10.3 Primitieve functies

Opgave 32:

a. $Opp(V_p) = \frac{1}{2} \cdot OP \cdot (OB + PQ) = \frac{1}{2} \cdot p \cdot (b + ap + b) = \frac{1}{2} p(2b + ap) = \frac{1}{2} ap^2 + bp$

b. $\frac{dO}{dp} = ap + b = f(x)$

Opgave 33:

a.

p	1	2	3	4	5
$O(p)$	1	8	27	64	125

b. $1^3 = 1 \quad 2^3 = 8 \quad 3^3 = 27 \quad 4^3 = 64 \quad 5^3 = 125$

c. $\frac{dO}{dp} = 3p^2$

d. $p^3 = 10$

$p = \sqrt[3]{10}$

Opgave 34:

a. $F(x) = (x^2 + 1)^6 + 1$

$F'(x) = 6(x^2 + 1)^5 \cdot 2x = 12x \cdot (x^2 + 1)^5$

b. $G(x) = (\frac{1}{2} x - \frac{1}{4}) \cdot e^{2x} - 2$

$G'(x) = \frac{1}{2} \cdot e^{2x} + (\frac{1}{2} x - \frac{1}{4}) \cdot 2e^{2x} = \frac{1}{2} e^{2x} + xe^{2x} - \frac{1}{2} e^{2x} = xe^{2x}$

c. $H(x) = 2\ln x + \ln^2 x + 3$

$H'(x) = \frac{2}{x} + 2\ln x \cdot \frac{1}{x} = \frac{2 + 2\ln x}{x}$

d. $J(x) = \frac{e^{3x} - 10}{2e^x} - 4 = \frac{1}{2} e^{2x} - 5e^{-x} - 4$

$J'(x) = e^{2x} + 5e^{-x} = \frac{e^{3x}}{e^x} + \frac{5}{e^x} = \frac{e^{3x} + 5}{e^x}$

Opgave 35:

a. $f(x) = x^4$

b. $g(x) = \frac{1}{4} \cdot 4e^{4x+1} = e^{4x+1}$

c. $h(x) = 3^x$

d. $j(x) = 1 \cdot \ln x + x \cdot \frac{1}{x} = \ln x + 1$

Opgave 36:

a. $F(x) = \frac{a}{n+1} \cdot x^{n+1} + c$

$F'(x) = \frac{a}{n+1} \cdot (n+1) \cdot x^n = ax^n$

b. $F(x)=\frac{g^x}{\ln g}+c$

$F'(x)=\frac{1}{\ln g}\cdot g^x\cdot \ln g=g^x$

c. $F(x)=e^x+c$

$F'(x)=e^x$

d. $F(x)=\ln|x|+c$

$F'(x)=\frac{1}{x}$

e. $F(x)=x\ln x-x+c$

$F'(x)=1\cdot\ln x+x\cdot\frac{1}{x}-1=\ln x+1-1=\ln x$

f. $F(x)=\frac{1}{\ln g}(x\ln x-x)$

$F'(x)=\frac{1}{\ln g}(1\cdot\ln x+x\cdot\frac{1}{x}-1)=\frac{1}{\ln g}\cdot\ln x=\frac{\ln x}{\ln g}={}^{g}\log x$

Opgave 37:

a. voor $n=-1$ is de noemer van $\frac{a}{n+1}$ nul

b. $F(x)=\ln|x|+c$

c. $[a\cdot F]'=a'\cdot F+a\cdot F'=0\cdot F+a\cdot F'=a\cdot F'=a\cdot f$

Opgave 38:

a. $F(x)=2x^3+c$

b. $F(x)=\frac{1}{2}x^4+x^5+c$

c. $f(x)=\frac{x^4-2x}{2x^3}=\frac{1}{2}x-\frac{1}{x^2}=\frac{1}{2}x-x^{-2}$

$F(x)=\frac{1}{4}x^2+x^{-1}=\frac{1}{4}x^2+\frac{1}{x}+c$

d. $F(x)=\frac{10^x}{\ln 10}+c$

e. $F(x)=\frac{5\cdot 2^x}{\ln 2}+c$

f. $f(x)=\frac{x^3+2}{x^4}=\frac{1}{x}+2x^{-4}$

$F(x)=\ln|x|-\frac{2}{3}x^{-3}=\ln|x|-\frac{2}{3x^3}+c$

Opgave 39:

a. $F(x)=\frac{1}{4}x^4-1\frac{1}{2}x^2+c$

b. $F(x)=5e^x+c$

c. $f(x)=\dfrac{x^4-6}{2x^3}=\tfrac{1}{2}x-3x^{-3}$

$F(x)=\tfrac{1}{4}x^2+1\tfrac{1}{2}x^{-2}=\tfrac{1}{4}x^2+\dfrac{3}{2x^2}+c$

d. $F(x)=\dfrac{3^x}{\ln 3}+\tfrac{1}{4}x^4+c$

e. $F(x)=2\cdot(x\ln x-x)=2x\ln x-2x+c$

f. $f(x)=\ln 2x=\ln 2+\ln x$

$F(x)=x\cdot\ln 2+x\ln x-x+c$

Opgave 40:

a. $F(x)=e^{x+1}+c$

b. $f(x)=\dfrac{8}{x^3}=8x^{-3}$

$F(x)=-4x^{-2}=\dfrac{-4}{x^2}+c$

c. $f(x)=\dfrac{-x^2+2x+3}{x^4}=-x^{-2}+2x^{-3}+3x^{-4}$

$F(x)=x^{-1}-x^{-2}-x^{-3}=\dfrac{1}{x}-\dfrac{1}{x^2}-\dfrac{1}{x^3}+c$

d. $f(x)=\ln x\sqrt{x}=\ln x^{1\frac{1}{2}}=1\tfrac{1}{2}\ln x$

$F(x)=1\tfrac{1}{2}(x\ln x-x)+c=1\tfrac{1}{2}x\ln x-1\tfrac{1}{2}x+c$

e. $f(x)={}^2\log\dfrac{1}{x}={}^2\log x^{-1}=-1\cdot{}^2\log x$

$F(x)=-\dfrac{1}{\ln 2}\cdot(x\ln x-x)+c=\dfrac{-x\ln x+x}{\ln 2}+c$

f. $f(x)=5\cdot\log 2x=5\cdot(\log 2+\log x)=5\log 2+5\log x$

$F(x)=5x\log 2+5\cdot\dfrac{1}{\ln 10}\cdot(x\ln x-x)=5x\log 2+\dfrac{5x\ln x-5x}{\ln 10}$

Opgave 41:

a. $F(x)=x^2-3x+c$

b. $F(1)=2$

$1-3+c=2$

$c=4$

$F(x)=x^2-3x+4$

c. $f(x)=2x-3=0$

$2x=3$

$x=1\tfrac{1}{2}$

$F(1\tfrac{1}{2})=2\tfrac{1}{4}-4\tfrac{1}{2}+c=0$

$c=2\tfrac{1}{4}$

$F(x)=x^2-3x+2\tfrac{1}{4}$

Opgave 42:

$f(x)=(x^2-1)^2=x^4-2x^2+1$

$F(x)=\frac{1}{5}x^5-\frac{2}{3}x^3+x+c$

$F(1)=\frac{1}{5}-\frac{2}{3}+1+c=7$

$c=6\frac{7}{15}$

$F(x)=\frac{1}{5}x^5-\frac{2}{3}x^3+x+6\frac{7}{15}$

Opgave 43:

a. $F(x)=\frac{1}{3}x^3+c$

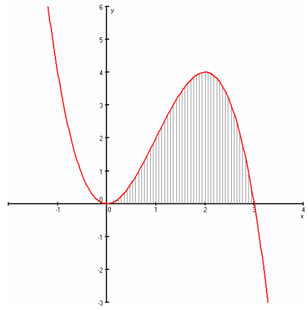
b. $F(0)=c=0$

c. $\frac{1}{3}x^3=10$

$x^3=30$

$x=\sqrt[3]{30}$

Opgave 44:



a. $3x^2-x^3=0$

$x^2(3-x)=0$

$x=0 \quad \vee \quad x=3$

$Opp(V)=\int_0^3(3x^2-x^3)dx=\left[x^3-\frac{1}{4}x^4\right]_0^3=6\frac{3}{4}$

b. $\int_0^p(3x^2-x^3)dx=\left[x^3-\frac{1}{4}x^4\right]_0^p=p^3-\frac{1}{4}p^4=3\frac{3}{8}$

$y_1=x^3-\frac{1}{4}x^4$ en $y_2=3{,}375$

intersect geeft $x=1{,}84$ dus $p=1{,}84$

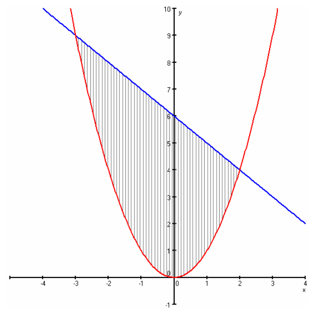
c. $Inh=\int_0^3\pi\cdot(3x^2-x^3)^2dx=\int_0^3\pi\cdot(9x^4-6x^5+x^6)dx=\pi\cdot\left[\frac{9}{5}x^5-x^6+\frac{1}{7}x^7\right]_0^3=20\frac{29}{35}\pi$

d. $Inh=\int_0^q\pi\cdot(3x^2-x^3)^2dx=\pi\cdot\left[\frac{9}{5}x^5-x^6+\frac{1}{7}x^7\right]_0^q=\pi\cdot(\frac{9}{5}q^5-q^6+\frac{1}{7}q^7)=10\frac{29}{70}\pi$

$y_1=\frac{9}{5}x^5-x^6+\frac{1}{7}x^7$ en $y_2=10\frac{29}{70}$

intersect geeft $x=1{,}91$ dus $q=1{,}91$

Opgave 45:



a. $x^2=6-x$

$x^2+x-6=0$

$(x+3)(x-2)=0$

$x=-3 \quad \vee \quad x=2$

$Opp(V)=\int_{-3}^2(6-x-x^2)dx=\left[6x-\frac{1}{2}x^2-\frac{1}{3}x^3\right]_{-3}^2=$

$=7\frac{1}{3}--13\frac{1}{2}=20\frac{5}{6}$

b. $\int_{-3}^{p}(6-x-x^2)\,dx=\left[6x-\frac{1}{2}x^2-\frac{1}{3}x^3\right]_{-3}^{p}=6p-\frac{1}{2}p^2-\frac{1}{3}p^3--13\frac{1}{2}=10\frac{5}{12}$

$-\frac{1}{3}p^3-\frac{1}{2}p^2+6p+3\frac{1}{12}=0$

$y_1=-\frac{1}{3}x^3-\frac{1}{2}x^2+6x+3\frac{1}{12}$

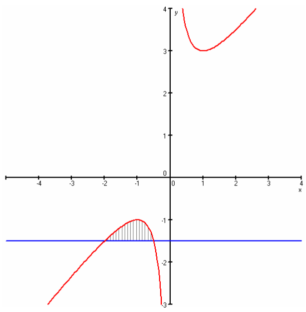
de optie zero geeft $x=-0{,}50$ dus $p=-0{,}50$

c. $Inh=\int_{-3}^{2}\pi\cdot((6-x)^2-(x^2)^2)\,dx=\int_{-3}^{2}\pi\cdot(36-12x+x^2-x^4)\,dx=$

$=\pi\cdot\left[36x-6x^2+\frac{1}{3}x^3-\frac{1}{5}x^5\right]_{-3}^{2}=\pi\cdot(44\frac{4}{15}--122\frac{2}{5})=166\frac{2}{3}\pi$

Opgave 46:

a. $f(x)=\frac{x^2+x+1}{x}=\frac{x^2}{x}+\frac{x}{x}+\frac{1}{x}=x+1+\frac{1}{x}$



$\frac{x^2+x+1}{x}=-1\frac{1}{2}$

$x^2+x+1=-1\frac{1}{2}x$

$x^2+2\frac{1}{2}x+1=0$

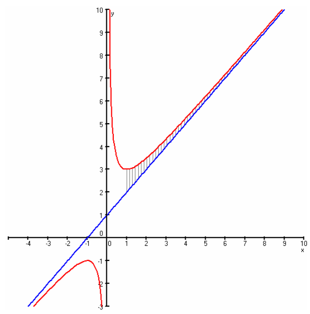
$(x+\frac{1}{2})(x+2)=0$

$x=-\frac{1}{2}\quad\vee\quad x=-2$

$Opp(V)=\int_{-2}^{-\frac{1}{2}}(f(x)--1\frac{1}{2})\,dx=\int_{-2}^{-\frac{1}{2}}(x+1+\frac{1}{x}+1\frac{1}{2})\,dx=\int_{-2}^{-\frac{1}{2}}(x+\frac{1}{x}+2\frac{1}{2})\,dx=$

$=\left[\frac{1}{2}x^2+\ln|x|+2\frac{1}{2}x\right]_{-2}^{-\frac{1}{2}}=\frac{1}{8}+\ln\frac{1}{2}-1\frac{1}{4}-(2+\ln 2-5)=1\frac{7}{8}+\ln\frac{1}{2}-\ln 2$

b. $Opp(W)=\int_{1}^{p}(f(x)-(x+1))\,dx=\int_{1}^{p}(x+1+\frac{1}{x}-x-1)\,dx=$

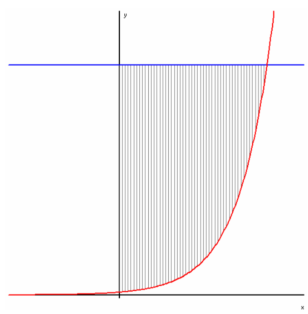


$=\int_{1}^{p}\frac{1}{x}\,dx=\left[\ln x\right]_{1}^{p}=\ln p-\ln 1=\ln p=2$

$p=e^2$

Opgave 47:

a. $3^x=81$



$3^x=3^4$

$x=4$

$Opp(V)=\int_{0}^{4}(81-3^x)\,dx=\left[81x-\frac{3^x}{\ln 3}\right]_{0}^{4}=$

$=324-\frac{81}{\ln 3}-(0-\frac{1}{\ln 3})=324-\frac{80}{\ln 3}$

b. $\int_{0}^{a}(81-3^x)\,dx=\left[81x-\frac{3^x}{\ln 3}\right]_{0}^{a}=81a-\frac{3^a}{\ln 3}-(0-\frac{1}{\ln 3})=81a-\frac{3^a}{\ln 3}+\frac{1}{\ln 3}=162-\frac{40}{\ln 3}$

$$y_1 = 81x - \frac{3^x}{\ln 3} + \frac{1}{\ln 3} \text{ en } y_2 = 162 - \frac{40}{\ln 3}$$

intersect geeft $x = 1{,}60$ dus $a = 1{,}60$

Opgave 48:

$$Opp(totaal) = 8 \cdot 1 + \int_1^8 \frac{8}{x^2}\,dx = 8 + \left[-\frac{8}{x}\right]_1^8 = 8 + -1 - -8 = 15$$

$$Opp(V_2) = \int_a^8 \frac{8}{x^2}\,dx = \left[-\frac{8}{x}\right]_a^8 = -1 - -\frac{8}{a} = -1 + \frac{8}{a} = 5$$

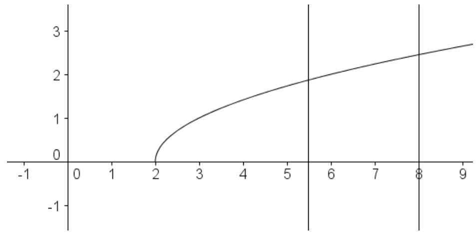
$$\frac{8}{a} = 6$$

$$a = \tfrac{4}{3}$$

Opgave 49:

$$Inh = \int_2^8 \pi \cdot (\sqrt{x-2})^2\,dx = \int_2^8 \pi \cdot (x-2)\,dx = \pi \cdot \left[\tfrac{1}{2}x^2 - 2x\right]_2^8 =$$

$$= \pi \cdot (32 - 16 - (2 - 4)) = 18\pi$$



Opgave 50:

a. $F(x) = a \cdot (3x+1)^6 + c = a \cdot u^6 + c$ met $u = 3x+1$ dus $u' = 3$

$F'(x) = 6a \cdot u^5 \cdot u' = 6a \cdot (3x+1)^5 \cdot 3 = 18a \cdot (3x+1)^5$

b. $18a = 1$

$a = \tfrac{1}{18}$

Opgave 51:

$y = \tfrac{1}{a} \cdot F(ax+b) + c = \tfrac{1}{a} \cdot F(u) + c$ met $u = ax+b$ dus $u' = a$

$y' = \tfrac{1}{a} \cdot F'(u) \cdot u' = \tfrac{1}{a} \cdot F'(ax+b) \cdot a = F'(ax+b) = f(ax+b)$

Opgave 52:

a. $f(x) = (2x-1)^6$

$F(x) = \tfrac{1}{7}(2x-1)^7 \cdot \tfrac{1}{2} + c = \tfrac{1}{14}(2x-1)^7 + c$

b. $g(x) = \dfrac{1}{(3x+4)^3} = (3x+4)^{-3}$

$G(x) = \tfrac{1}{3} \cdot -\tfrac{1}{2}(3x+4)^{-2} + c = -\tfrac{1}{6}(3x+4)^{-2} + c = -\dfrac{1}{6(3x+4)^2} + c$

c. $h(x) = 4\sqrt{3-2x} = 4(3-2x)^{\frac{1}{2}}$

$H(x) = 4 \cdot -\tfrac{1}{2} \cdot \tfrac{1}{1\frac{1}{2}}(3-2x)^{1\frac{1}{2}} + c = -\tfrac{4}{3}(3-2x)^{1\frac{1}{2}} + c = -\tfrac{4}{3}(3-2x)\sqrt{3-2x} + c$

d. $j(x)=\dfrac{2}{\sqrt{1-x}}=2(1-x)^{-\frac{1}{2}}$

$J(x)=2\cdot 2\cdot -1\cdot (1-x)^{\frac{1}{2}}+c=-4(1-x)^{\frac{1}{2}}+c=-4\sqrt{1-x}+c$

Opgave 53:

a. $f(x)=\dfrac{1}{x-1}$

$F(x)=\ln|x-1|+c$

b. $f(x)=\dfrac{3}{2x-5}$

$F(x)=3\cdot\ln|2x-5|\cdot\frac{1}{2}+c=1\frac{1}{2}\ln|2x-5|+c$

c. $f(x)=e^{4x-1}$

$F(x)=\frac{1}{4}e^{4x-1}+c$

d. $f(x)=\ln(4x-1)$

$F(x)=\frac{1}{4}\cdot\big((4x-1)\ln|4x-1|-(4x-1)\big)+c$

e. $f(x)=(2x+1)\sqrt{2x+1}=(2x+1)^{1\frac{1}{2}}$

$F(x)=\frac{1}{2\frac{1}{2}}\cdot\frac{1}{2}(2x+1)^{2\frac{1}{2}}+c=\frac{1}{5}(2x+1)^{2}\cdot\sqrt{2x+1}+c$

f. $f(x)=2^{3x}$

$F(x)=\frac{1}{3}\cdot 2^{3x}\cdot\frac{1}{\ln 2}+c=\dfrac{2^{3x}}{3\ln 2}+c$

g. $f(x)=3^{2-5x}$

$F(x)=-\frac{1}{5}\cdot 3^{2-5x}\cdot\frac{1}{\ln 3}+c=-\dfrac{3^{2-5x}}{5\ln 3}+c$

h. $f(x)={}^{2}\log(5x+3)=\dfrac{\ln(5x+3)}{\ln 2}=\frac{1}{\ln 2}\cdot\ln(5x+3)$

$F(x)=\frac{1}{\ln 2}\cdot\frac{1}{5}\cdot\big((5x+3)\ln(5x+3)-(5x+3)\big)+c=\frac{1}{5\ln 2}\cdot\big((5x+3)\ln(5x+3)-(5x+3)\big)+c$

Opgave 54:

a. $\dfrac{5}{2x+1}=\dfrac{5}{10-4x}$

$2x+1=10-4x$

$6x=9$

$x=1\frac{1}{2}$

$$OppV=\int_0^{1\frac{1}{2}}\frac{5}{10-4x}\,dx+\int_{1\frac{1}{2}}^{2}\frac{5}{2x+1}\,dx=$$

$$=\left[5\cdot-\tfrac{1}{4}\ln|10-4x|\right]_0^{1\frac{1}{2}}+\left[5\cdot\tfrac{1}{2}\ln|2x+1|\right]_{1\frac{1}{2}}^{2}$$

$$=-1\tfrac{1}{4}\ln 4--1\tfrac{1}{4}\ln 10+2\tfrac{1}{2}\ln 5-2\tfrac{1}{2}\ln 4$$

$$=1\tfrac{1}{4}\ln 10+2\tfrac{1}{2}\ln 5-3\tfrac{3}{4}\ln 4$$

b. $Inh(L) = \pi \cdot \int_0^{1\frac{1}{2}} \left(\frac{5}{10-4x} \right)^2 dx + \pi \cdot \int_{1\frac{1}{2}}^{2} \left(\frac{5}{2x+1} \right)^2 dx$

$$= \pi \cdot \int_0^{1\frac{1}{2}} 25(10-4x)^{-2}\, dx + \pi \cdot \int_{1\frac{1}{2}}^{2} 25(2x+1)^{-2}\, dx$$

$$= \pi \cdot \left[\tfrac{25}{4}(10-4x)^{-1} \right]_0^{1\frac{1}{2}} + \pi \cdot \left[-\tfrac{25}{2}(2x+1)^{-1} \right]_{1\frac{1}{2}}^{2}$$

$$= \pi \cdot \left[\frac{25}{4(10-4x)} \right]_0^{1\frac{1}{2}} + \pi \cdot \left[\frac{-25}{2(2x+1)} \right]_{1\frac{1}{2}}^{2}$$

$$= \pi \cdot (\tfrac{25}{16} - \tfrac{25}{40}) - \pi \cdot (-\tfrac{25}{10} + \tfrac{25}{8})$$

$$= \tfrac{25}{16}\pi$$

Opgave 55:

a. de kettingregel is bij deze functie geen getal maar een functie: $u' = 8x$ en je kunt de kettingregel nu niet ‘opheffen’ door te delen door de functie

b. $G'(x) = 1\frac{1}{2}a(4x^2-1)^{\frac{1}{2}} \cdot 8x = 12ax\sqrt{4x^2-1}$

er is geen waarde van a waarvoor geldt: $G'(x) = \sqrt{4x^2-1}$

c. $H(x) = a(4x^2-1)^{1\frac{1}{2}} + c$

$H'(x) = 1\frac{1}{2}a(4x^2-1)^{\frac{1}{2}} \cdot 8x = 12ax\sqrt{4x^2-1}$

$12a = 1$

$a = \frac{1}{12}$

$H(x) = \frac{1}{12}(4x^2-1)\sqrt{4x^2-1} + c$