7.6 Diagnostische toets

Opgave 1:

a. $f'(x) = (2x+3)(3-7x) + (x^2+3x)\cdot -7 = (2x+3)(3-7x) - 7(x^2+3x)$

b. $g'(x) = 6x(3x^2+4) + (3x^2+4)\cdot 6x = 12x(3x^2+4)$

Opgave 2:

a. $f'(x) = \dfrac{(x^2+2)\cdot 3 - (3x-7)\cdot 2x}{(x^2+2)^2} = \dfrac{3x^2+6-6x^2+14x}{(x^2+2)^2} = \dfrac{-3x^2+14x+6}{(x^2+2)^2}$

b. $g'(x) = 3 - \dfrac{(x+4)\cdot 0 - 2\cdot 1}{(x+4)^2} = 3 + \dfrac{2}{(x+4)^2}$

Opgave 3:

$f(x) = \dfrac{x^2-9}{3x+2} = 0$

$x^2 - 9 = 0$

$x^2 = 9$

$x = -3 \quad \vee \quad x = 3$

$f'(x) = \dfrac{(3x+2)\cdot 2x - (x^2-9)\cdot 3}{(3x+2)^2} = \dfrac{6x^2+4x-3x^2+27}{(3x+2)^2} = \dfrac{3x^2+4x+27}{(3x+2)^2}$

$f'(-3) = \frac{6}{7}$	$f'(3) = \frac{6}{11}$
$y = \frac{6}{7}x + b$ door $(-3,0)$	$y = \frac{6}{11}x + b$ door $(3,0)$
$0 = -\frac{18}{7} + b$	$0 = \frac{18}{11} + b$
$b = \frac{18}{7} = 2\frac{4}{7}$	$b = -\frac{18}{11} = -1\frac{7}{11}$
$y = \frac{6}{7}x + 2\frac{4}{7}$	$y = \frac{6}{11}x - 1\frac{7}{11}$

Opgave 4:

a. $f(x) = \dfrac{2}{x^5} = 2x^{-5}$

$f'(x) = -10x^{-6} = -\dfrac{10}{x^6}$

b. $g(x) = \dfrac{x^5+2}{x^3} = \dfrac{x^5}{x^3} + \dfrac{2}{x^3} = x^2 + 2x^{-3}$

$g'(x) = 2x - 6x^{-4} = 2x - \dfrac{6}{x^4}$

c. $h(x) = \dfrac{3}{x} - \dfrac{x}{3} = 3x^{-1} - \frac{1}{3}x$

$h'(x) = -3x^{-2} - \frac{1}{3} = -\dfrac{3}{x^2} - \frac{1}{3}$

Opgave 5:

a. $f(x) = x^3 + \sqrt[3]{x^2} = x^3 + x^{\frac{2}{3}}$

$f'(x) = 3x^2 + \frac{2}{3}x^{-\frac{1}{3}} = 3x^2 + \dfrac{2}{3x^{\frac{1}{3}}} = 3x^2 + \dfrac{2}{3 \cdot \sqrt[3]{x}}$

b. $g(x) = x^3 \cdot \sqrt[3]{x^2} = x^3 \cdot x^{\frac{2}{3}} = x^{3\frac{2}{3}}$

$g'(x) = 3\frac{2}{3}x^{2\frac{2}{3}} = 3\frac{2}{3}x^2 \cdot \sqrt[3]{x^2}$

c. $h(x) = \dfrac{x\sqrt{x}}{x^3 + 1}$

$h'(x) = \dfrac{(x^3+1) \cdot 1\frac{1}{2}\sqrt{x} - x\sqrt{x} \cdot 3x^2}{(x^3+1)^2} = \dfrac{1\frac{1}{2}x^3 \cdot \sqrt{x} + 1\frac{1}{2}\sqrt{x} - 3x^2 \cdot \sqrt{x}}{(x^3+1)^2} =$

$= \dfrac{-1\frac{1}{2}x^3 \cdot \sqrt{x} + 1\frac{1}{2}\sqrt{x}}{(x^3+1)^2} = \dfrac{-3x^3 \cdot \sqrt{x} + 3\sqrt{x}}{2(x^3+1)^2}$

Opgave 6:

$y_A = -2$

$f(x) = \dfrac{x^2 - 3}{x^2\sqrt{x}} = \dfrac{x^2 - 3}{x^{2\frac{1}{2}}} = x^{-\frac{1}{2}} - 3x^{-2\frac{1}{2}}$

$f'(x) = -\frac{1}{2}x^{-1\frac{1}{2}} + 7\frac{1}{2}x^{-3\frac{1}{2}} = -\dfrac{1}{2x\sqrt{x}} + \dfrac{15}{2x^3 \cdot \sqrt{x}}$

$f'(1) = 7$

$y = 7x + b$ door $(1,-2)$

$-2 = 7 + b$

$b = -9$

$y = 7x - 9$

snijpunt x-as: $7x - 9 = 0$

$7x = 9$

$x = \frac{9}{7}$ dus $(\frac{9}{7},0)$

snijpunt y-as: $(0,-9)$

$Opp(\Delta)BC) = \frac{1}{2} \cdot \frac{9}{7} \cdot 9 = \frac{81}{14}$

Opgave 7:

a. $f(x) = 3(x^2 + 4x)^4 = 3u^4$ met $u = x^2 + 4x$ dus $u' = 2x + 4$

$f'(x) = 12u^3 \cdot u' = 12(x^2 + 4x)^3 \cdot (2x + 4)$

b. $g(x) = (x^2 + 2)\sqrt{x^2 + 2} = (x^2 + 2)^{1\frac{1}{2}} = u^{1\frac{1}{2}}$ met $u = x^2 + 2$ dus $u' = 2x$

$g'(x) = 1\frac{1}{2}u^{\frac{1}{2}} \cdot u' = 1\frac{1}{2}(x^2 + 2)^{\frac{1}{2}} \cdot 2x = 3x\sqrt{x^2 + 2}$

c. $h(x) = \dfrac{3}{(2x^3 + 2)^5} = 3(2x^3 + 2)^{-5} = 3u^{-5}$ met $u = 2x^3 + 2$ dus $u' = 6x^2$

$h'(x) = -15u^{-6} \cdot u' = -15(2x^3 + 2)^{-6} \cdot 6x^2 = \dfrac{-90x^2}{(2x^3 + 2)^6}$

Opgave 8:

a. $f(x) = 2x^2(x^2 - 4x)^5$

$f'(x) = 4x(x^2 - 4x)^5 + 2x^2 \cdot 5(x^2 - 4x)^4 \cdot (2x - 4) =$

$= 4x(x^2 - 4x)^5 + 10x^2(x^2 - 4x)^4(2x - 4)$

b. $g(x) = (x^3 + x)\sqrt{x^3 + 2}$

$g'(x) = (3x^2 + 1)\sqrt{x^3 + 2} + (x^3 + x) \cdot \dfrac{1}{2\sqrt{x^3 + 2}} \cdot 3x^2 =$

$= (3x^2 + 1)\sqrt{x^3 + 2} + \dfrac{3x^2(x^3 + x)}{2\sqrt{x^3 + 2}}$

c. $h(x) = \dfrac{3x^2 + 6x}{(2x^3 + 2)^5}$

$h'(x) = \dfrac{(2x^3 + 2)^5 \cdot (6x + 6) - (3x^2 + 6x) \cdot 5(2x^3 + 2)^4 \cdot 6x^2}{(2x^3 + 2)^{10}} =$

$= \dfrac{(2x^3 + 2)(6x + 6) - 30x^2(3x^2 + 6x)}{(2x^3 + 2)^6}$

$= \dfrac{12x^4 + 12x^3 + 12x + 12 - 90x^4 - 180x^3}{(2x^3 + 2)^6}$

$= \dfrac{-78x^4 - 168x^3 + 12x + 12}{(2x^3 + 2)^6}$

Opgave 9:

a. $f'(x) = 1 \cdot \sqrt{50 - x^2} + x \cdot \dfrac{1}{2\sqrt{50 - x^2}} \cdot -2x = \sqrt{50 - x^2} - \dfrac{x^2}{\sqrt{50 - x^2}} =$

$= \dfrac{50 - x^2}{\sqrt{50 - x^2}} - \dfrac{x^2}{\sqrt{50 - x^2}} = \dfrac{50 - 2x^2}{\sqrt{50 - x^2}} = 0$

$50 - 2x^2 = 0$

$-2x^2 = -50$

$x^2 = 25$

$x = -5 \quad \vee \quad x = 5$

$(-5, -25) \quad (5, 25)$

b. $y_A = 7$

$f'(1) = 6\frac{6}{7}$

$y = 6\frac{6}{7}x + b$ door $(1,7)$

$7 = 6\frac{6}{7} + b$

$b = \frac{1}{7}$

$y = 6\frac{6}{7}x + \frac{1}{7}$

Opgave 10:

a. $f'(x)=\dfrac{(x^2+1)\cdot -1--x\cdot 2x}{(x^2+1)^2}=\dfrac{-x^2-1+2x^2}{(x^2+1)^2}=\dfrac{x^2-1}{(x^2+1)^2}=0$

$x^2-1=0$

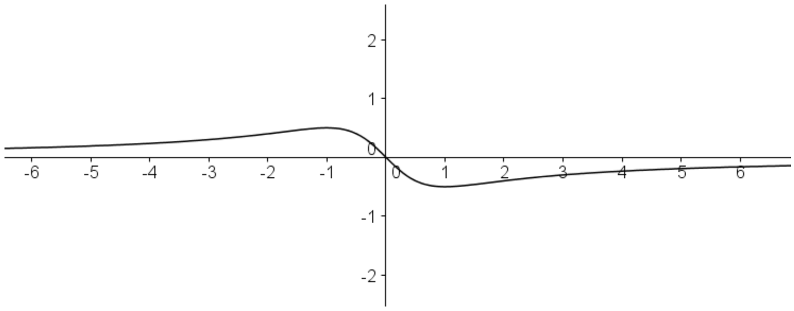
$x^2=1$

$x=-1 \quad \vee \quad x=1$

max $f(-1)=\frac{1}{2}$

min $f(1)=-\frac{1}{2}$

$B_f=[-\frac{1}{2},\frac{1}{2}]$



b. $f'(x)=\dfrac{x^2-1}{(x^2+1)^2}=\frac{3}{25}$

$3(x^2+1)^2=25(x^2-1)$

$3x^4+6x^2+3=25x^2-25$

$3x^4-19x^2+28=0$

$x^2=\dfrac{19\pm\sqrt{25}}{6}=\dfrac{19\pm 5}{6}$

$x^2=\frac{19+5}{6}=4 \quad \vee \quad x^2=\frac{19-5}{6}=2\frac{1}{3}$

$x=2 \quad \vee \quad x=-2 \quad \vee \quad x=\sqrt{2\frac{1}{3}} \quad \vee \quad x=-\sqrt{2\frac{1}{3}}$

Opgave 11:

$f_p'(x)=-x^2+2px+3=0$

$2px=x^2-3$

$p=\frac{1}{2}x-\dfrac{3}{2x}$ voor $x\neq 0$

$y=-\frac{1}{3}x^3+(\frac{1}{2}x-\dfrac{3}{2x})x^2+3x-4$

$y=-\frac{1}{3}x^3+\frac{1}{2}x^3-1\frac{1}{2}x+3x-4$

$y=\frac{1}{6}x^3+1\frac{1}{2}x-4$

Opgave 12:

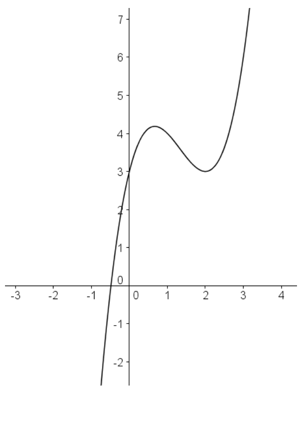
a. $f'(x)=3x^2-8x+4=0$

$x=\dfrac{8\pm\sqrt{16}}{6}=\dfrac{8\pm 4}{6}$

$x=\frac{8+4}{6}=2 \quad \vee \quad x=\frac{8-4}{6}=\frac{2}{3}$

max $f(\frac{2}{3})=4\frac{5}{27}$

min $f(2)=3$



b. $p<3 \quad \vee \quad p>4\frac{5}{27}$

c. $p=3 \quad \vee \quad p=4\frac{5}{27}$

d. $3<p<4\frac{5}{27}$

Opgave 13:

$$f'(x)=\frac{(x+1)(2x+2)-(x^2+2x+3)\cdot 1}{(x+1)^2}=\frac{2x^2+4x+2-x^2-2x-3}{(x+1)^2}=\frac{x^2+2x-1}{(x+1)^2}=\tfrac{1}{2}$$

$2(x^2+2x-1)=(x+1)^2$

$2x^2+4x-2=x^2+2x+1$

$x^2+2x-3=0$

$(x+3)(x-1)=0$

$x=-3 \quad \vee \quad x=1$

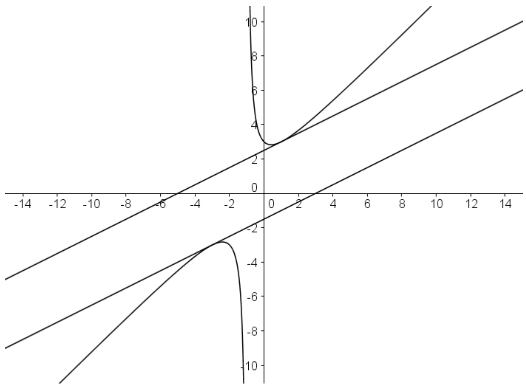
$y=-3 \qquad y=3$

$y=\tfrac{1}{2}x+p$ door $(-3,-3)$ $\qquad y=\tfrac{1}{2}x+p$ door $(1,3)$

$-3=-1\tfrac{1}{2}+p \qquad 3=\tfrac{1}{2}+p$

$p=-1\tfrac{1}{2} \qquad p=2\tfrac{1}{2}$

dus $p\le -1\tfrac{1}{2} \quad \vee \quad p\ge 2\tfrac{1}{2}$

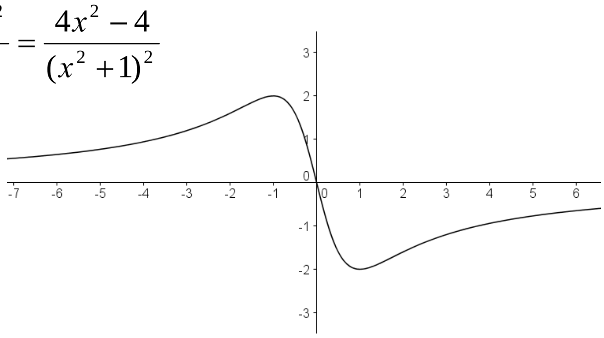


Opgave 14:

$$f'(x)=\frac{(x^2+1)\cdot -4 - -4x\cdot 2x}{(x^2+1)^2}=\frac{-4x^2-4+8x^2}{(x^2+1)^2}=\frac{4x^2-4}{(x^2+1)^2}$$

$f'(0)=-4$

$-4<a<0$



Opgave 15:

a. $f_p'(x)=-x^2+4x+p$

$f_p'(1)=-1+4+p=0$

$p=-3$

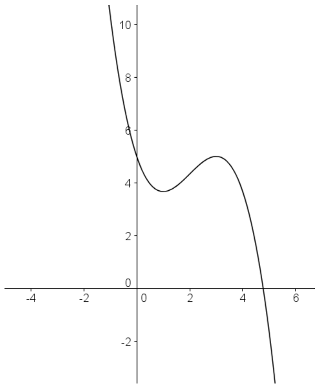
$f_{-3}'(x)=-x^2+4x-3=0$

$x^2-4x+3=0$

$(x-1)(x-3)=0$

$x=1 \quad \vee \quad x=3$

max $f_{-3}(3)=5$



b. $f_p'(x)=-x^2+4x+p$

$D=16+4p>0$

$4p>-16$

$p>-4$

Opgave 16:

$$f_p'(x)=\frac{(x+1)\cdot 2\cdot \frac{1}{2\sqrt{x}}-(2\sqrt{x}+p)\cdot 1}{(x+1)^2}=\frac{(x+1)\cdot\frac{1}{\sqrt{x}}-2\sqrt{x}-p}{(x+1)^2}$$

$$f_p'(4) = \frac{2\tfrac{1}{2} - 4 - p}{25} = -0{,}1$$

$$-1\tfrac{1}{2} - p = -2\tfrac{1}{2}$$

$$-p = -1$$

$$p = 1$$

$$f_1(x) = \frac{2\sqrt{x} + 1}{x + 1}$$

$$y_A = f_1(4) = 1$$

$y = -0{,}1x + q$ door $(4{,}1)$

$$1 = -0{,}4 + q$$

$$q = 1{,}4$$