7.3 De kettingregel

Opgave 25:

a. $f(x)=(x^2-5x)^2=x^4-10x^3+25x^2$

$f'(x)=4x^3-30x^2+50x$

b. $2(x^2-5x)\cdot\left[x^2-5x\right]'=2(x^2-5x)(2x-5)=(x^2-5x)(4x-10)=$

$=4x^3-10x^2-20x^2+50x=4x^3-30x^2+50x=f'(x)$

Opgave 26:

$y_2=y_3$

Opgave 27:

a. $f(x)=-2(2x+1)^4=-2u^4$ met $u=2x+1$ dus $u'=2$

$f'(x)=-8u^3\cdot u'=-8(2x+1)^3\cdot 2=-16(2x+1)^3$

b. $g(x)=\dfrac{1}{(3x-2)^2}=(3x-2)^{-2}=u^{-2}$ met $u=3x-2$ dus $u'=3$

$g'(x)=-2u^{-3}\cdot u'=-\dfrac{2}{u^3}\cdot u'=-\dfrac{2}{(3x-2)^3}\cdot 3=-\dfrac{6}{(3x-2)^3}$

c. $h(x)=\sqrt{2x^2+4x}=\sqrt{u}$ met $u=2x^2+4x$ dus $u'=4x+4$

$h'(x)=\dfrac{1}{2\sqrt{u}}\cdot u'=\dfrac{1}{2\sqrt{2x^2+4x}}\cdot(4x+4)=\dfrac{2x+2}{\sqrt{2x^2+4x}}$

d. $j(x)=\dfrac{1}{\sqrt{4x-1}}=(4x-1)^{-\frac{1}{2}}=u^{-\frac{1}{2}}$ met $u=4x-1$ dus $u'=4$

$j'(x)=-\frac{1}{2}u^{-1\frac{1}{2}}\cdot u'=\dfrac{-1}{2u\sqrt{u}}\cdot u'=\dfrac{-1}{2(4x-1)\sqrt{4x-1}}\cdot 4=\dfrac{-2}{(4x-1)\sqrt{4x-1}}$

e. $k(x)=(x^2+3)\sqrt{x^2+3}=(x^2+3)^{1\frac{1}{2}}=u^{1\frac{1}{2}}$ met $u=x^2+3$ dus $u'=2x$

$k'(x)=1\frac{1}{2}u^{\frac{1}{2}}\cdot u'=1\frac{1}{2}\sqrt{x^2+3}\cdot 2x=3x\sqrt{x^2+3}$

f. $l(x)=\dfrac{1}{\sqrt{x^2+2x+3}}=(x^2+2x+3)^{-\frac{1}{2}}=u^{-\frac{1}{2}}$ met $u=x^2+2x+3$ dus $u'=2x+2$

$l'(x)=-\frac{1}{2}u^{-1\frac{1}{2}}\cdot u'=\dfrac{-1}{2u\sqrt{u}}\cdot u'=\dfrac{-1}{2(x^2+2x+3)\sqrt{x^2+2x+3}}\cdot(2x+2)=$

$=\dfrac{-x-1}{(x^2+2x+3)\sqrt{x^2+2x+3}}$

Opgave 28:

a. $f(x)=4(x^3+7x-2)^2=4u^2$ met $u=x^3+7x-2$ dus $u'=3x^2+7$

$f'(x)=8u\cdot u'=8(x^3+7x-2)(3x^2+7)$

b. $g(x)=-\dfrac{6}{(x^2+3x)^3}=-\dfrac{6}{u^3}=-6u^{-3}$ met $u=x^2+3x$ dus $u'=2x+3$

$$g'(x) = 18u^{-4} \cdot u' = \frac{18u'}{u^4} = \frac{18(2x+3)}{(x^2+3x)^4}$$

c. $h(x) = \sqrt[3]{x^3+3x} = \sqrt[3]{u} = u^{\frac{1}{3}}$ met $u = x^3 + 3x$ dus $u' = 3x^2 + 3$

$$h'(x) = \tfrac{1}{3}u^{-\frac{2}{3}} \cdot u' = \frac{1}{3u^{\frac{2}{3}}} \cdot u' = \frac{u'}{3 \cdot \sqrt[3]{u^2}} = \frac{3x^2+3}{3 \cdot \sqrt[3]{(x^3+3x)^2}} = \frac{x^2+1}{\sqrt[3]{(x^3+3x)^2}}$$

d. $j(x) = \dfrac{1}{(4-x)\sqrt{4-x}} = \dfrac{1}{(4-x)^{1\frac{1}{2}}} = \dfrac{1}{u^{1\frac{1}{2}}} = u^{-1\frac{1}{2}}$ met $u = 4 - x$ dus $u' = -1$

$$j'(x) = -1\tfrac{1}{2}u^{-2\frac{1}{2}} \cdot u' = \frac{-3}{2u^{2\frac{1}{2}}} \cdot u' = \frac{-3u'}{2u^2\sqrt{u}} = \frac{3}{2(4-x)^2 \cdot \sqrt{4-x}}$$

e. $k(x) = 5 \cdot \sqrt{2x^4 + x^2} + 4x^2 = 5 \cdot \sqrt{u} + 4x^2 = 5 \cdot u^{\frac{1}{2}} + 4x^2$ met $u = 2x^4 + x^2$ dus

$u' = 8x^3 + 2x$

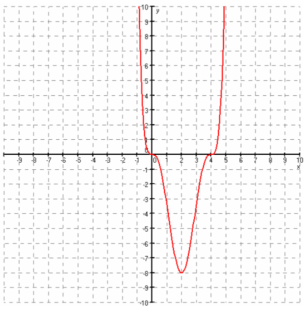
$$k'(x) = 2\tfrac{1}{2}u^{-\frac{1}{2}} \cdot u' + 8x = \frac{5u'}{2\sqrt{u}} + 8x = \frac{5(8x^3+2x)}{2\sqrt{2x^4+x^2}} + 8x = \frac{5(4x^3+x)}{\sqrt{2x^4+x^2}} + 8x$$

f. $l(x) = \dfrac{x^2+4}{\sqrt{x^2+4}} = \dfrac{u}{\sqrt{u}} = \sqrt{u} = u^{\frac{1}{2}}$ met $u = x^2 + 4$ dus $u' = 2x$

$$l'(x) = \tfrac{1}{2}u^{-\frac{1}{2}} \cdot u' = \frac{1}{2u^{\frac{1}{2}}} \cdot u' = \frac{u'}{2 \cdot \sqrt{u}} = \frac{2x}{2 \cdot \sqrt{x^2+4}} = \frac{x}{\sqrt{x^2+4}}$$

Opgave 29:

a.



b. $f(x) = (\tfrac{1}{2}x^2 - 2x)^3 = u^3$ met $u = \tfrac{1}{2}x^2 - 2x$ dus $u' = x - 2$

$f'(x) = 3u^2 \cdot u' = 3(\tfrac{1}{2}x^2 - 2x)^2 \cdot (x-2) = 0$

$\tfrac{1}{2}x^2 - 2x = 0 \quad \vee \quad x - 2 = 0$

$\tfrac{1}{2}x(x-4) = 0 \quad \vee \quad x = 2$

$x = 0 \quad \vee \quad x = 2 \quad \vee \quad x = 4$

c. $y_A = f(6) = 216$

$rc = f'(6) = 432$

$y = 432x + b$ door $(6,216)$

$216 = 2592 + b$

$-2376 = b$

$y = 432x - 2376$

Opgave 30:

a. $f(x)=\sqrt{x^2+9}-x^2+5x$

$f'(x)=\frac{1}{2\sqrt{x^2+9}}\cdot 2x-2x+5=\frac{x}{\sqrt{x^2+9}}-2x+5$

$rc=f'(4)=-2\frac{1}{5}$

$y_A=f(4)=9$

$y=-2\frac{1}{5}x+b$ door $(4,9)$

$9=-8\frac{4}{5}+b$

$17\frac{4}{5}=b$

$y=-2\frac{1}{5}x+17\frac{4}{5}$

b. $f'(3)=\frac{3}{\sqrt{18}}-6+5\neq 0$ dus niet

c. $f'(x)=\frac{x}{\sqrt{x^2+9}}-2x+5=5$

$\frac{x}{\sqrt{x^2+9}}=2x$

$2x\sqrt{x^2+9)}=x$

$x=0 \quad \vee \quad 2\sqrt{x^2+9}=1$

$x=0 \quad \vee \quad \sqrt{x^2+9}=\frac{1}{2}$

$x=0 \quad \vee \quad x^2+9=\frac{1}{4}$

$x=0 \quad \vee \quad x^2=-8\frac{3}{4}$ kan niet

$y=f(0)=3$

$y=5x+b$ door $(0,3)$

$3+0=b$

$b=3$

$y=5x+3$

Opgave 31:

$f(x)=x\cdot\sqrt{2x+1}=g(x)\cdot h(x)$ met $g(x)=x$ en $h(x)=\sqrt{2x+1}$

$h(x)=\sqrt{2x+1}=\sqrt{u}$ met $u=2x+1$

Opgave 32:

a. $f(x)=x\cdot\sqrt{3x+1}$`

$f'(x)=1\cdot\sqrt{3x+1}+x\cdot\frac{1}{2\cdot\sqrt{3x+1}}\cdot 3=\sqrt{3x+1}+\frac{3x}{2\cdot\sqrt{3x+1}}=\frac{2(3x+1)}{2\cdot\sqrt{3x+1}}+\frac{3x}{2\cdot\sqrt{3x+1}}$

$=\frac{6x+2+3x}{2\cdot\sqrt{3x+1}}=\frac{9x+2}{2\cdot\sqrt{3x+1}}$

b. $g(x)=\frac{\sqrt{x^2+1}}{2x+1}$

$$g'(x) = \frac{(2x+1)\cdot\frac{1}{2\cdot\sqrt{x^2+1}}\cdot 2x - \sqrt{x^2+1}\cdot 2}{(2x+1)^2} = \frac{\frac{x(2x+1)}{\sqrt{x^2+1}} - 2\cdot\sqrt{x^2+1}}{(2x+1)^2} =$$

$$= \frac{\frac{x(2x+1)}{\sqrt{x^2+1}} - \frac{2(x^2+1)}{\sqrt{x^2+1}}}{(2x+1)^2} = \frac{x(2x+1)-2(x^2+1)}{(2x+1)^2\cdot\sqrt{x^2+1}} = \frac{2x^2+x-2x^2-2}{(2x+1)^2\cdot\sqrt{x^2+1}} =$$

$$= \frac{x-2}{(2x+1)^2\cdot\sqrt{x^2+1}}$$

c. $h(x) = x\cdot(3x+1)^3$

$h'(x) = 1\cdot(3x+1)^3 + 9x(3x+1)^2 = (3x+1)^3 + 9x(3x+1)^2$

d. $k(x) = \frac{x^2-1}{\sqrt{4x+1}}$

$$k'(x) = \frac{\sqrt{4x+1}\cdot 2x - (x^2-1)\cdot\frac{1}{2\cdot\sqrt{4x+1}}\cdot 4}{4x+1} = \frac{2x\cdot\sqrt{4x+1} - \frac{2(x^2-1)}{\sqrt{4x+1}}}{4x+1} =$$

$$= \frac{\frac{2x(4x+1)}{\sqrt{4x+1}} - \frac{2(x^2-1)}{\sqrt{4x+1}}}{4x+1} = \frac{2x(4x+1)-2(x^2-1)}{(4x+1)\cdot\sqrt{4x+1}} = \frac{8x^2+2x-2x^2+2}{(4x+1)\cdot\sqrt{4x+1}} =$$

$$= \frac{6x^2+2x+2}{(4x+1)\cdot\sqrt{4x+1}}$$

Opgave 33:

$f(x) = \frac{1}{2}x\cdot\sqrt{3x+1}$

$y_A = f(8) = 20$

$f'(x) = \frac{1}{2}\cdot\sqrt{3x+1} + \frac{1}{2}x\cdot\frac{1}{2\cdot\sqrt{3x+1}}\cdot 3 = \frac{1}{2}\cdot\sqrt{3x+1} + \frac{3x}{4\cdot\sqrt{3x+1}}$

$rc = f'(8) = 3\frac{7}{10}$

$y = 3\frac{7}{10}x + b$ door $(8,20)$

$20 = 29\frac{3}{5} + b$

$-9\frac{3}{5} = b$

$y = 3\frac{7}{10}x - 9\frac{3}{5}$

Opgave 34:

$f(x) = \frac{x+1}{\sqrt{x^2+4}}$

a. $$f'(x) = \frac{\sqrt{x^2+4}\cdot 1 - (x+1)\cdot\frac{1}{2\sqrt{x^2+4}}\cdot 2x}{x^2+4} = \frac{\sqrt{x^2+4} - \frac{x(x+1)}{\sqrt{x^2+4}}}{x^2+4} =$$

$$= \frac{\frac{x^2+4}{\sqrt{x^2+4}} - \frac{x^2+x}{\sqrt{x^2+4}}}{x^2+4} = \frac{\frac{x^2+4-x^2-x}{\sqrt{x^2+4}}}{x^2+4} = \frac{4-x}{(x^2+4)\cdot\sqrt{x^2+4}} = 0$$

$4 - x = 0$

$x = 4$

$y = f(4) = \frac{5}{\sqrt{20}} = \frac{5}{2\cdot\sqrt{5}} = \frac{1}{2}\sqrt{5}$ dus $(4, \frac{1}{2}\sqrt{5})$

b. $A = (0, \frac{1}{2})$

$rc = f'(0) = \frac{1}{2}$

$y = \frac{1}{2}x + b$ door $(0, \frac{1}{2})$

$\frac{1}{2} = 0 + b$

$b = \frac{1}{2}$

$y = \frac{1}{2}x + \frac{1}{2}$

snijpunt x-as: $\frac{1}{2}x + \frac{1}{2} = 0$

$\frac{1}{2}x = -\frac{1}{2}$

$x = -1$

$B = (-1,0)$

$Opp(\Delta OAB) = \frac{1}{2}\cdot OB\cdot OA = \frac{1}{2}\cdot 1\cdot\frac{1}{2} = \frac{1}{4}$

Opgave 35:

a. $g(x) = \frac{x+6}{\sqrt{8x+9}} = (x+6)\cdot(8x+9)^{-\frac{1}{2}} = h(x)\cdot j(x)$ met $h(x) = x+6$ en $j(x) = (8x+9)^{-\frac{1}{2}}$

dus $j(x) = (8x+9)^{-\frac{1}{2}} = u^{-\frac{1}{2}}$ met $u = 8x+9$ dus $u' = 8$

$j'(x) = -\frac{1}{2}u^{-1\frac{1}{2}}\cdot u' = -\frac{1}{2}\cdot(8x+9)^{-1\frac{1}{2}}\cdot 8$

$g'(x) = h'(x)\cdot j(x) + h(x)\cdot j'(x) = 1\cdot(8x+9)^{-\frac{1}{2}} + (x+6)\cdot -\frac{1}{2}(8x+9)^{-1\frac{1}{2}}\cdot 8 =$

$$= \frac{1}{\sqrt{8x+9}} - \frac{4(x+6)}{(8x+9)\sqrt{8x+9}} = \frac{8x+9}{(8x+9)\sqrt{8x+9}} - \frac{4x+24}{(8x+9)\sqrt{8x+9}} =$$

$$= \frac{8x+9-4x-24}{(8x+9)\sqrt{8x+9}} = \frac{4x-15}{(8x+9)\sqrt{8x+9}}$$

b. $f(x) = \frac{x+1}{\sqrt{x^2+4}} = (x+1)\cdot(x^2+4)^{-\frac{1}{2}}$

$$f'(x) = 1\cdot(x^2+4)^{-\frac{1}{2}} + (x+1)\cdot -\frac{1}{2}(x^2+4)^{-1\frac{1}{2}}\cdot 2x = \frac{1}{\sqrt{x^2+4}} - \frac{x(x+1)}{(x^2+4)\sqrt{x^2+4}} =$$

$$\frac{x^2+4}{(x^2+4)\sqrt{x^2+4}} - \frac{x^2+x}{(x^2+4)\sqrt{x^2+4}} = \frac{x^2+4-x^2-x}{(x^2+4)\sqrt{x^2+4}} = \frac{4-x}{(x^2+4)\sqrt{x^2+4}}$$