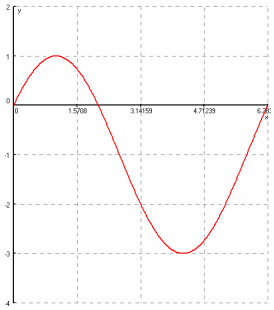
a. $f(x) = -1 + 2\cos(x - \frac{1}{3}\pi)$

evenwichts.as: -1

amplitude: 2

periode: 2π

beginpunt: $(\frac{1}{3}\pi,1)$



b. $-1 + 2\cos(x - \frac{1}{3}\pi) = -1$

$2\cos(x - \frac{1}{3}\pi) = 0$

$\cos(x - \frac{1}{3}\pi) = 0$

$x - \frac{1}{3}\pi = \frac{1}{2}\pi + k \cdot \pi$

$x = \frac{5}{6}\pi + k \cdot \pi$

$x = \frac{5}{6}\pi \quad \vee \quad x = 1\frac{5}{6}\pi$

$(\frac{5}{6}\pi,-1)$ en $(1\frac{5}{6}\pi,-1)$

c. $-1 + 2\cos(x - \frac{1}{3}\pi) = 1$

$2\cos(x - \frac{1}{3}\pi) = 2$

$\cos(x - \frac{1}{3}\pi) = 1$

$x - \frac{1}{3}\pi = 0$

$x = \frac{1}{3}\pi$

$(\frac{1}{3}\pi,1)$

$-1 + 2\cos(x - \frac{1}{3}\pi) = -3$

$2\cos(x - \frac{1}{3}\pi) = -2$

$\cos(x - \frac{1}{3}\pi) = -1$

$x - \frac{1}{3}\pi = \pi$

$x = 1\frac{1}{3}\pi$

$(1\frac{1}{3}\pi,-3)$

d. $-1 + 2\cos(x - \frac{1}{3}\pi) = 0$

$2\cos(x - \frac{1}{3}\pi) = 1$

$\cos(x - \frac{1}{3}\pi) = \frac{1}{2}$

$x - \frac{1}{3}\pi = \frac{1}{3}\pi + k \cdot 2\pi \quad \vee \quad x - \frac{1}{3}\pi = -\frac{1}{3}\pi + k \cdot 2\pi$

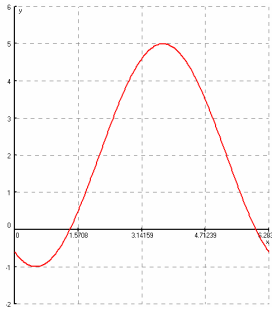
$x = \frac{2}{3}\pi + k \cdot 2\pi \quad \vee \quad x = 0 + k \cdot 2\pi$

$x = 0 \quad \vee \quad x = \frac{2}{3}\pi \quad \vee \quad x = 2\pi$ x

Opgave 16:

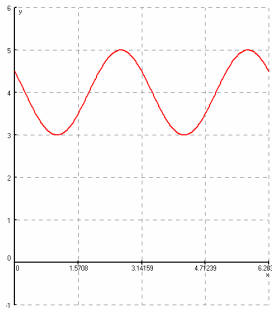
a. $f(x) = 2 - 3\sin(x + \frac{1}{3}\pi)$

evenwichts as: 2
amplitude: 3
periode: 2π
beginpunt dalend door $(-\frac{1}{3}\pi, 2)$



b. $g(x) = 4 + \cos(2x - 1\frac{2}{3}\pi) = 4 + \cos 2(x - \frac{5}{6}\pi)$

evenwichts as: 4
amplitude: 1
periode: π
beginpunt: $(\frac{5}{6}\pi, 5)$



Opgave 17:

$V = 30 + 10\sin(\frac{2}{7}\pi(t-2))$

a. evenwichts as: 30
amplitude: 10
periode: $\frac{2\pi}{\frac{2}{7}\pi} = 7$

beginpunt: $(2, 30)$

b. $y_1 = 30 + 10\sin(\frac{2}{7}\pi(x-2))$ en $y_2 = 25$

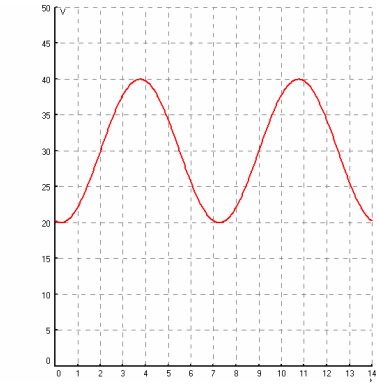
intersect geeft:

$x = 1{,}42 \quad \vee \quad x = 6{,}08 \quad \vee \quad x = 8{,}42 \quad \vee \quad x = 13{,}08$

$1{,}42 < t < 6{,}08 \quad \vee \quad 8{,}42 < t < 13{,}08$

c. dat is in het beginpunt, dus op $t = 2$

$\left[\frac{dV}{dt}\right]_{t=2} = 8{,}98$



Opgave 18:

a. minimum $= -35$
maximum $= 65$
evenwichts as: 15
amplitude: 50
periode: 30 dus $c = \frac{2\pi}{30} = \frac{1}{15}\pi$
beginpunt: $t = 25$ dalend door evenwichts-as
$N = 15 - 50\sin\frac{1}{15}\pi(t - 25)$

b. $N = 15 + 50\cos\frac{1}{15}\pi(t - 2{,}5)$

Opgave 19:

minimum $= -1{,}65$
maximum $= 3{,}65$
evenwichts as: 1
amplitude: 2,65

periode: π dus $c = \frac{2\pi}{\pi} = 2$
beginpunt: $x = 2{,}19$
$h(x) = 1 + 2{,}65 \sin 2(x - 2{,}19)$