7.4 Raaklijnen en toppen

Opgave 36:

a. $5-2\sqrt{x}=2$

$-2\sqrt{x}=-3$

$\sqrt{x}=1\frac{1}{2}$

$x=2\frac{1}{4}$

b. $6+x\sqrt{x}=10$

$x\sqrt{x}=4$

$x^3=16$

$x=\sqrt[3]{16}$

c. $\dfrac{5x^2-10}{x^2-4}=0$

$5x^2-10=0$

$5x^2=10$

$x^2=2$

$x=\sqrt{2} \quad \vee \quad x=-\sqrt{2}$

d. $x^4-5x^2+4=0$

stel $p=x^2$ dan $p^2-5p+4=0$

$(p-1)(p-4)=0$

$p=1 \quad \vee \quad p=4$

$x^2=1 \quad \vee \quad x^2=4$

$x=1 \quad \vee \quad x=-1 \quad \vee \quad x=2 \quad \vee \quad x=-2$

e. $x^3-8x\sqrt{x}+12=0$

stel $p=x\sqrt{x}$ dan $p^2-8p+12=0$

$(p-2)(p-6)=0$

$p=2 \quad \vee \quad p=6$

$x\sqrt{x}=2 \quad \vee \quad x\sqrt{x}=6$

$x^3=4 \quad \vee \quad x^3=36$

$x=\sqrt[3]{4} \quad \vee \quad x=\sqrt[3]{36}$

f. $\dfrac{5x^2-10}{(x^2-4)^2}=1\frac{2}{5}$

$\dfrac{5x^2-10}{(x^2-4)^2}=\frac{7}{5}$

$7(x^2-4)^2=5(5x^2-10)$

$7(x^4-8x^2+16)=25x^2-50$

$7x^4-56x^2+112=25x^2-50$

$7x^4-81x^2+162=0$

stel $p=x^2$ dan $7p^2-81p+162=0$

$$p = \frac{81 \pm \sqrt{2025}}{14} = \frac{81 \pm 45}{14}$$
$p = \frac{81+45}{14} = 9 \quad \vee \quad p = \frac{81-45}{14} = \frac{18}{7}$
$x^2 = 9 \quad \vee \quad x^2 = \frac{18}{7}$
$x = 3 \quad \vee \quad x = -3 \quad \vee \quad x = \sqrt{\frac{18}{7}} \quad \vee \quad x = -\sqrt{\frac{18}{7}}$

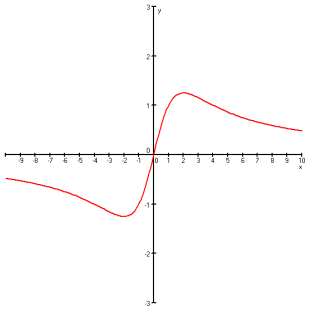
Opgave 37:

$f(x) = 6x - 2x\sqrt{x}$
$f'(x) = 6 - 3\sqrt{x} = 0$
$-3\sqrt{x} = -6$
$\sqrt{x} = 2$
$x = 4$
$T(4,8)$

Opgave 38:

a. $f(x) = \frac{5x}{x^2+4}$

$$f'(x) = \frac{(x^2+4)\cdot 5 - 5x \cdot 2x}{(x^2+4)^2} = \frac{5x^2+20-10x^2}{(x^2+4)^2} = \frac{20-5x^2}{(x^2+4)^2} = 0$$



$20 - 5x^2 = 0$
$-5x^2 = -20$
$x^2 = 4$
$x = 2 \quad \vee \quad x = -2$
min $f(-2) = -1\frac{1}{4}$
max $f(2) = 1\frac{1}{4}$
$B_f = [-1\frac{1}{4}, 1\frac{1}{4}]$

b. $f'(x) = \frac{20-5x^2}{(x^2+4)^2} = \frac{3}{5}$

$3(x^2+4)^2 = 5(20-5x^2)$
$3(x^4+8x^2+16) = 100 - 25x^2$
$3x^4 + 24x^2 + 48 = 100 - 25x^2$
$3x^4 + 49x^2 - 52 = 0$
stel $p = x^2$ dan $3p^2 + 49p - 52 = 0$

$$p = \frac{-49 \pm \sqrt{3025}}{6} = \frac{-49 \pm 55}{6}$$

$p = \frac{-49+55}{6} = 1 \quad \vee \quad p = \frac{-49-55}{6} = -17\frac{1}{3}$
$x^2 = 1 \quad \vee \quad x^2 = -17\frac{1}{3}$
$x = 1 \quad \vee \quad x = -1$

Opgave 39:

a. $8 - 2x \geq 0$

$-2x \geq -8$

$x \leq 4$ dus $D_f = \langle \leftarrow, 4]$

b. $f'(x) = 1 \cdot \sqrt{8-2x} + x \cdot \frac{1}{2 \cdot \sqrt{8-2x}} \cdot -2 = \sqrt{8-2x} - \frac{x}{\sqrt{8-2x}}$

$= \frac{8-2x}{\sqrt{8-2x}} - \frac{x}{\sqrt{8-2x}} = \frac{8-3x}{\sqrt{8-2x}}$

c. $f'(x) = \frac{8-3x}{\sqrt{8-2x}} = 0$

$8-3x = 0$

$-3x = -8$

$x = 2\frac{2}{3}$

$y = 2\frac{2}{3} \cdot \sqrt{2\frac{2}{3}} = \frac{8}{3} \cdot \sqrt{\frac{24}{9}} = \frac{8}{3} \cdot \frac{2}{3} \cdot \sqrt{6} = \frac{16}{9}\sqrt{6}$

top= $(2\frac{2}{3}, 1\frac{7}{9}\sqrt{6})$

$B_f = \langle \leftarrow, 1\frac{7}{9}\sqrt{6}]$

d. $\frac{8-3x}{\sqrt{8-2x}} = 1$

$8-3x = \sqrt{8-2x}$

$9x^2 - 48x + 64 = 8 - 2x$

$9x^2 - 46x + 56 = 0$

$x = \frac{-46 \pm \sqrt{100}}{18} = \frac{46 \pm 10}{18}$

$x = \frac{46-10}{18} = 2 \quad \vee \quad x = \frac{46+10}{18} = 3\frac{1}{9}$ (vervalt)

$A(2,4)$

Opgave 40:

a. $y_A = f(0) = -3$

$f'(x) = 2 \cdot \sqrt{9-2x} + 2x \cdot \frac{1}{2 \cdot \sqrt{9-2x}} \cdot -2 = 2 \cdot \sqrt{9-2x} - \frac{2x}{\sqrt{9-2x}}$

$rc = f'(0) = 6$

$y = 6x + b$ door $(0,-3)$

$b = -3$

$y = 6x - 3$

b. $f'(x) = 2 \cdot \sqrt{9-2x} - \frac{2x}{\sqrt{9-2x}} = 0$

$2 \cdot \sqrt{9-2x} = \frac{2x}{\sqrt{9-2x}}$

$2(9-2x) = 2x$

$18 - 4x = 2x$

$-6x = -18$

$x = 3$

max $f(3) = 6\sqrt{3} - 3$

c. $9-2x \geq 0$

$-2x \geq -9$

$x \leq 4\frac{1}{2}$

$D_f = \langle \leftarrow, 4\frac{1}{2}]$

$B_f = \langle \leftarrow, 6\sqrt{3}-3]$

d. $f'(x) = 2\sqrt{9-2x} - \dfrac{2x}{\sqrt{9-2x}} = 1\frac{1}{2}$

$2(9-2x) - 2x = 1\frac{1}{2}\sqrt{9-2x}$

$18-4x-2x = 1\frac{1}{2}\sqrt{9-2x}$

$18-6x = 1\frac{1}{2}\sqrt{9-2x}$

$12-4x = \sqrt{9-2x}$

$16x^2 - 96x + 144 = 9-2x$

$16x^2 - 94x + 135 = 0$

$x = \dfrac{94 \pm \sqrt{196}}{32} = \dfrac{94 \pm 14}{32}$

$x = \frac{94-14}{32} = 2\frac{1}{2} \quad \vee \quad x = \frac{94+14}{32} = 3\frac{3}{8}$ (vervalt)

$B(2\frac{1}{2}, 7)$

Opgave 41:

a. $f(x) = \dfrac{x^3+2}{\sqrt{x}} = x^{2\frac{1}{2}} + 2x^{-\frac{1}{2}}$

$f'(x) = 2\frac{1}{2}x^{1\frac{1}{2}} - x^{-1\frac{1}{2}} = 2\frac{1}{2}x\sqrt{x} - \dfrac{1}{x\sqrt{x}} = \dfrac{2\frac{1}{2}x^3}{x\sqrt{x}} - \dfrac{1}{x\sqrt{x}} = \dfrac{2\frac{1}{2}x^3-1}{x\sqrt{x}} = 0$

$2\frac{1}{2}x^3 - 1 = 0$

$2\frac{1}{2}x^3 = 1$

$x^3 = \frac{2}{5}$

$x = \sqrt[3]{\frac{2}{5}}$

$y = \dfrac{2\frac{2}{5}}{\sqrt{\sqrt[3]{\frac{2}{5}}}} = \dfrac{2\frac{2}{5}}{\sqrt{(\frac{2}{5})^{\frac{1}{3}}}} = \dfrac{2\frac{2}{5}}{((\frac{2}{5})^{\frac{1}{3}})^{\frac{1}{2}}} = \dfrac{2\frac{2}{5}}{(\frac{2}{5})^{\frac{1}{6}}} = \dfrac{2\frac{2}{5}}{\sqrt[6]{\frac{2}{5}}}$

$a = 2\frac{2}{5} \quad b = 6 \quad c = \frac{2}{5}$

b. $\dfrac{2\frac{1}{2}x^3-1}{x\sqrt{x}} = 1\frac{1}{2}$

$2\frac{1}{2}x^3 - 1 = 1\frac{1}{2}x\sqrt{x}$

$5x^3 - 2 = 3x\sqrt{x}$

$5x^3 - 3x\sqrt{x} - 2 = 0$

stel $p = x\sqrt{x}$ dan $5p^2 - 3p - 2 = 0$

$p = \dfrac{3 \pm \sqrt{49}}{10} = \dfrac{3 \pm 7}{10}$

$p = \frac{3+7}{10} = 1 \quad \vee \quad p = \frac{3-7}{10} = -\frac{2}{5}$

$x\sqrt{x}=1 \quad \vee \quad x\sqrt{x}=-\frac{2}{5}$ (kan niet)

$x^3=1$

$x=1$

$y=3$

$y=1\frac{1}{2}x+b$ door (1,3)

$3=1\frac{1}{2}+b$

$b=1\frac{1}{2}$

$y=1\frac{1}{2}x+1\frac{1}{2}$

Opgave 42:

a. $f'(x)=\dfrac{(x\sqrt{x}+1)\cdot 9-9x\cdot 1\frac{1}{2}\sqrt{x}}{(x\sqrt{x}+1)^2}=\dfrac{9x\sqrt{x}+9-13\frac{1}{2}x\sqrt{x}}{(x\sqrt{x}+1)^2}=\dfrac{9-4\frac{1}{2}x\sqrt{x}}{(x\sqrt{x}+1)^2}$

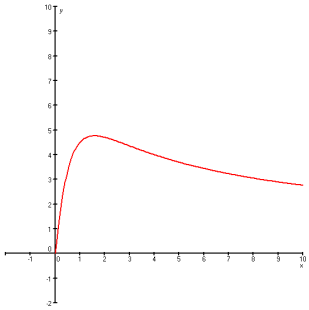
$f'(4)=-\frac{1}{3}$

$y=-\frac{1}{3}x+b$ door (4,4)

$4=-\frac{4}{3}+b$

$b=5\frac{1}{3}$

$y=-\frac{1}{3}x+5\frac{1}{3}$



b. $f'(x)=\dfrac{9-4\frac{1}{2}x\sqrt{x}}{(x\sqrt{x}+1)^2}=0$

$9-4\frac{1}{2}x\sqrt{x}=0$

$-4\frac{1}{2}x\sqrt{x}=-9$

$x\sqrt{x}=2$

$x^3=4$

$x=\sqrt[3]{4}=\sqrt[3]{2^2}=2^{\frac{2}{3}}$

$\max f(\sqrt[3]{4})=\dfrac{9\cdot\sqrt[3]{4}}{2^{\frac{2}{3}}\cdot\sqrt{2^{\frac{2}{3}}}+1}=\dfrac{9\cdot\sqrt[3]{4}}{2^{\frac{2}{3}}\cdot 2^{\frac{1}{3}}+1}=\dfrac{9\cdot\sqrt[3]{4}}{2+1}=\dfrac{9\cdot\sqrt[3]{4}}{3}=3\cdot\sqrt[3]{4}$

c. $f'(x)=\dfrac{9-4\frac{1}{2}x\sqrt{x}}{(x\sqrt{x}+1)^2}=\frac{9}{8}$

$9(x\sqrt{x}+1)^2=8(9-4\frac{1}{2}x\sqrt{x})$

$(x\sqrt{x}+1)^2=8(1-\frac{1}{2}x\sqrt{x})$

$x^3+2x\sqrt{x}+1=8-4x\sqrt{x}$

$x^3+6x\sqrt{x}-7=0$

stel $p=x\sqrt{x}$ dan $p^2+6p-7=0$

$(p-1)(p+7)=0$

$p=1 \quad \vee \quad p=-7$

$x\sqrt{x}=1 \quad \vee \quad x\sqrt{x}=-7$ (kan niet)

$x^3=1$

$x=1$ dus $B(1,4\frac{1}{2})$

Opgave 43:

a. $\frac{dN}{dt} = 90 - 60\sqrt{t}$

$\left[\frac{dN}{dt}\right]_{t=1} = 30$

Om 8 uur neemt het aantal auto's dat per minuut passeert toe met 30 per uur.

b. $\frac{dN}{dt} = 90 - 60\sqrt{t} = 0$

$-60\sqrt{t} = -90$

$\sqrt{t} = 1{,}5$

$t = 2{,}25$ dus om 9.15 uur

c. de afname is 1 per 2 minuten, dus 30 per uur

$\frac{dN}{dt} = 90 - 60\sqrt{t} = -30$

$-60\sqrt{t} = -120$

$\sqrt{t} = 2$

$t = 4$ dus om 11.00 uur

Opgave 44:

a. $f_2(x) = x^3 + 2x^2$ dus $f_2'(x) = 3x^2 + 4x$

$f_5(x) = x^3 + 5x^2$ dus $f_5'(x) = 3x^2 + 10x$

b. $f_p'(x) = 3x^2 + 2px$

Opgave 45:

$f_p'(x) = -\frac{1}{2}x + p = 0$

$p = \frac{1}{2}x$

Opgave 46:

$f_p'(x) = x^2 + 2px = 0$

$2px = -x^2$

$p = -\frac{1}{2}x$ voor $x \neq 0$; $x = 0$ geeft $y = 5$

$y = \frac{1}{3}x^3 - \frac{1}{2}x \cdot x^2 + 5$

$y = -\frac{1}{6}x^3 + 5$ ook $(0,5)$ ligt op deze grafiek

Opgave 47:

$f_p'(x) = \frac{(x^2+4)\cdot p - px \cdot 2x}{(x^2+4)^2} = \frac{px^2 + 4p - 2px^2}{(x^2+4)^2} = \frac{4p - px^2}{(x^2+4)^2} = 0$

$4p - px^2 = 0$

$p(4 - x^2) = 0$

$p = 0 \quad \vee \quad x^2 = 4$

dus $x = 2 \quad \vee \quad x = -2$

als $p = 0$ geldt $f(x) = 0$; dit is een horizontale lijn , dus geen toppen

Opgave 48:

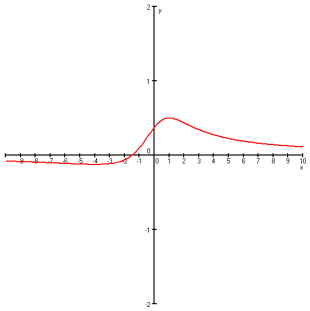
a. $f'_p(x) = \dfrac{(x^2+4)\cdot 1-(x+p)\cdot 2x}{(x^2+4)^2} = \dfrac{x^2+4-2x^2-2px}{(x^2+4)^2} = \dfrac{-x^2-2px+4}{(x^2+4)^2}$

$f'_p(1) = \dfrac{-2p+3}{25} = 0$

$-2p+3=0$

$-2p=-3$

$p = 1\frac{1}{2}$



$f'_{1\frac{1}{2}}(x) = \dfrac{-x^2-3x+4}{(x^2+4)^2} = 0$

$-x^2-3x+4=0$

$x^2+3x-4=0$

$(x+4)(x-1)=0$

$x=-4 \quad \vee \quad x=1$

$\min f(-4) = -\frac{1}{8}$

b. $f'_p(x) = \dfrac{-x^2-2px+4}{(x^2+4)^2} = 0$

$-x^2-2px+4=0$

$-2px = x^2-4$

$p = -\frac{1}{2}x + \dfrac{2}{x}$ voor $x \neq 0$; $x=0$ geeft $y = \frac{1}{4}p$

$y = \dfrac{x-\frac{1}{2}x+\frac{2}{x}}{x^2+4} = \dfrac{\frac{1}{2}x+\frac{2}{x}}{x^2+4} = \dfrac{\frac{x^2}{2x}+\frac{4}{2x}}{x^2+4} = \dfrac{\frac{x^2+4}{2x}}{x^2+4} = \dfrac{1}{2x}$