10.2 Oppervlakten en inhouden

Opgave 11:

a. $y_1 = 10x - x^2$

$Opp(U) == fnInt(Y_1, X, 1, 8) = 144{,}667$

$y_2 = x + 8$

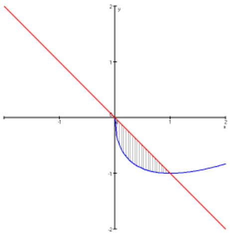
$Opp(V) = fnInt(Y_2, X, 1, 8) = 87{,}5$

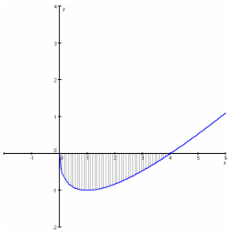
b. $Opp(W) = Opp(U) - Opp(V) = 144{,}67 - 87{,}5 = 57{,}17$

c. $y_3 = y_1 - y_2$

$$\int_1^8 (f(x) - g(x))dx = fnInt(Y_3, X, 1, 8) = 57{,}17 = Opp(W)$$

Opgave 12:





a. $y_1 = x - 2\sqrt{x}$

$y_2 = -x$

$y_3 = y_2 - y_1$

intersect van y_1 en y_2 geeft $x = 0 \quad \vee \quad x = 1$

$$Opp(V) = \int_0^1 (-x - (x - 2\sqrt{x}))dx = fnInt(Y_3, X, 0, 1) = 0{,}33$$

b. zero geeft $x = 0 \quad \vee \quad x = 4$

$$Opp(W) = \int_0^4 (x - 2\sqrt{x})dx = fnInt(Y_1, X, 0, 4) = 2{,}67$$

Opgave 13:

$y_1 = x^3 - 3x$ en $y_2 = 1 - \frac{1}{2}x$

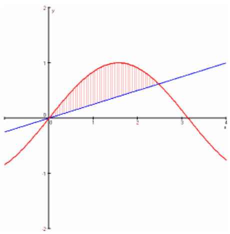
intersect geeft $x = -1{,}320 \quad \vee \quad x = -0{,}432 \quad \vee \quad x = 1{,}752$

$y_3 = y_1 - y_2$ en $y_4 = y_2 - y_1$

$$Opp = \int_{-1,320}^{-0,432} (f(x) - g(x))dx + \int_{-0,432}^{1,752} (g(x) - f(x))dx =$$

$$= fnInt(Y_3, X, -1.320, -0.432) + fnInt(Y_4, X, -0.432, 1.752) = 3{,}75$$

Opgave 14:



$y_1 = \sin x$

$$Opp(V) = \int_0^{\pi} \sin x dx = fnInt(Y_1, X, 0, \pi) = 2$$

$y_2 = \frac{1}{4}x$

intersect geeft $x = 2{,}475$

$y_3 = y_1 - y_2$

$$Opp(I) = \int_0^{2,475} (\sin x - \tfrac{1}{4}x)dx = fnInt(Y_3, X, 0, 2.475) = 1{,}02 \neq \tfrac{1}{2} \cdot 2 \text{ dus niet}$$

Opgave 15:

a. $y_1 = \sin x$

$$Opp(V) = \int_0^{\pi} \sin x dx = fnInt(Y_1, X, 0, \pi) = 2 \text{ dus ook } Opp(W) = 2$$

b. $\int_0^{2\pi} \sin x dx = 0$

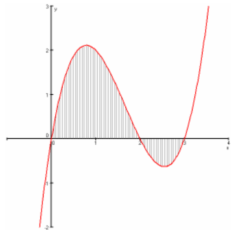
Omdat W onder de x-as ligt is $\int_{\pi}^{2\pi} \sin x dx = -2$ want de hoogte van je rechthoekjes is nu negatief.

c. $y_2 = abs(y_1)$

$$\int_0^{2\pi} |f(x)| dx = fnInt(Y_2, X, 0, 2\pi) = 4 = Opp(V) + Opp(W)$$

Opgave 16:

$y_1 = x^3 - 5x^2 + 6x$



de optie zero geeft $x = 0 \quad \vee \quad x = 2 \quad \vee \quad x = 3$

$y_2 = abs(y_1)$

$$Opp(V + W) = \int_0^3 |x^3 - 5x^2 + 6x| dx = fnInt(Y_2, X, 0, 3) = 3{,}08$$

Opgave 17:

$$Opp(I + II) = \int_{-1{,}320}^{1{,}752} |f(x) - g(x)| dx$$

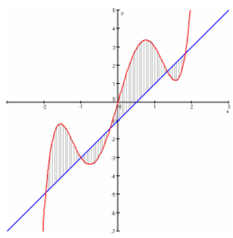
Als je $|f(x) - g(x)|$ neemt hoef je niet te kijken of de grafiek van f boven de grafiek van g ligt of andersom.

Opgave 18:

$y_1 = x^5 - 5x^3 + 7x$ en $y_2 = 2x - 1$

intersect geeft $x = -1{,}956 \quad \vee \quad x = -1 \quad \vee \quad x = -0{,}209 \quad \vee \quad x = 1{,}338 \quad \vee \quad x = 1{,}827$

$y_3 = abs(y_1 - y_2)$



$$Opp = \int_{-1{,}956}^{1{,}827} y_3 dx = fnInt(Y_3, X, -1.956, 1{,}827) = 5{,}48$$

Opgave 19:

a. $f(3) = 19$

$Inh = \pi \cdot 19^2 \cdot 2 = 722\pi$

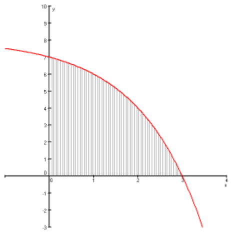
$f(5) = 35$

$Inh = \pi \cdot 35^2 \cdot 2 = 2450\pi$

b. $Inh \approx 242\pi + 722\pi + 2450\pi = 3414\pi$

Opgave 20:

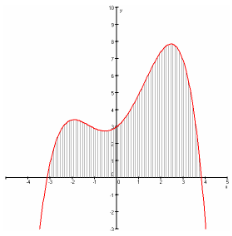
$8-2^x=0$
$2^x=8$
$2^x=2^3$
$x=3$



$y_1=8-2^x$ en $y_2=\pi\cdot{y_1}^2$

$Inh=\int_0^3 \pi\cdot f^2(x)dx = fnInt(Y_2,X,0,3)=238{,}33$

Opgave 21:



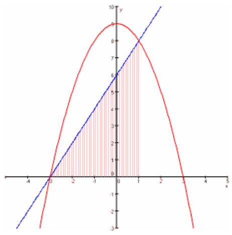
$y_1=-0{,}1x^4+x^2+x+3$

de optie zero geeft $x=-3{,}14 \quad \vee \quad x=3{,}83$

$y_2=\pi\cdot{y_1}^2$

$Inh=\int_{-3{,}14}^{3{,}83} \pi\cdot f^2(x)dx = fnInt(Y_2,X,-3.14,3.83)=487{,}49$

Opgave 22:



$y_1=9-x^2$ en $y_2=2x+6$

intersect geeft $x=-3 \quad \vee \quad x=1$

$y_3=\pi\cdot{y_2}^2$ en $y_4=\pi\cdot{y_1}^2$

$Inh=\int_{-3}^{1} \pi\cdot(2x+6)^2\,dx+\int_{-1}^{3} \pi\cdot(9-x^2)^2\,dx$

$=fnInt(Y_3,X,-3,1)+fnInt(Y_4,X,1,3)=268{,}083+170{,}903=438{,}99$

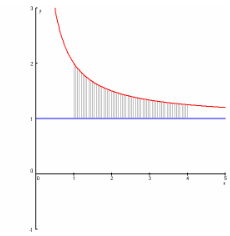
Opgave 23:

a. $g(x)=2+\frac{3}{x}-2=\frac{3}{x}$

b. $y_1=\frac{3}{x}$ en $y_2=\pi\cdot{y_1}^2$

$Inh=\int_1^3 \pi\cdot\left(\frac{3}{x}\right)^2 dx = fnInt(Y_2,X,1,3)=18{,}85$

Opgave 24:



a. $f(x)=\frac{x+1}{x}=\frac{x}{x}+\frac{1}{x}=1+\frac{1}{x}$

$y_1=1+\frac{1}{x}$ en $y_2=1$

$y_3=\pi\cdot({y_1}^2-{y_2}^2)$

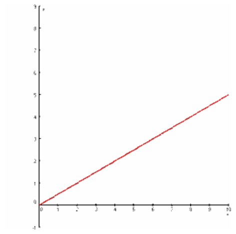
$Inh=\int_1^4 \pi\cdot((1+\tfrac{1}{x})^2-1^2)\,dx = fnInt(Y_3,X,1,4)=11{,}07$

b. transleer V 1 omlaag en wentel het nieuwe vlakdeel om de x-as

$g(x) = \frac{1}{x}$ dus neem $y_1 = \frac{1}{x}$

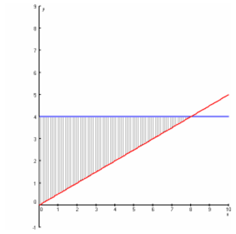
$Inh = \int_1^4 \pi \cdot (\tfrac{1}{x})^2\,dx = fnInt(Y_2, X, 1, 4) = 2{,}36$ met $y_2 = \pi \cdot {y_1}^2$

Opgave 25:



a. $y_1 = \tfrac{1}{2}x$ en $y_2 = \pi \cdot {y_1}^2$

$Inh = \int_0^8 \pi \cdot (\tfrac{1}{2}x)^2\,dx = fnInt(Y_2, X, 0, 8) = 134{,}04$



b. $y_1 = \tfrac{1}{2}x$ en $y_2 = 4$

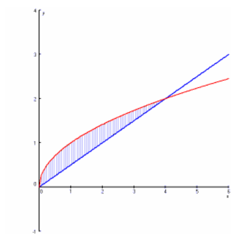
$y_3 = \pi \cdot ({y_2}^2 - {y_1}^2)$

$Inh = \int_0^8 \pi \cdot (4^2 - (\tfrac{1}{2}x)^2)\,dx = fnInt(Y_3, X, 0, 8) = 268{,}08$

Opgave 26:

$Inh = \pi \cdot 2^2 \cdot 5 - \pi \cdot 1^2 \cdot 5 = 20\pi - 5\pi = 15\pi$

Opgave 27:

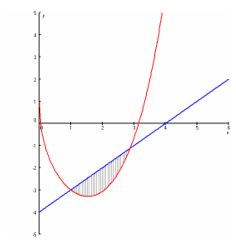


$y_1 = \sqrt{x}$ en $y_2 = \tfrac{1}{2}x$

$y_3 = \pi \cdot ({y_1}^2 - {y_2}^2)$

$h = \int_0^4 \pi \cdot ((\sqrt{x})^2 - (\tfrac{1}{2}x)^2)dx = fnInt(Y_3, X, 0, 4) = 8{,}4$

Opgave 28:



$y_1 = 2^x - 5\sqrt{x}$ en $y_2 = x - 4$

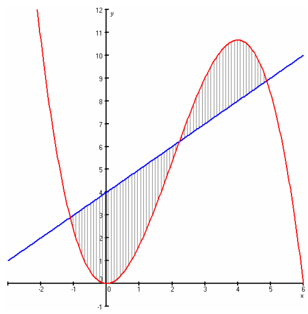
intersect geeft $x = 1 \quad \vee \quad x = 2{,}88$

$y_3 = \pi \cdot ({y_1}^2 - {y_2}^2)$

$Inh = \int_1^{2{,}88} \pi \cdot ((2^x - 5\sqrt{x})^2 - (x-4)^2)\,dx =$

$= fnInt(Y_3, X, 1, 2.88) = 20{,}9$

Opgave 29:



$y_1 = -\tfrac{1}{3}x^3 + 2x^2$ en $y_2 = x + 4$

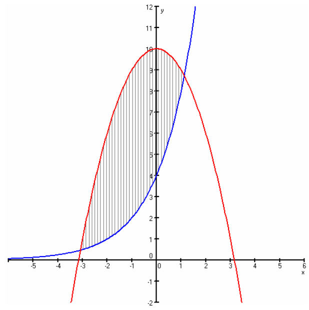
intersect geeft $x = -1{,}11 \quad \vee \quad x = 2{,}22 \quad \vee \quad x = 4{,}88$

$y_3 = \pi \cdot ({y_2}^2 - {y_1}^2)$ en $y_4 = \pi \cdot ({y_1}^2 - {y_2}^2)$

$$Inh = \int_{-1,11}^{2,22} \pi \cdot ((x+4)^2 - (-\tfrac{1}{3}x^3 + 2x^2)^2)\,dx + \int_{2,22}^{4,88} \pi \cdot ((-\tfrac{1}{3}x^3 + 2x^2)^2 - (x+4)^2)\,dx =$$

$$= fnInt(Y_3, X, -1.11, 2.22) + fnInt(Y_4, X, 2.22, 4.88) = 155,882 + 267,005 = 422,88$$

Opgave 30:



a. $y_1 = 10 - x^2$ en $y_2 = 2^{x+2}$

intersect geeft $x = -3,09 \quad \vee \quad x = 1,13$

$y_3 = \pi \cdot ({y_1}^2 - {y_2}^2)$

$$Inh = \int_{-3,09}^{1,13} \pi \cdot ((10 - x^2)^2 - (2^{x+2})^2)\,dx =$$

$$= fnInt(Y_3, X, -3.09, 1.13) = 682,59$$

b. transleer vlakdeel V 10 omlaag

$h(x) = -x^2$ en $i(x) = 2^{x+2} - 10$

$y_1 = -x^2$ en $y_2 = 2^{x+2} - 10$

$y_3 = \pi \cdot ({y_1}^2 - {y_2}^2)$

$$Inh = \int_{-3,09}^{1,13} \pi \cdot ((2^{x+2} - 10)^2 - (-x^2)^2)\,dx = fnInt(Y_3, X, -3.09, 1.13) = 569,80$$

Opgave 31:

a. de x-as ligt op of onder het laagste punt van de grafiek van f.

b. de x-as ligt op of boven het hoogste punt van de grafiek van g.