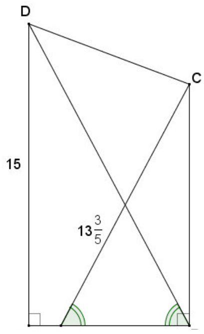
HOOFDSTUK 8: Vermoedens en bewijzen.

8.1 Gelijkvormige en congruente driehoeken

Opgave 1:

ΔADE en ΔABC vormen een snavelfiguur met vergrotingsfactor 2

Opgave 2:



a. $\left.\begin{array}{l}\angle BAD = \angle EBC \\ \angle ABD = \angle BEC\end{array}\right\}\ \Delta ABD \sim \Delta BEC \quad (hh)$

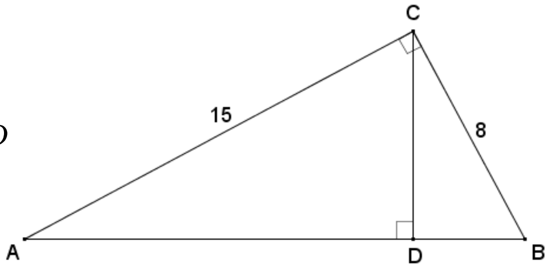
b. $BD = \sqrt{AB^2 + AD^2} = \sqrt{8^2 + 15^2} = \sqrt{289} = 17$

$\frac{AB}{BE} = \frac{BD}{EC}$ dus $\frac{8}{BE} = \frac{17}{13\frac{3}{5}}$

dus $BE = \frac{8 \cdot 13\frac{3}{5}}{17} = 6\frac{2}{5}$

$AE = AB - BE = 8 - 6\frac{2}{5} = 1\frac{3}{5}$

Opgave 3:



a. in ΔACD geldt: $\angle ACD = 90° - \angle CAD$

$\angle C = 90°$

dus $\angle BCD = 90° - \angle ACD =$

$= 90° - (90° - \angle CAD)$

$= 90° - 90° + \angle CAD$

$= \angle CAD$

in ΔBCD geldt: $\angle CBD = 90° - \angle BCD = 90° - \angle CAD = \angle ACD$

$\left.\begin{array}{l}\angle ADC = \angle ACB \\ \angle CAD = \angle BAC\end{array}\right\}\ \Delta ACD \sim \Delta ABC \quad (hh)$

$\left.\begin{array}{l}\angle ADC = \angle CDB \\ \angle CAD = \angle BCD\end{array}\right\}\ \Delta ACD \sim \Delta CBD \quad (hh)$

$\left.\begin{array}{l}\angle BDC = \angle BCA \\ \angle DBC = \angle CBA\end{array}\right\}\ \Delta BCD \sim \Delta BAC \quad (hh)$

b. $AB = \sqrt{AC^2 + BC^2} = \sqrt{15^2 + 8^2} = \sqrt{289} = 17$

$\Delta BCD \sim \Delta BAC$ dus $\frac{BC}{BA} = \frac{CD}{AC}$

$\frac{8}{17} = \frac{CD}{15}$

$CD = \frac{8 \cdot 15}{17} = 7\frac{1}{17}$

$\Delta ACD \sim \Delta ABC$ dus $\frac{AD}{AC} = \frac{AC}{AB}$

$\frac{AD}{15} = \frac{15}{17}$

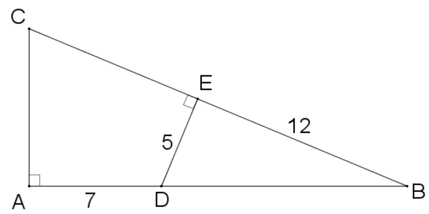
$AD = \frac{15 \cdot 15}{17} = 13\frac{4}{17}$

$\Delta BCD \infty \Delta BAC$ dus $\frac{BD}{BC} = \frac{BC}{BA}$

$\frac{BD}{8} = \frac{8}{17}$

$BD = \frac{8 \cdot 8}{17} = 3\tfrac{13}{17}$

Opgave 4:



$\left.\begin{array}{l}\angle DBE = \angle CBA \\ \angle BED = \angle BAC\end{array}\right\}$ $\Delta BED \infty \Delta BAC \quad (hh)$

$BD = \sqrt{BE^2 + DE^2} = \sqrt{12^2 + 5^2} = \sqrt{169} = 13$

dus $AB = 20$

$\frac{AC}{ED} = \frac{AB}{EB}$ dus $\frac{AC}{5} = \frac{20}{12}$

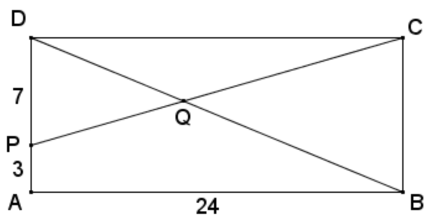
$AC = \frac{5 \cdot 20}{12} = 8\tfrac{1}{3}$

$BC = \sqrt{AB^2 + AC^2} = \sqrt{20^2 + (8\tfrac{1}{3})^2} = \sqrt{469\tfrac{4}{9}} = 21\tfrac{2}{3}$

dus $CE = 21\tfrac{2}{3} - 12 = 9\tfrac{2}{3}$

Opgave 5:

$\left.\begin{array}{l}\angle PQD = \angle CQB \ (overstaande\ hoeken) \\ \angle QPD = \angle QCB \ (z-hoeken)\end{array}\right\}$ $\Delta PQD \infty \Delta CQB \quad (hh)$



$CP = \sqrt{DP^2 + CD^2} = \sqrt{7^2 + 24^2} = \sqrt{625} = 25$

$CQ = 25 - PQ$

$\frac{PQ}{CQ} = \frac{PD}{CB}$ dus $\frac{PQ}{25 - PQ} = \frac{7}{10}$

$10PQ = 7(25 - PQ)$

$10PQ = 175 - 7PQ$

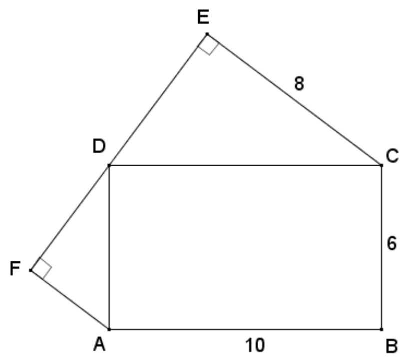
$17PQ = 175$

$PQ = 10\tfrac{5}{17}$

Opgave 6:

a. $\angle ADF = 180° - 90° - \angle CDE = 90° - \angle CDE = \angle DCE$

$\left.\begin{array}{l}\angle ADF = \angle DCE \\ \angle AFD = \angle DEC\end{array}\right\}$ $\Delta ADF \infty \Delta DCE \quad (hh)$



$DE = \sqrt{CD^2 - CE^2} = \sqrt{10^2 - 8^2} = \sqrt{36} = 6$

$\frac{AF}{DE} = \frac{AD}{DC}$ dus $\frac{AF}{6} = \frac{6}{10}$

$AF = \frac{6 \cdot 6}{10} = 3\tfrac{3}{5}$

b. $\angle BSC = \angle DSG$ (overstaande hoeken)

$\angle SDG = 180° - 90° - \angle DSG$

$= 90° - \angle DSG$

$= 90° - \angle BSC$

$= \angle SBC$

$\left.\begin{array}{l}\angle CDE = \angle SBC \\ \angle CED = \angle SCB\end{array}\right\} \quad \Delta CDE \infty \Delta SBC$

$\frac{DE}{BC} = \frac{CE}{SC}$ dus $\frac{6}{6} = \frac{8}{SC}$

$SC = 8$ dus $DS = 2$

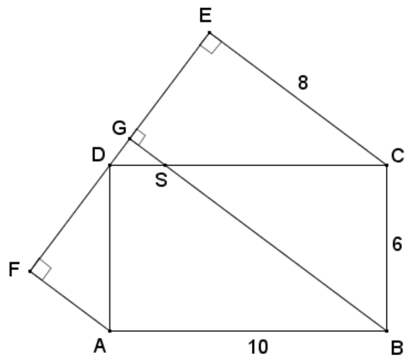
$BS = \sqrt{BC^2 + CS^2} = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$

$\left.\begin{array}{l}\angle SDG = \angle SBC \\ \angle SGD = \angle SCB\end{array}\right\} \quad \Delta SDG \infty \Delta SBC$

$\frac{SD}{SB} = \frac{SG}{SC}$ dus $\frac{2}{10} = \frac{SG}{8}$

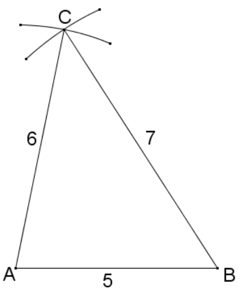
$SG = \frac{8 \cdot 2}{10} = 1\frac{3}{5}$

$BG = BS + SG = 10 + 1\frac{3}{5} = 11\frac{3}{5}$

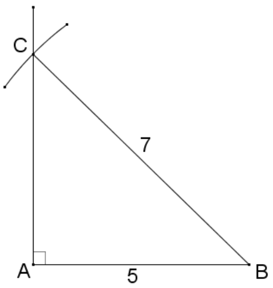


Opgave 7:

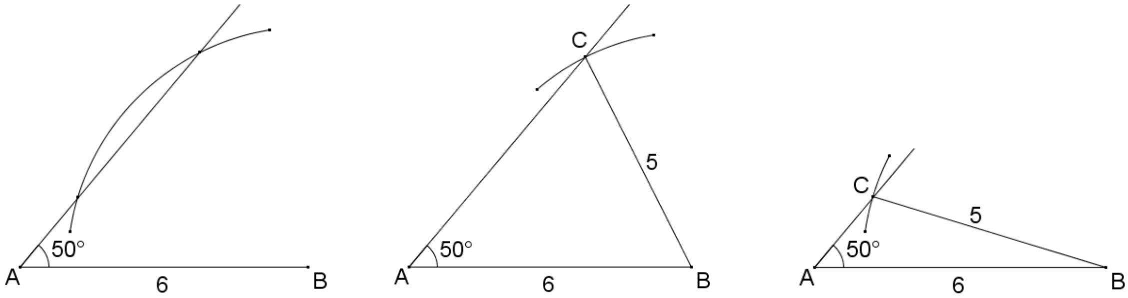
a. één driehoek mogelijk



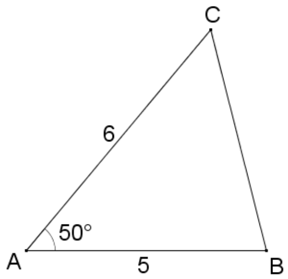
b. één driehoek mogelijk



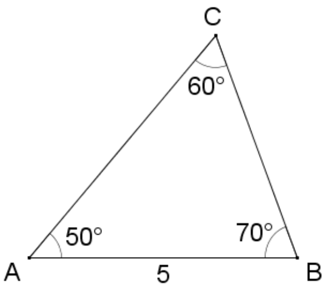
c. twee driehoeken mogelijk
de cirkel met middelpunt B en straal 5 snijdt de lijn AC op twee plaatsen, dus zijn er twee driehoeken mogelijk



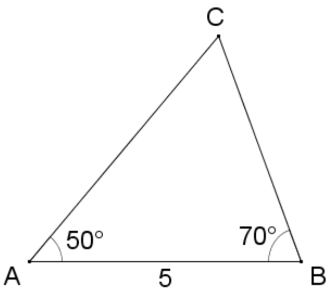
d. één driehoek mogelijk



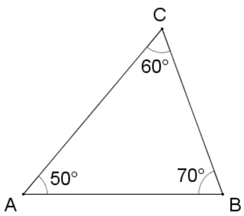
e. één driehoek mogelijk
$\angle B = 180° - \angle A - \angle C = 180° - 50° - 60° = 70°$

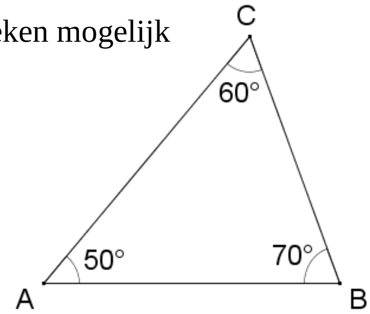


f. één driehoek mogelijk



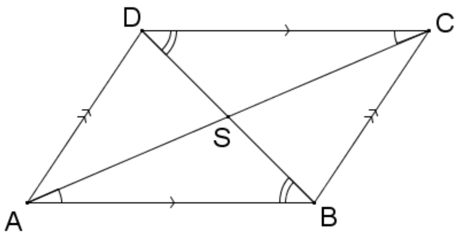
g. er zijn oneindig veel driehoeken mogelijk





Opgave 8:

Teken vierhoek *ABCD* en de diagonalen *AC* en *BD*.
Punt *S* is het snijpunt van *AC* en *BD*.

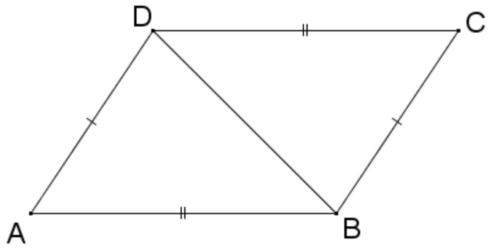


$$\left.\begin{array}{l} \angle BAS = \angle DCS \ (z-hoeken) \\ AB = CD \ (zie\ voorbeeld) \\ \angle ABS = \angle CDS \ (z-hoeken) \end{array}\right\} \quad \Delta ABS \cong \Delta CDS \quad (hzh)$$

dus $AS = CS$ en $BS = DS$
dus de diagonalen *AC* en *BD* delen elkaar middendoor

Opgave 9:

Teken vierhoek *ABCD* met $AB = CD$ en $BC = AD$.
Teken de diagonaal *BD*.

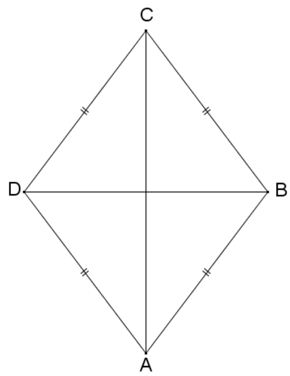


$$\left.\begin{array}{l} AB = CD \\ BD = DB \\ AD = CB \end{array}\right\} \quad \Delta ABD \cong \Delta CDB \quad (zzz)$$

dus $\angle A = \angle C$

Opgave 10:

a. Teken vierhoek *ABCD* met $AB = BC = CD = AD$.
Teken de diagonalen *AC* en *BD*.



$$\left.\begin{array}{l} AB = CD \\ AC = CA \\ BC = DA \end{array}\right\} \quad \Delta ABC \cong \Delta CDA \quad (zzz)$$

dus $\angle BAC = \angle DCA$
dus $\angle BAC$ en $\angle DCA$ zijn *z*-hoeken
dus $AB \,//\, CD$

$$\left.\begin{array}{l} AB = CD \\ BD = DB \\ AD = CB \end{array}\right\} \quad \Delta ABD \cong \Delta CDB \quad (zzz)$$

dus $\angle ADB = \angle CBD$
dus $\angle ADB$ en $\angle CDB$ zijn *z*-hoeken
dus $AD \,//\, BC$

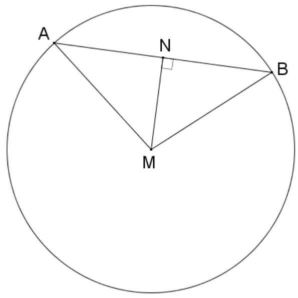
b. $AB = BC$ dus ΔABC is gelijkbenig, dus $\angle BAC = \angle BCA$
$AD = CD$ dus ΔACD is gelijkbenig, dus $\angle DAC = \angle DCA$
$\angle BAC = \angle DCA$ (zie opgave 10a)
Hieruit volgt dat $\angle BAC = \angle BCA = \angle DCA = \angle DAC$

$AB = AD$ dus ΔABD is gelijkbenig, dus $\angle ABD = \angle ADB$
$BC = CD$ dus ΔBCD is gelijkbenig, dus $\angle CBD = \angle CDB$
$\angle ADB = \angle CBD$ (zie opgave 10a)
Hieruit volgt dat $\angle ABD = \angle ADB = \angle CBD = \angle CDB$
Dus de diagonalen delen de hoeken van de vierhoek middendoor.

Opgave 11:

$$\left.\begin{array}{l} AM = BM \ (straal\ cirkel) \\ MN = MN \\ \angle ANM = \angle BNM = 90° \end{array}\right\} \quad \Delta AMN \cong \Delta BMN \quad (zzr)$$

dus $AN = BN$



Opgave 12:

$\angle ACQ = 60°$ (gelijkzijdige driehoek)

$\angle BCP = 60°$ (gelijkzijdige driehoek)

$$\left.\begin{array}{l} \angle BCQ = \angle BCA + \angle ACQ = \angle BCA + 60° \\ \angle ACP = \angle ACB + \angle BCP = \angle ACB + 60° \end{array}\right\} \quad \angle BCQ = \angle ACP$$

$$\left.\begin{array}{l} QC = AC \\ \angle QCB = \angle ACP \\ CB = CP \end{array}\right\} \quad \Delta QCB \cong \Delta ACP \quad (zhz)$$

dus $QB = AP$