6.2 Goniometrische vergelijkingen.

Opgave 15:

Bij $(0,1)$: $\ldots, -3\frac{1}{2}\pi, -1\frac{1}{2}\pi, \frac{1}{2}\pi, 2\frac{1}{2}\pi, 4\frac{1}{2}\pi, \ldots$

Bij $(0,-1)$: $\ldots, -4\frac{1}{2}\pi, -2\frac{1}{2}\pi, -\frac{1}{2}\pi, 1\frac{1}{2}\pi, 3\frac{1}{2}\pi, \ldots$

Opgave 16:

a. $\sin(3x - \frac{1}{2}\pi) = 0$

$3x - \frac{1}{2}\pi = 0 + k \cdot \pi$

$3x = \frac{1}{2}\pi + k \cdot \pi$

$x = \frac{1}{6}\pi + k \cdot \frac{1}{3}\pi$

b. $\cos(\frac{1}{2}x - \frac{1}{6}\pi) = 0$

$\frac{1}{2}x - \frac{1}{6}\pi = \frac{1}{2}\pi + k \cdot \pi$

$\frac{1}{2}x = \frac{2}{3}\pi + k \cdot \pi$

$x = 1\frac{1}{3}\pi + k \cdot 2\pi$

c. $\sin^2 x - \sin x = 0$

$\sin x(\sin x - 1) = 0$

$\sin x = 0 \quad \vee \quad \sin x = 1$

$x = 0 + k \cdot \pi \quad \vee \quad x = \frac{1}{2}\pi + k \cdot 2\pi$

d. $\cos^2 2x + \cos 2x = 0$

$\cos 2x(\cos 2x + 1) = 0$

$\cos 2x = 0 \quad \vee \quad \cos 2x = -1$

$2x = \frac{1}{2}\pi + k \cdot \pi \quad \vee \quad 2x = \pi + k \cdot 2\pi$

$x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi \quad \vee \quad x = \frac{1}{2}\pi + k \cdot \pi$

Opgave 17:

a. $\sin 2x = 1$

$2x = \frac{1}{2}\pi + k \cdot 2\pi$

$x = \frac{1}{4}\pi + k \cdot \pi$

$\sin 2x = -1$

$2x = -\frac{1}{2}\pi + k \cdot 2\pi$

$x = -\frac{1}{4}\pi + k \cdot \pi$

b. $\ldots, -1\frac{1}{4}\pi, -\frac{1}{4}\pi, \frac{3}{4}\pi, 1\frac{3}{4}\pi, \ldots$

c. als je de twee rijtjes oplossingen samenvoegt krijg je:

$\ldots, -1\frac{3}{4}\pi, -1\frac{1}{4}\pi, -\frac{3}{4}\pi, -\frac{1}{4}\pi, \frac{1}{4}\pi, \frac{3}{4}\pi, 1\frac{1}{4}\pi, 1\frac{3}{4}\pi, \ldots$

Dat is: $x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$

Opgave 18:

a. $\cos^2(x - \frac{1}{5}\pi) = 1$

$\cos(x - \frac{1}{5}\pi) = 1 \quad \vee \quad \cos(x - \frac{1}{5}\pi) = -1$

$x - \frac{1}{5}\pi = 0 + k \cdot 2\pi \quad \vee \quad x - \frac{1}{5}\pi = \pi + k \cdot 2\pi$

$x = \frac{1}{5}\pi + k \cdot 2\pi \quad \vee \quad x = 1\frac{1}{5}\pi + k \cdot 2\pi$

samen $x = \frac{1}{5}\pi + k \cdot \pi$

b. $\sin^2(2x - \frac{1}{4}\pi) = 1$

$\sin(2x - \frac{1}{4}\pi) = 1 \quad \vee \quad \sin(2x - \frac{1}{4}\pi) = -1$

$2x - \frac{1}{4}\pi = \frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad 2x - \frac{1}{4}\pi = -\frac{1}{2}\pi + k \cdot 2\pi$

$2x = \frac{3}{4}\pi + k \cdot 2\pi \quad \vee \quad 2x = -\frac{1}{4}\pi + k \cdot 2\pi$

$x = \frac{3}{8}\pi + k \cdot \pi \quad \vee \quad x = -\frac{1}{8}\pi + k \cdot \pi$

samen $x = \frac{3}{8}\pi + k \cdot \frac{1}{2}\pi$

c. $\sin^3 x - \sin x = 0$

$\sin x(\sin^2 x - 1) = 0$

$\sin x = 0 \quad \vee \quad \sin^2 x = 1$

$\sin x = 0 \quad \vee \quad \sin x = 1 \quad \vee \quad \sin x = -1$

$x = 0 + k \cdot \pi \quad \vee \quad x = \frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad x = -\frac{1}{2}\pi + k \cdot 2\pi$

samen $x = 0 + k \cdot \frac{1}{2}\pi$

d. $\cos^3 2x - \cos 2x = 0$

$\cos 2x(\cos^2 2x - 1) = 0$

$\cos 2x = 0 \quad \vee \quad \cos^2 2x = 1$

$\cos 2x = 0 \quad \vee \quad \cos 2x = 1 \quad \vee \quad \cos 2x = -1$

$2x = \frac{1}{2}\pi + k \cdot \pi \quad \vee \quad 2x = 0 + k \cdot 2\pi \quad \vee \quad 2x = \pi + k \cdot 2\pi$

samen $2x = 0 + k \cdot \frac{1}{2}\pi$

dus $x = 0 + k \cdot \frac{1}{4}\pi$

Opgave 19:

a. $\sin(4x - \frac{1}{3}\pi) = 1$

$4x - \frac{1}{3}\pi = \frac{1}{2}\pi + k \cdot 2\pi$

$4x = \frac{5}{6}\pi + k \cdot 2\pi$

$x = \frac{5}{24}\pi + k \cdot \frac{1}{2}\pi$

b. $\cos 4\pi x = 1$

$4\pi x = 0 + k \cdot 2\pi$

$x = 0 + k \cdot \frac{1}{2}$

c. $\sin^2(\frac{1}{4}\pi x) = 1$

$\sin \frac{1}{4}\pi x = 1 \quad \vee \quad \sin \frac{1}{4}\pi x = -1$

$\frac{1}{4}\pi x = \frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad \frac{1}{4}\pi x = -\frac{1}{2}\pi + k \cdot 2\pi$

samen $\frac{1}{4}\pi x = \frac{1}{2}\pi + k \cdot \pi$

dus $x = 2 + k \cdot 4$

d. $\sin 2x \cdot \cos 2x + \sin 2x = 0$

$\sin 2x(\cos 2x + 1) = 0$

$\sin 2x = 0 \quad \vee \quad \cos 2x = -1$

$2x = 0 + k \cdot \pi \quad \vee \quad 2x = \pi + k \cdot 2\pi$

$x = 0 + k \cdot \frac{1}{2}\pi \quad \vee \quad x = \frac{1}{2}\pi + k \cdot \pi$

dus $x = 0 + k \cdot \frac{1}{2}\pi$

Opgave 20:

a. $\sin \frac{1}{6}\pi = \frac{1}{2}$

b. $\sin 2\frac{1}{6}\pi = \sin \frac{1}{6}\pi = \frac{1}{2}$

$\sin 4\frac{1}{6}\pi = \sin \frac{1}{6}\pi = \frac{1}{2}$

c. $\sin \frac{5}{6}\pi = \frac{1}{2}$

d. $\sin 2\frac{5}{6}\pi = \sin \frac{5}{6}\pi = \frac{1}{2}$

$\sin -1\frac{1}{6}\pi = \sin \frac{5}{6}\pi = \frac{1}{2}$

Opgave 21:

a. $2\sin \frac{1}{2}x = 1$

$\sin \frac{1}{2}x = \frac{1}{2}$

$\frac{1}{2}x = \frac{1}{6}\pi + k \cdot 2\pi \quad \vee \quad \frac{1}{2}x = \frac{5}{6}\pi + k \cdot 2\pi$

$x = \frac{1}{3}\pi + k \cdot 4\pi \quad \vee \quad x = \frac{5}{3}\pi + k \cdot 4\pi$

b. $2\cos(x - \frac{1}{3}\pi) = 1$

$\cos(x - \frac{1}{3}\pi) = \frac{1}{2}$

$x - \frac{1}{3}\pi = \frac{1}{3}\pi + k \cdot 2\pi \quad \vee \quad x - \frac{1}{3}\pi = -\frac{1}{3}\pi + k \cdot 2\pi$

$x = \frac{2}{3}\pi + k \cdot 2\pi \quad \vee \quad x = 0 + k \cdot 2\pi$

c. $2\sin(2x - \frac{1}{4}\pi) = -\sqrt{3}$

$\sin(2x - \frac{1}{4}\pi) = -\frac{1}{2}\sqrt{3}$

$2x - \frac{1}{4}\pi = -\frac{1}{3}\pi + k \cdot 2\pi \quad \vee \quad 2x - \frac{1}{4}\pi = 1\frac{1}{3}\pi + k \cdot 2\pi$

$2x = -\frac{1}{12}\pi + k \cdot 2\pi \quad \vee \quad 2x = 1\frac{7}{12}\pi + k \cdot 2\pi$

$x = -\frac{1}{24}\pi + k \cdot \pi \quad \vee \quad x = \frac{19}{24}\pi + k \cdot \pi$

d. $2\cos(3x - \pi) = -1$

$\cos(3x - \pi) = -\frac{1}{2}$

$3x - \pi = \frac{2}{3}\pi + k \cdot 2\pi \quad \vee \quad 3x - \pi = -\frac{2}{3}\pi + k \cdot 2\pi$

$3x = 1\frac{2}{3}\pi + k \cdot 2\pi \quad \vee \quad 3x = \frac{1}{3}\pi + k \cdot 2\pi$

$x = \frac{5}{9}\pi + k \cdot \frac{2}{3}\pi \quad \vee \quad x = \frac{1}{9}\pi + k \cdot \frac{2}{3}\pi$

Opgave 22:

a. $2\sin(2x - \frac{1}{6}\pi) = \sqrt{2}$

$\sin(2x - \frac{1}{6}\pi) = \frac{1}{2}\sqrt{2}$

$2x - \frac{1}{6}\pi = \frac{1}{4}\pi + k \cdot 2\pi \quad \vee \quad 2x - \frac{1}{6}\pi = \frac{3}{4}\pi + k \cdot 2\pi$

$2x = \frac{5}{12}\pi + k \cdot 2\pi \quad \vee \quad 2x = \frac{11}{12}\pi + k \cdot 2\pi$

$x = \frac{5}{24}\pi + k \cdot \pi \quad \vee \quad x = \frac{11}{24}\pi + k \cdot \pi$

$x = \frac{5}{24}\pi \quad \vee \quad x = \frac{11}{24}\pi \quad \vee \quad x = 1\frac{5}{24}\pi \quad \vee \quad x = 1\frac{11}{24}\pi$

b. $2\cos(3x - \frac{1}{2}\pi) = \sqrt{3}$

$\cos(3x - \frac{1}{2}\pi) = \frac{1}{2}\sqrt{3}$

$3x - \frac{1}{2}\pi = \frac{1}{6}\pi \quad \vee \quad 3x - \frac{1}{2}\pi = -\frac{1}{6}\pi$

$3x = \frac{2}{3}\pi + k \cdot 2\pi \quad \vee \quad 3x = \frac{1}{3}\pi + k \cdot 2\pi$

$x = \frac{2}{9}\pi + k \cdot \frac{2}{3}\pi \quad \vee \quad x = \frac{1}{9}\pi + k \cdot \frac{2}{3}\pi$

$x = \frac{1}{9}\pi \quad \vee \quad x = \frac{2}{9}\pi \quad \vee \quad x = \frac{7}{9}\pi \quad \vee \quad x = \frac{8}{9}\pi \quad \vee \quad x = 1\frac{4}{9}\pi \quad \vee \quad x = 1\frac{5}{9}\pi$

c. $\sin \frac{2}{3}x = -\frac{1}{2}\sqrt{2}$

$\frac{2}{3}x = -\frac{1}{4}\pi + k \cdot 2\pi \quad \vee \quad \frac{2}{3}x = 1\frac{1}{4}\pi + k \cdot 2\pi$

$x = -\frac{3}{8}\pi + k \cdot 3\pi \quad \vee \quad x = 1\frac{7}{8}\pi + k \cdot 3\pi$

$x = 1\frac{7}{8}\pi$

d. $\cos \frac{1}{2}x = -\frac{1}{2}\sqrt{3}$

$\frac{1}{2}x = \frac{5}{6}\pi + k \cdot 2\pi \quad \vee \quad \frac{1}{2}x = 1\frac{1}{6}\pi + k \cdot 2\pi$

$x = 1\frac{2}{3}\pi + k \cdot 4\pi \quad \vee \quad x = 2\frac{1}{3}\pi + k \cdot 4\pi$

$x = 1\frac{2}{3}\pi$

Opgave 23:

a. $2\sin^2 x = 1$

$\sin^2 x = \frac{1}{2}$

$\sin x = \sqrt{\frac{1}{2}} = \frac{1}{2}\sqrt{2} \quad \vee \quad \sin x = -\frac{1}{2}\sqrt{2}$

b. $\sin x = \frac{1}{2}\sqrt{2}$

$x = \frac{1}{4}\pi + k \cdot 2\pi \quad \vee \quad x = \frac{3}{4}\pi + k \cdot 2\pi$

$\sin x = -\frac{1}{2}\sqrt{2}$

$x = 1\frac{1}{4}\pi + k \cdot 2\pi \quad \vee \quad x = 1\frac{3}{4}\pi + k \cdot 2\pi$

c. de oplossingen verschillen steeds $\frac{1}{2}\pi$ van elkaar, dus $x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$

d. $x = \frac{1}{4}\pi$ is een oplossing en iedere andere oplossing ligt steeds $\frac{1}{2}\pi$ verder (of terug).

Opgave 24:

a. $2\cos^2(\frac{1}{2}x) = 1$

$\cos^2(\frac{1}{2}x) = \frac{1}{2}$

$\cos \frac{1}{2}x = \frac{1}{2}\sqrt{2} \quad \vee \quad \cos \frac{1}{2}x = -\frac{1}{2}\sqrt{2}$

$\frac{1}{2}x = \frac{1}{4}\pi + k \cdot 2\pi \quad \vee \quad \frac{1}{2}x = -\frac{1}{4}\pi + k \cdot 2\pi \quad \vee \quad \frac{1}{2}x = \frac{3}{4}\pi + k \cdot 2\pi \quad \vee \quad \frac{1}{2}x = -\frac{3}{4}\pi + k \cdot 2\pi$

$\frac{1}{2}x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$

$x = \frac{1}{2}\pi + k \cdot \pi$

b. $4\sin^2(x - \frac{1}{6}\pi) = 1$

$\sin^2(x - \frac{1}{6}\pi) = \frac{1}{4}$

$\sin(x - \frac{1}{6}\pi) = \frac{1}{2} \quad \vee \quad \sin(x - \frac{1}{6}\pi) = -\frac{1}{2}$

$x - \frac{1}{6}\pi = \frac{1}{6}\pi + k \cdot 2\pi \quad \vee \quad x - \frac{1}{6}\pi = \frac{5}{6}\pi + k \cdot 2\pi \quad \vee \quad x - \frac{1}{6}\pi = 1\frac{1}{6}\pi + k \cdot 2\pi \quad \vee$

$x - \frac{1}{6}\pi = 1\frac{5}{6}\pi + k \cdot 2\pi$

$x = \frac{1}{3}\pi + k \cdot 2\pi \quad \vee \quad x = \pi + k \cdot 2\pi \quad \vee \quad x = 1\frac{1}{3}\pi + k \cdot 2\pi \quad \vee \quad x = 2\pi + k \cdot 2\pi$

dus $x = \frac{1}{3}\pi + k \cdot \pi \quad \vee \quad x = \pi + k \cdot \pi$

c. $4\cos^2(x + \frac{1}{4}\pi) = 3$

$\cos^2(x + \frac{1}{4}\pi) = \frac{3}{4}$

$\cos(x+\frac{1}{4}\pi)=\sqrt{\frac{3}{4}}=\frac{1}{2}\sqrt{3} \quad \vee \quad \cos(x+\frac{1}{4}\pi)=-\frac{1}{2}\sqrt{3}$

$x+\frac{1}{4}\pi=\frac{1}{6}\pi+k\cdot 2\pi \quad \vee \quad x+\frac{1}{4}\pi=-\frac{1}{6}\pi+k\cdot 2\pi \quad \vee \quad x+\frac{1}{4}\pi=\frac{5}{6}\pi+k\cdot 2\pi \quad \vee$

$x+\frac{1}{4}\pi=-\frac{5}{6}\pi+k\cdot 2\pi$

$x=-\frac{1}{12}\pi+k\cdot 2\pi \quad \vee \quad x=-\frac{5}{12}\pi+k\cdot 2\pi \quad \vee \quad x=\frac{7}{12}\pi+k\cdot 2\pi \quad \vee$

$x=-1\frac{1}{12}\pi+k\cdot 2\pi$

dus $x=-\frac{1}{12}\pi+k\cdot\pi \quad \vee \quad x=\frac{7}{12}\pi+k\cdot\pi$

d. $4\sin^3 x-\sin x=0$

$\sin x(4\sin^2 x-1)=0$

$\sin x=0 \quad \vee \quad 4\sin^2 x=1$

$\sin x=0 \quad \vee \quad \sin^2 x=\frac{1}{4}$

$\sin x=0 \quad \vee \quad \sin x=\frac{1}{2} \quad \vee \quad \sin x=-\frac{1}{2}$

$x=0+k\cdot\pi \quad \vee \quad x=\frac{1}{6}\pi+k\cdot\pi \quad \vee \quad x=\frac{5}{6}\pi+k\cdot\pi$

e. $2\cos^2 x=\cos x+1$

$2\cos^2 x-\cos x-1=0$

stel $p=\cos x$

$2p^2-p-1=0$

$p=\frac{1\pm\sqrt{1+8}}{4}=\frac{1\pm 3}{4}$

$p=\frac{1+3}{4}=1 \quad \vee \quad p=\frac{1-3}{4}=-\frac{1}{2}$

$\cos x=1 \quad \vee \quad \cos x=-\frac{1}{2}$

$x=0+k\cdot 2\pi \quad \vee x=\frac{2}{3}\pi+k\cdot 2\pi \quad \vee \quad x=1\frac{1}{3}\pi+k\cdot 2\pi$

dus $x=0+k\cdot\frac{2}{3}\pi$

f. $\cos^2 x-\cos x+\frac{1}{4}=0$

$(\cos x-\frac{1}{2})^2=0$

$\cos x=\frac{1}{2}$

$x=\frac{1}{3}\pi+k\cdot 2\pi \quad \vee \quad x=-\frac{1}{3}\pi+k\cdot 2\pi$

Opgave 25:

a. $\sin\frac{1}{2}\pi x=\frac{1}{2}\sqrt{3}$

$\frac{1}{2}\pi x=\frac{1}{3}\pi+k\cdot 2\pi \quad \vee \quad \frac{1}{2}\pi x=\frac{2}{3}\pi+k\cdot 2\pi$

$x=\frac{2}{3}+k\cdot 4 \quad \vee \quad x=1\frac{1}{3}+k\cdot 4$

$x=\frac{2}{3} \quad \vee \quad x=1\frac{1}{3} \quad \vee \quad x=4\frac{2}{3} \quad \vee \quad x=5\frac{1}{3} \quad \vee \quad x=8\frac{2}{3} \quad \vee \quad x=9\frac{1}{3}$

b. $\cos\frac{1}{3}\pi x=-\frac{1}{2}\sqrt{3}$

$\frac{1}{3}\pi x=\frac{5}{6}\pi+k\cdot 2\pi \quad \vee \quad \frac{1}{3}\pi x=1\frac{1}{6}\pi+k\cdot 2\pi$

$x=2\frac{1}{2}+k\cdot 6 \quad \vee \quad x=3\frac{1}{2}+k\cdot 6$

$x=2\frac{1}{2} \quad \vee \quad x=3\frac{1}{2} \quad \vee \quad x=8\frac{1}{2} \quad \vee \quad x=9\frac{1}{2}$

c. $4\sin^2(\frac{1}{5}\pi x)=1$

$\sin^2(\frac{1}{5}\pi x)=\frac{1}{4}$

$\sin(\frac{1}{5}\pi x)=\frac{1}{2} \quad \vee \quad \sin(\frac{1}{5}\pi x)=-\frac{1}{2}$

$\frac{1}{5}\pi x = \frac{1}{6}\pi + k \cdot \pi \quad \vee \quad \frac{1}{5}\pi x = \frac{5}{6}\pi + k \cdot \pi$

$x = \frac{5}{6} + k \cdot 5 \quad \vee \quad x = 4\frac{1}{6} + k \cdot 5$

$x = \frac{5}{6} \quad \vee \quad x = 4\frac{1}{6} \quad \vee \quad x = 5\frac{5}{6} \quad \vee \quad x = 9\frac{1}{6}$

d. $2\cos^2(0{,}1\pi x) + \cos(0{,}1\pi x) = 1$

$2\cos^2(0{,}1\pi x) + \cos(0{,}1\pi x) - 1 = 0$

stel $p = \cos(0{,}1\pi x)$

$2p^2 + p - 1 = 0$

$p = \frac{-1 \pm \sqrt{1+8}}{4} = \frac{-1 \pm 3}{4}$

$p = \frac{-1+3}{4} = \frac{1}{2} \quad \vee \quad p = \frac{-1-3}{4} = -1$

$\cos(0{,}1\pi x) = \frac{1}{2} \quad \vee \quad \cos(0{,}1\pi x) = -1$

$0{,}1\pi x = \frac{1}{3}\pi + k \cdot 2\pi \quad \vee \quad 0{,}1\pi x = -\frac{1}{3}\pi + k \cdot 2\pi \quad \vee \quad x = \pi + k \cdot 2\pi$

$x = 3\frac{1}{3} + k \cdot 20 \quad \vee \quad x = -3\frac{1}{3} + k \cdot 20 \quad \vee \quad x = 10 + k \cdot 20$

$x = 3\frac{1}{3} \quad \vee \quad x = 10$

Opgave 26:

a. $x = 0{,}775 + k \cdot 2\pi \quad \vee \quad x = \pi - 0{,}775 = 2{,}366 + k \cdot 2\pi$

b. $\cos x = 0{,}8$

$x = 0{,}644 + k \cdot 2\pi \quad \vee \quad x = -0{,}644 + k \cdot 2\pi$

Opgave 27:

a. $\sin x = -0{,}85$

$x = -1{,}016 + k \cdot 2\pi \quad \vee \quad x = \pi - -1{,}016 = 4{,}158 + k \cdot 2\pi$

b. $\cos\frac{1}{2}x = 0{,}25$

$\frac{1}{2}x = 1{,}318 + k \cdot 2\pi \quad \vee \quad \frac{1}{2}x = -1{,}318 + k \cdot 2\pi$

$x = 2{,}636 + k \cdot 4\pi \quad \vee \quad x = -2{,}363 + k \cdot 2\pi$

c. $\sin(x+2) = 0{,}9$

$x + 2 = 1{,}120 + k \cdot 2\pi \quad \vee \quad x + 2 = \pi - 1{,}120 = 2{,}022 + k \cdot 2\pi$

$x = -0{,}880 + k \cdot 2\pi \quad \vee \quad x = 0{,}022 + k \cdot 2\pi$

d. $\cos(2x+1) = -0{,}4$

$2x + 1 = 1{,}982 + k \cdot 2\pi \quad \vee \quad 2x + 1 = -1{,}982 + k \cdot 2\pi$

$2x = 0{,}982 + k \cdot 2\pi \quad \vee \quad 2x = -2{,}982 + k \cdot 2\pi$

$x = 0{,}491 + k \cdot \pi \quad \vee \quad x = -1{,}491 + k \cdot \pi$

Opgave 28:

a. $2\sin 1{,}75x = 1{,}4$

$\sin 1{,}75x = 0{,}7$

$1{,}75x = 0{,}775 + k \cdot 2\pi \quad \vee \quad 1{,}75x = \pi - 0{,}775 = 2{,}366 + k \cdot 2\pi$

$x = 0{,}443 + k \cdot 3{,}59 \quad \vee \quad x = 1{,}352 + k \cdot 3{,}59$

$x = 0{,}443 \quad \vee \quad x = 1{,}352 \quad \vee \quad x = 4{,}033 \quad \vee \quad x = 4{,}943$

b. $\cos^2 0{,}95x = 0{,}86$

$\cos 0{,}95x = 0{,}927 \quad \vee \quad \cos 0{,}95x = -0{,}927$

$0{,}95x = 0{,}383 + k \cdot 2\pi \quad \vee \quad 0{,}95x = -0{,}383 + k \cdot 2\pi \quad \vee \quad 0{,}95x = 2{,}757 + k \cdot 2\pi \quad \vee$
$0{,}95x = -2{,}757 + k \cdot 2\pi$
$x = 0{,}404 + k \cdot 6{,}614 \quad \vee \quad x = -0{,}404 + k \cdot 6{,}614 \quad \vee \quad x = 2{,}902 + k \cdot 6{,}614 \quad \vee$
$x = -2{,}902 + k \cdot 6{,}614$
$x = 0{,}404 \quad \vee \quad x = 2{,}902 \quad \vee \quad x = 3{,}712 \quad \vee \quad x = 6{,}210$

Opgave 29:

a. $\sin 3x = \sin \frac{1}{6}\pi$
$3x = \frac{1}{6}\pi + k \cdot 2\pi \quad \vee \quad 3x = \frac{5}{6}\pi + k \cdot 2\pi$
$x = \frac{1}{18}\pi + k \cdot \frac{2}{3}\pi \quad \vee \quad x = \frac{5}{18}\pi + k \cdot \frac{2}{3}\pi$

b. $\cos 3x = \cos \frac{1}{6}\pi$
$3x = \frac{1}{6}\pi + k \cdot 2\pi \quad \vee \quad 3x = -\frac{1}{6}\pi + k \cdot 2\pi$
$x = \frac{1}{18}\pi + k \cdot \frac{2}{3}\pi \quad \vee \quad x = -\frac{1}{18}\pi + k \cdot \frac{2}{3}\pi$

Opgave 30:

a. $\sin(x+1) = \sin(2x+3)$
$x+1 = 2x+3 + k \cdot 2\pi \quad \vee \quad x+1 = \pi - (2x+3) + k \cdot 2\pi$
$-x = 2 + k \cdot 2\pi \quad \vee \quad x+1 = \pi - 2x - 3 + k \cdot 2\pi$
$x = -2 + k \cdot 2\pi \quad \vee \quad 3x = \pi - 4 + k \cdot 2\pi$
$x = -2 + k \cdot 2\pi \quad \vee \quad x = \frac{1}{3}\pi - \frac{4}{3} + k \cdot \frac{2}{3}\pi$

b. $\cos(2x-1) = \cos(x+1)$
$2x-1 = x+1 + k \cdot 2\pi \quad \vee \quad 2x-1 = -x-1 + k \cdot 2\pi$
$x = 2 + k \cdot 2\pi \quad \vee \quad 3x = 0 + k \cdot 3\pi$
$x = 2 + k \cdot 2\pi \quad \vee \quad x = 0 + k \cdot \frac{2}{3}\pi$

c. $\sin(2x - \frac{1}{2}\pi) = \sin(x + \frac{1}{3}\pi)$
$2x - \frac{1}{2}\pi = x + \frac{1}{3}\pi + k \cdot 2\pi \quad \vee \quad 2x - \frac{1}{2}\pi = \pi - (x + \frac{1}{3}\pi) + k \cdot 2\pi$
$x = \frac{5}{6}\pi + k \cdot 2\pi \quad \vee \quad 2x - \frac{1}{2}\pi = \pi - x - \frac{1}{3}\pi + k \cdot 2\pi$
$x = \frac{5}{6}\pi + k \cdot 2\pi \quad \vee \quad 3x = 1\frac{1}{6}\pi + k \cdot 2\pi$
$x = \frac{5}{6}\pi + k \cdot 2\pi \quad \vee \quad x = \frac{7}{18}\pi + k \cdot \frac{2}{3}\pi$

d. $\cos(x - \frac{1}{3}\pi) = \cos 2x$
$x - \frac{1}{3}\pi = 2x + k \cdot 2\pi \quad \vee \quad x - \frac{1}{3}\pi = -2x + k \cdot 2\pi$
$-x = \frac{1}{3}\pi + k \cdot 2\pi \quad \vee \quad 3x = \frac{1}{3}\pi + k \cdot 2\pi$
$x = -\frac{1}{3}\pi + k \cdot 2\pi \quad \vee \quad x = \frac{1}{9}\pi + k \cdot \frac{2}{3}\pi$

e. $\sin 2\pi x = \sin \pi (x-1)$
$2\pi x = \pi x - \pi + k \cdot 2\pi \quad \vee \quad 2\pi x = \pi - (\pi x - \pi) + k \cdot 2\pi$
$\pi x = -\pi + k \cdot 2\pi \quad \vee \quad 2\pi x = \pi - \pi x + \pi + k \cdot 2\pi$
$x = -1 + k \cdot 2 \quad \vee \quad 3\pi x = 0 + k \cdot 2\pi$
$x = -1 + k \cdot 2 \quad \vee \quad x = 0 + k \cdot \frac{2}{3}$

f. $\cos \frac{1}{2}\pi x = \cos \pi (x-2)$
$\frac{1}{2}\pi x = \pi x - 2\pi + k \cdot 2\pi \quad \vee \quad \frac{1}{2}\pi x = -\pi x + 2\pi + k \cdot 2\pi$
$-\frac{1}{2}\pi x = 0 + k \cdot 2\pi \quad \vee \quad 1\frac{1}{2}\pi x = 0 + k \cdot 2\pi$

$x = 0 + k \cdot 4 \quad \vee \quad x = 0 + k \cdot \frac{4}{3}$

dus $x = 0 + k \cdot \frac{4}{3}$

Opgave 31:

a. $\sin(2x - \frac{1}{3}\pi) = \sin(x + \frac{1}{4}\pi)$

$2x - \frac{1}{3}\pi = x + \frac{1}{4}\pi + k \cdot 2\pi \quad \vee \quad 2x - \frac{1}{3}\pi = \pi - (x + \frac{1}{4}\pi) + k \cdot 2\pi$

$x = \frac{7}{12}\pi + k \cdot 2\pi \quad \vee \quad 2x - \frac{1}{3}\pi = \pi - x - \frac{1}{4}\pi + k \cdot 2\pi$

$x = \frac{7}{12}\pi + k \cdot 2\pi \quad \vee \quad 3x = 1\frac{1}{12}\pi + k \cdot 2\pi$

$x = \frac{7}{12}\pi + k \cdot 2\pi \quad \vee \quad x = \frac{13}{36}\pi + k \cdot \frac{2}{3}\pi$

$x = \frac{13}{36}\pi \quad \vee \quad x = \frac{7}{12}\pi \quad \vee \quad x = 1\frac{1}{36}\pi \quad \vee \quad x = 1\frac{25}{36}\pi$

b. $\cos(3x + \frac{1}{2}\pi) = \cos(2x - \frac{1}{4}\pi)$

$3x + \frac{1}{2}\pi = 2x - \frac{1}{4}\pi + k \cdot 2\pi \quad \vee \quad 3x + \frac{1}{2}\pi = -2x + \frac{1}{4}\pi + k \cdot 2\pi$

$x = -\frac{3}{4}\pi + k \cdot 2\pi \quad \vee \quad 5x = -\frac{1}{4}\pi + k \cdot 2\pi$

$x = -\frac{3}{4}\pi + k \cdot 2\pi \quad \vee \quad x = -\frac{1}{20}\pi + k \cdot \frac{2}{5}\pi$

$x = \frac{7}{20}\pi \vee \quad x = \frac{15}{20}\pi \quad \vee \quad x = 1\frac{3}{20}\pi \quad \vee \quad x = 1\frac{1}{4}\pi \quad \vee \quad x = 1\frac{11}{20}\pi \quad \vee \quad x = 1\frac{19}{20}\pi$