HOOFDSTUK 7: Differentiaalrekening

7.1 De afgeleide van gebroken functies

Opgave 1:

a. $p(x) = x^2(3x-7) = 3x^3 - 7x^2$

$p'(x) = 9x^2 - 14x$

$f'(x) = 2x$

$g'(x) = 3$

b. $f'(x) \cdot g'(x) = 2x \cdot 3 = 6x \neq p'(x)$

c. $f'(x) \cdot g(x) + f(x) \cdot g'(x) = 2x \cdot (3x-7) + x^2 \cdot 3 = 6x^2 - 14x + 3x = 9x^2 - 14x = p'(x)$

Opgave 2:

a. $f'(x) = -6x(2+7x) + (2-3x^2) \cdot 7$

b. $g'(x) = 2(2x-5) + (2x-5) \cdot 2$

c. $h'(x) = (2x-3)(x^3 + x^2 + x) + (x^2 - 3x)(3x^2 + 2x + 1)$

d. $j'(x) = 6x(3x^2 - 4) + (3x^2 - 4) \cdot 6x$

Opgave 3:

a. $p' = [f \cdot g]' \cdot h + [f \cdot g] \cdot h' = (f' \cdot g + f \cdot g') \cdot h + f \cdot g \cdot h' =$
$f' \cdot g \cdot h + f \cdot g' \cdot h + f \cdot g \cdot h'$

b. $p' = f' \cdot g \cdot h \cdot j + f \cdot g' \cdot h \cdot j + f \cdot g \cdot h' \cdot j + f \cdot g \cdot h \cdot j'$

Opgave 4:

$q(x) = \dfrac{t(x)}{n(x)}$

Kruiselings vermenigvuldigen geeft $q(x) \cdot n(x) = t(x)$

Differentiëren van het linker- en rechterlid geeft

$q'(x) \cdot n(x) + q(x) \cdot n'(x) = t'(x)$

$q'(x) \cdot n(x) = t'(x) - q(x) \cdot n'(x)$

$q'(x) = \dfrac{t'(x) - q(x) \cdot n'(x)}{n(x)}$

Substitutie van $q(x) = \dfrac{t(x)}{n(x)}$ geeft $q'(x) = \dfrac{t'(x) - \dfrac{t(x)}{n(x)} \cdot n'(x)}{n(x)}$

Teller en noemer vermenigvuldigen met $n(x)$ geeft $q'(x) = \dfrac{n(x) \cdot t'(x) - t(x) \cdot n'(x)}{(n(x))^2}$

Opgave 5:

a. $f'(x) = \dfrac{(x+5)-(x-2)}{(x+5)^2} = \dfrac{x+5-x+2}{(x+5)^2} = \dfrac{7}{(x+5)^2}$

b. $g'(x) = \dfrac{(-x+6) \cdot 3 - (3x+2) \cdot -1}{(-x+6)^2} = \dfrac{-3x+18+3x+2}{(-x+6)^2} = \dfrac{20}{(-x+6)^2}$

c. $h'(x) = \dfrac{(2x-1)\cdot 0 - 2\cdot 2}{(2x-1)^2} = \dfrac{-4}{(2x-1)^2}$

d. $j(x) = \dfrac{6x-9}{3} = 2x-3$

$j'(x) = 2$

Opgave 6:

a. $f'(x) = \dfrac{(2x^2+1)\cdot 3x^2 - x^3\cdot 4x}{(2x^2+1)^2} = \dfrac{6x^4+3x^2-4x^4}{(2x^2+1)^2} = \dfrac{2x^4+3x^2}{(2x^2+1)^2}$

b. $g'(x) = \dfrac{(3-x^2)\cdot 1 - (x-2)\cdot -2x}{(3-x^2)^2} = \dfrac{3-x^2+2x^2-4x}{(3-x^2)^2} = \dfrac{x^2-4x+3}{(3-x^2)^2}$

c. $h'(x) = \dfrac{(x-2)\cdot -2x - (3-x^2)\cdot 1}{(x-2)^2} + 3x^2 = \dfrac{-2x^2+4x-3+x^2}{(x-2)^2} + 3x^2 = \dfrac{-x^2+4x-3}{(x-2)^2} + 3x^2$

d. $j'(x) = 1 - \dfrac{(x+4)\cdot 0 - 2\cdot 1}{(x+4)^2} = 1 + \dfrac{2}{(x+4)^2}$

Opgave 7:

a. $\dfrac{x^2-4}{2x+5} = 0$

$x^2 - 4 = 0$

$x^2 = 4$

$x = 2 \quad \vee \quad x = -2$

$f'(x) = \dfrac{(2x+5)\cdot 2x - (x^2-4)\cdot 2}{(2x+5)^2} = \dfrac{4x^2+10x-2x^2+8}{(2x+5)^2} = \dfrac{2x^2+10x+8}{(2x+5)^2}$

in $(-2,0)$	in $(2,0)$
$rc = f'(-2) = -4$	$rc = f'(2) = \frac{4}{9}$
$y = -4x + b$ door $(-2,0)$	$y = \frac{4}{9}x + b$ door $(2,0)$
$0 = 8 + b$	$0 = \frac{8}{9} + b$
$b = -8$	$b = -\frac{8}{9}$
$y = -4x - 8$	$y = \frac{4}{9}x - \frac{8}{9}$

b. $y_C = f(0) = -\frac{4}{5}$

$rc = f'(0) = \frac{8}{25}$

$y = \frac{8}{25}x + b$ door $(0,-\frac{4}{5})$

$-\frac{4}{5} = 0 + b$

$b = -\frac{4}{5}$

$y = \frac{8}{25}x - \frac{4}{5}$

c. $f'(x) = \dfrac{2x^2+10x+8}{(2x+5)^2} = 0$

$2x^2 + 10x + 8 = 0$

$x^2 + 5x + 4 = 0$

$(x+1)(x+4) = 0$

$x = -1 \quad \vee \quad x = -4$

$y = -1 \qquad y = -4$

dus $(-1,-1)$ en $(-4,-4)$

Opgave 8:

$y_A = \frac{5}{4}$

$\dfrac{2x-5}{x^2-4} = 0$

$2x - 5 = 0$

$2x = 5$

$x_B = 2\frac{1}{2}$

$f'(x) = \dfrac{(x^2-4)\cdot 2 - (2x-5)\cdot 2x}{(x^2-4)^2} = \dfrac{2x^2 - 8 - 4x^2 + 10x}{(x^2-4)^2} = \dfrac{-2x^2 + 10x - 8}{(x^2-4)^2}$

$rc = f'(0) = -\frac{1}{2}$

$y = -\frac{1}{2}x + b$ door $(0, \frac{5}{4})$

$b = \frac{5}{4}$

$y = -\frac{1}{2}x + \frac{5}{4}$

$-\frac{1}{2}x + \frac{5}{4} = 0$

$-\frac{1}{2}x = -\frac{5}{4}$

$x = 2\frac{1}{2}$ dus de bewering is waar