

7.2 De afgeleide van machtfuncties

Opgave 9:

a. $\frac{1}{x^3} = x^{-3}$
 $\frac{5}{x^4} = 5x^{-4}$
 $\frac{1}{3x^2} = \frac{1}{3}x^{-2}$

b. $x^{-4} = \frac{1}{x^4}$
 $3x^{-2} = \frac{3}{x^2}$
 $\frac{1}{7}x^{-6} = \frac{1}{7x^6}$

Opgave 10:

a. $\frac{x^3 + 5x^2}{x} = x^2 + 5x$
 $\frac{4x^2 + 7x}{x^3} = \frac{4x^2}{x^3} + \frac{7x}{x^3} = 4x^{-1} + 7x^{-2}$
 $\frac{2x^5 + 5x^2}{3x^4} = \frac{2x^5}{3x^4} + \frac{5x^2}{3x^4} = \frac{2}{3}x + \frac{5}{3}x^{-2}$

b. $\frac{1}{2x} + \frac{2}{x^2} = \frac{x}{2x^2} + \frac{4}{2x^2} = \frac{x+4}{2x^2}$
 $\frac{1}{2}x + \frac{3}{x^2} = \frac{x^3}{2x^2} + \frac{6}{2x^2} = \frac{x^3+6}{2x^2}$
 $\frac{2}{3}x^2 - \frac{3}{4x} = \frac{8x^3}{12x} - \frac{9}{12x} = \frac{8x^3-9}{12x}$

Opgave 11:

a. $\left[\frac{1}{x^2}\right]' = \frac{x^2 \cdot 0 - 1 \cdot 2x}{x^4} = -\frac{2x}{x^4} = -\frac{2}{x^3}$

b. $[x^{-2}]' = \left[\frac{1}{x^2}\right]' = -\frac{2}{x^3} = -2x^{-3}$

c. $[x^{-5}]' = \left[\frac{1}{x^5}\right]' = \frac{x^5 \cdot 0 - 1 \cdot 5x^4}{x^{10}} = -\frac{5x^4}{x^{10}} = -5x^{-6}$

Opgave 12:

- a. omdat de noemer uit één term bestaat kun je uitdelen waardoor je losse termen krijgt die je stuk voor stuk kunt differentiëren.
- b. $g(x)$ en $h(x)$

Opgave 13:

a. $f(x) = \frac{1}{x^6} = x^{-6}$

$$f'(x) = -6x^{-7} = -\frac{6}{x^7}$$

b. $g(x) = 5 - \frac{3}{x^2} = 5 - 3x^{-2}$

$$g'(x) = 6x^{-3} = \frac{6}{x^3}$$

c. $h(x) = ax^4 - \frac{b}{x^4} = ax^4 - bx^{-4}$

$$h'(x) = 4ax^3 + 4bx^{-5} = 4ax^3 + \frac{4b}{x^5}$$

Opgave 14:

a. $f'(x) = \frac{3x^2 \cdot 2 - (2x-1) \cdot 6x}{9x^4} = \frac{6x^2 - 12x^2 + 6x}{9x^4} = \frac{-6x^2 + 6x}{9x^4} = \frac{-2x + 2}{3x^3}$

b. $g'(x) = \frac{(2x-1) \cdot 6x - 3x^2 \cdot 2}{(2x-1)^2} = \frac{12x^2 - 6x - 6x^2}{(2x-1)^2} = \frac{6x^2 - 6x}{(2x-1)^2}$

c. $h'(x) = \frac{x^3 \cdot 18x^5 - (3x^6 - 3) \cdot 3x^2}{x^6} = \frac{18x^8 - 9x^8 + 9x^2}{x^6} = \frac{9x^8 + 9x^2}{x^6} = \frac{9x^6 + 9}{x^4}$

Opgave 15:

a. $y_A = \frac{2}{3}$

$$f'(x) = \frac{(x^2 - 1) \cdot 1 - x \cdot 2x}{(x^2 - 1)^2} = \frac{x^2 - 1 - 2x^2}{(x^2 - 1)^2} = \frac{-x^2 - 1}{(x^2 - 1)^2}$$

$$rc = f'(2) = -\frac{5}{9}$$

$$y = -\frac{5}{9}x + b \text{ door } (2, \frac{2}{3})$$

$$\frac{2}{3} = -\frac{10}{9} + b$$

$$b = 1\frac{7}{9}$$

$$y = -\frac{5}{9}x + 1\frac{7}{9}$$

b. $y_B = 1\frac{1}{2}$

$$g'(x) = \frac{x \cdot 2x - (x^2 - 1) \cdot 1}{x^2} = \frac{2x^2 - x^2 + 1}{x^2} = \frac{x^2 + 1}{x^2}$$

$$rc = g'(2) = \frac{5}{4}$$

$$y = \frac{5}{4}x + b \text{ door } (2, 1\frac{1}{2})$$

$$1\frac{1}{2} = 2\frac{1}{2} + b$$

$$b = -1$$

$$y = \frac{5}{4}x - 1$$

c. $\frac{x^2 - 1}{x} = 0$

$$x^2 - 1 = 0$$

$$x^2 = 1$$

$$x = 1 \vee x = -1$$

$$rc = g'(1) = 2$$

$$y = 2x + b \text{ door } (1,0)$$

$$0 = 2 + b$$

$$b = -2$$

$$y = 2x - 2$$

$$rc = g'(-1) = 2$$

$$y = 2x + b \text{ door } (-1,0)$$

$$0 = -2 + b$$

$$b = 2$$

$$y = 2x + 2$$

Opgave 16:

$$\text{a. } \frac{x^2}{\sqrt{x}} = \frac{x^2}{x^{\frac{1}{2}}} = x^{1\frac{1}{2}}$$

$$\sqrt[3]{x^2} = x^{\frac{2}{3}}$$

$$\frac{x^2 \cdot \sqrt{x}}{x^4} = \frac{x^2 \cdot x^{\frac{1}{2}}}{x^4} = \frac{x^{2\frac{1}{2}}}{x^4} = x^{-1\frac{1}{2}}$$

$$\text{b. } x^{\frac{1}{5}} = \sqrt[5]{x}$$

$$x^{2\frac{1}{2}} = x^2 \cdot x^{\frac{1}{2}} = x^2 \cdot \sqrt{x}$$

$$x^{-1\frac{1}{3}} = \frac{1}{x^{\frac{1}{3}}} = \frac{1}{x \cdot x^{\frac{1}{3}}} = \frac{1}{x \cdot \sqrt[3]{x}}$$

Opgave 17:

$$\text{a. } [x]' = [x^{\frac{1}{2}} \cdot x^{\frac{1}{2}}]' = [x^{\frac{1}{2}}]' \cdot x^{\frac{1}{2}} + x^{\frac{1}{2}} \cdot [x^{\frac{1}{2}}]' = 2 \cdot x^{\frac{1}{2}} \cdot [x^{\frac{1}{2}}]' = 1$$

$$\text{b. } [x^{\frac{1}{2}}]' = \frac{1}{2x^{\frac{1}{2}}} = \frac{1}{2}x^{-\frac{1}{2}}$$

Opgave 18:

$$\text{a. } f(x) = x + \sqrt{x} = x + x^{\frac{1}{2}}$$

$$f'(x) = 1 + \frac{1}{2}x^{-\frac{1}{2}} = 1 + \frac{1}{2x^{\frac{1}{2}}} = 1 + \frac{1}{2 \cdot \sqrt{x}}$$

$$\text{b. } g(x) = x \cdot \sqrt[3]{x} = x^1 \cdot x^{\frac{1}{3}} = x^{\frac{4}{3}}$$

$$g'(x) = 1\frac{1}{3}x^{\frac{1}{3}} = 1\frac{1}{3} \cdot \sqrt[3]{x}$$

$$\text{c. } h(x) = \frac{1}{\sqrt{x}} = \frac{1}{x^{\frac{1}{2}}} = x^{-\frac{1}{2}}$$

$$h'(x) = -\frac{1}{2}x^{-\frac{1}{2}} = \frac{-1}{2x^{\frac{1}{2}}} = \frac{-1}{2x \cdot \sqrt{x}}$$

$$\text{d. } j(x) = x^3 \cdot \sqrt[5]{x^3} = x^3 \cdot x^{\frac{3}{5}} = x^{\frac{18}{5}}$$

$$j'(x) = 3\frac{3}{5}x^{\frac{13}{5}} = 3\frac{3}{5}x^2 \cdot x^{\frac{3}{5}} = 3\frac{3}{5}x^2 \cdot \sqrt[5]{x^3}$$

$$\text{e. } k(x) = x^2 \cdot \sqrt[4]{x} = x^2 \cdot x^{\frac{1}{4}} = x^{2\frac{1}{4}}$$

$$k'(x) = 2\frac{1}{4}x^{\frac{3}{4}} = 2\frac{1}{4}x^1 \cdot x^{\frac{1}{4}} = 2\frac{1}{4}x\sqrt[4]{x}$$

$$\text{f. } l(x) = (x^2 + 1)(1 + \sqrt{x}) = x^2 + x^2 \cdot \sqrt{x} + 1 + \sqrt{x} = x^2 + x^{2\frac{1}{2}} + 1 + x^{\frac{1}{2}}$$

$$l'(x) = 2x + 2 \cdot \frac{1}{2} x^{\frac{1}{2}} + \frac{1}{2} x^{-\frac{1}{2}} = 2x + 2 \cdot \frac{1}{2} x \cdot \sqrt{x} + \frac{1}{2x^{\frac{1}{2}}} = 2x + 2 \cdot \frac{1}{2} x \cdot \sqrt{x} + \frac{1}{2\sqrt{x}}$$

Opgave 19:

a. $f(x) = (x \cdot \sqrt{x} - 3)^2 = x^3 - 6x \cdot \sqrt{x} + 9 = x^3 - 6x^{\frac{3}{2}} + 9$

$$f'(x) = 3x^2 - 9x^{\frac{1}{2}} = 3x^2 - 9\sqrt{x}$$

b. $g'(x) = \frac{(x+1) \cdot 1 \cdot \frac{1}{2}\sqrt{x} - x\sqrt{x} \cdot 1}{(x+1)^2} = \frac{1 \cdot \frac{1}{2}x\sqrt{x} + 1 \cdot \frac{1}{2}\sqrt{x} - x\sqrt{x}}{(x+1)^2} = \frac{\frac{1}{2}x\sqrt{x} + 1 \cdot \frac{1}{2}\sqrt{x}}{(x+1)^2}$

c. $h'(x) = \frac{(x^2+2) \cdot \frac{1}{\sqrt{x}} - 2\sqrt{x} \cdot 2x}{(x^2+2)^2} = \frac{\frac{x^2}{\sqrt{x}} + \frac{2}{\sqrt{x}} - 4x\sqrt{x}}{(x^2+2)^2} = \frac{x\sqrt{x} + \frac{2}{\sqrt{x}} - 4x\sqrt{x}}{(x^2+2)^2} =$

$$\frac{\frac{2}{\sqrt{x}} - 3x\sqrt{x}}{(x^2+2)^2} = \frac{\frac{2}{\sqrt{x}} - \frac{3x^2}{\sqrt{x}}}{(x^2+2)^2} = \frac{\frac{2-3x^2}{\sqrt{x}}}{(x^2+2)^2} = \frac{2-3x^2}{\sqrt{x} \cdot (x^2+2)^2}$$

Opgave 20:

a. $f(x) = \frac{x+1}{\sqrt{x}} = \frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}} = x^{\frac{1}{2}} + x^{-\frac{1}{2}}$

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}} = \frac{1}{2\sqrt{x}} - \frac{1}{2x\sqrt{x}} = \frac{x}{2x\sqrt{x}} - \frac{1}{2x\sqrt{x}} = \frac{x-1}{2x\sqrt{x}}$$

b. $g(x) = \frac{x+1}{x\sqrt{x}} = \frac{x}{x\sqrt{x}} + \frac{1}{x\sqrt{x}} = x^{-\frac{1}{2}} + x^{-\frac{3}{2}}$

$$g'(x) = -\frac{1}{2}x^{-\frac{3}{2}} - 1 \cdot \frac{1}{2}x^{-\frac{5}{2}} = \frac{-1}{2x^{\frac{3}{2}}} - \frac{3}{2x^{\frac{5}{2}}} = \frac{-1}{2x\sqrt{x}} - \frac{3}{2x^2\sqrt{x}} = \frac{-x}{2x^2\sqrt{x}} - \frac{3}{2x^2\sqrt{x}} = \frac{-x-3}{2x^2\sqrt{x}}$$

c. $h(x) = \frac{x^2+2}{2\sqrt{x}} = \frac{x^2}{2\sqrt{x}} + \frac{2}{2\sqrt{x}} = \frac{1}{2}x^{\frac{3}{2}} + x^{-\frac{1}{2}}$

$$h'(x) = \frac{3}{4}x^{\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}} = \frac{3}{4}\sqrt{x} - \frac{1}{2x\sqrt{x}} = \frac{3x^2}{4x\sqrt{x}} - \frac{2}{4x\sqrt{x}} = \frac{3x^2-2}{4x\sqrt{x}}$$

Opgave 21:

$$y_A = f\left(\frac{1}{8}\right) = \frac{1}{4} \quad y_B = f(8) = 4$$

$$f(x) = \sqrt[3]{x^2} = x^{\frac{2}{3}}$$

$$f'(x) = \frac{2}{3}x^{-\frac{1}{3}} = \frac{2}{3 \cdot \sqrt[3]{x}}$$

$$f'\left(\frac{1}{8}\right) = \frac{4}{3}$$

$$f'(8) = \frac{1}{3}$$

$$y = \frac{4}{3}x + b \text{ door } \left(\frac{1}{8}, \frac{1}{4}\right)$$

$$y = \frac{1}{3}x + b \text{ door } (8, 4)$$

$$\frac{1}{4} = \frac{1}{6} + b$$

$$4 = \frac{8}{3} + b$$

$$b = \frac{1}{12}$$

$$b = \frac{4}{3}$$

$$y = \frac{4}{3}x + \frac{1}{12}$$

$$y = \frac{1}{3}x + \frac{4}{3}$$

$$\frac{4}{3}x + \frac{1}{12} = \frac{1}{3}x + \frac{4}{3}$$

$$x = 1\frac{1}{4}$$

$$y = 1\frac{3}{4}$$

$$\text{Dus } C = (1\frac{1}{4}, 1\frac{3}{4})$$

Opgave 22:

$$\text{a. } f(x) = x\sqrt{x} - 3x = x^{\frac{3}{2}} - 3x$$

$$f'(x) = 1\frac{1}{2}x^{\frac{1}{2}} - 3 = 1\frac{1}{2}\sqrt{x} - 3$$

$$f'(0) = -3$$

$$y = -3x + b \text{ door } (0,0)$$

$$b = 0$$

$$y = -3x$$

$$\text{b. } f'(x) = 1\frac{1}{2}\sqrt{x} - 3 = 3$$

$$1\frac{1}{2}\sqrt{x} = 6$$

$$\sqrt{x} = 4$$

$$x = 16$$

$$y = f(16) = 16$$

$$y = 3x + b \text{ door } (16,16)$$

$$16 = 48 + b$$

$$-32 = b$$

$$y = 3x - 32$$

Opgave 23:

$$y_A = f(4) = 8$$

$$f'(x) = \frac{(x+1) \cdot 7\frac{1}{2}\sqrt{x} - 5x\sqrt{x} \cdot 1}{(x+1)^2} = \frac{7\frac{1}{2}x\sqrt{x} + 7\frac{1}{2}\sqrt{x} - 5x\sqrt{x}}{(x+1)^2} = \frac{2\frac{1}{2}x\sqrt{x} + 7\frac{1}{2}\sqrt{x}}{(x+1)^2}$$

$$f'(4) = 1\frac{2}{5}$$

$$y = 1\frac{2}{5}x + b \text{ door } (4,8)$$

$$8 = 5\frac{3}{5} + b$$

$$2\frac{2}{5} = b$$

$$y = 1\frac{2}{5}x + 2\frac{2}{5}$$

$$1\frac{2}{5}x + 2\frac{2}{5} = 0$$

$$1\frac{2}{5}x = -2\frac{2}{5}$$

$$x = -1\frac{5}{7}$$

$$A(4,8) \text{ en } B(-1\frac{5}{7}, 0)$$

$$\text{Opp}(\triangle ABC) = \frac{1}{2} \cdot 1\frac{5}{7} \cdot 8 = 6\frac{6}{7}$$

Opgave 24:

$$\text{a. } s'(t) = 15\sqrt{t}$$

$$s'(1) = 15 \text{ m/s}$$

$$\text{b. } v = 108 \frac{\text{km}}{\text{uur}} = 30 \frac{\text{m}}{\text{s}}$$

$$15\sqrt{t} = 30$$

$$\sqrt{t} = 2$$

$$t = 4 \text{ sec}$$

c. $s(9) = 270$

$$s'(9) = 45 \frac{m}{s}$$

$$\text{afstand} = 270 + 51 \cdot 45 = 2565 \text{ meter}$$