10.4 Toepassingen van integralen

Opgave 56:

a. $Inh = \int_0^6 \pi \cdot (\frac{1}{2}x)^2 \, dx = \pi \cdot \int_0^6 \frac{1}{4}x^2 \, dx = \pi \cdot \left[\frac{1}{12}x^3\right]_0^6 = 18\pi$

b. kegel , $r = 3 \quad h = 6$

Opgave 57:

$Inh = \frac{1}{3}\pi \cdot 4^2 \cdot 6 - \frac{1}{3}\pi \cdot (\frac{2}{3}p)^2 \cdot p = 32\pi - \frac{4}{27}\pi p^3 = 24\pi$

$-\frac{4}{27}\pi p^3 = -8\pi$

$p^3 = 54$

$p = \sqrt[3]{54} = \sqrt[3]{27} \cdot \sqrt[3]{2} = 3 \cdot \sqrt[3]{2}$

Opgave 58:

$Inh = \frac{1}{3}\pi \cdot 12^2 \cdot 8 - \frac{1}{3}\pi \cdot (1\frac{1}{2}p)^2 \cdot p = 384\pi - \frac{3}{4}\pi p^3 = 378\pi$

$-\frac{3}{4}\pi p^3 = -6\pi$

$p^3 = 8$

$p = \sqrt[3]{8} = 2$

Opgave 59:

$$Inh = \int_{-R}^{R} \pi \cdot y^2 dx = \pi \cdot \int_{-R}^{R} (R^2 - x^2)\, dx = \pi \cdot \left[R^2 x - \frac{1}{3}x^3\right]_{-R}^{R} =$$
$$= \pi \cdot (R^3 - \frac{1}{3}R^3 - (-R^3 + \frac{1}{3}R^3)) = \frac{4}{3}\pi R^3$$

Opgave 60:

$$Inh = \int_{\frac{1}{3}R}^{R} \pi \cdot y^2 dx = \pi \cdot \int_{\frac{1}{3}R}^{R} (R^2 - x^2)\, dx = \pi \cdot \left[R^2 x - \frac{1}{3}x^3\right]_{\frac{1}{3}R}^{R} =$$
$$= \pi \cdot (R^3 - \frac{1}{3}R^3 - (\frac{1}{3}R^3 - \frac{1}{81}R^3)) = \frac{28}{81}\pi R^3$$

Opgave 61:

$Inh_B = \frac{4}{3}\pi \cdot 6^3 = 288\pi$

$$Inh_{V_P} = \int_{-3}^{p} \pi \cdot y^2 dx = \pi \cdot \int_{-3}^{p} (36 - x^2)\, dx = \pi \cdot \left[36x - \frac{1}{3}x^3\right]_{-3}^{p} =$$
$$= \pi \cdot (36p - \frac{1}{3}p^3 - (-108 + 9)) = \pi \cdot (36p - \frac{1}{3}p^3 + 99)$$

neem $y_1 = \pi \cdot (36x - \frac{1}{3}x^3 + 99)$ en $y_2 = 144\pi$

intersect geeft $x = 1{,}27$ dus $p = 1{,}27$

Opgave 62:

$k: \quad y = -\frac{3}{4}x + 6\frac{1}{4} = 0$

$-\frac{3}{4}x = -6\frac{1}{4}$

$x = 8\frac{1}{3}$

$Inh_{kegel} = \frac{1}{3}\pi \cdot 4^2 \cdot 5\frac{1}{3} = 28\frac{4}{9}\pi$

$$Inh_L = 28\tfrac{4}{9}\pi - \int_3^5 \pi \cdot y^2 dx = 28\tfrac{4}{9}\pi - \int_3^5 \pi \cdot (25 - x^2)\, dx = 28\tfrac{4}{9}\pi - \pi \cdot \left[25x - \tfrac{1}{3}x^3\right]_3^5 =$$

$$= 28\tfrac{4}{9}\pi - \pi \cdot (125 - \tfrac{125}{3} - (75 - 9)) = 28\tfrac{4}{9}\pi - 17\tfrac{1}{3}\pi = 11\tfrac{1}{9}\pi$$

Opgave 63:

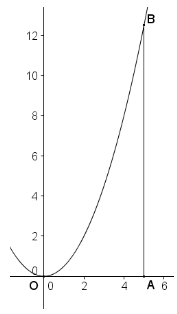
a. $AB = \sqrt{(\Delta y)^2 + (\Delta x)^2} = \sqrt{6^2 + 2^2} = \sqrt{40} = 6{,}32$

b. $C(3{,}4\frac{1}{2})$

$AC + CB = \sqrt{1^2 + (2\frac{1}{2})^2} + \sqrt{1^2 + (3\frac{1}{2})^2} = \sqrt{7\frac{1}{4}} + \sqrt{13\frac{1}{4}} = 6{,}33$

ja

Opgave 64:



$OA = 5 \quad OB = 12\frac{1}{2}$

$f(x) = \frac{1}{2}x^2$

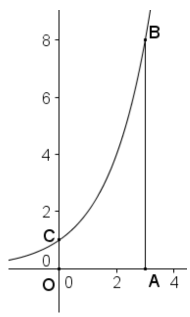
$f'(x) = x$

neem $y_1 = \sqrt{1 + x^2}$

$boog\ OB = \int_0^5 \sqrt{1 + x^2}\, dx = fnInt(Y_1, X, 0{,}5) = 13{,}90$

$omtrek = 5 + 12{,}5 + 13{,}90 = 31{,}40$

Opgave 65:



$OA = 3 \quad OB = 8 \quad OC = 1$

$f(x) = 2^x$

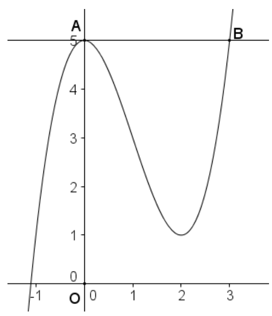
$f'(x) = 2^x \cdot \ln 2$

neem $y_1 = \sqrt{1 + (2^x \cdot \ln 2)^2}$

$boog\ BC = \int_0^3 \sqrt{1 + (2^x \cdot \ln 2)^2}\, dx = fnInt(Y_1, X, 0{,}3) = 7{,}79$

$omtrek = 3 + 8 + 1 + 7{,}79 = 19{,}79$

Opgave 66:



$x^3 - 3x^2 + 5 = 5$

$x^3 - 3x^2 = 0$

$x^2(x - 3) = 0$

$x = 0 \quad \vee \quad x = 3$

$AB = 3$

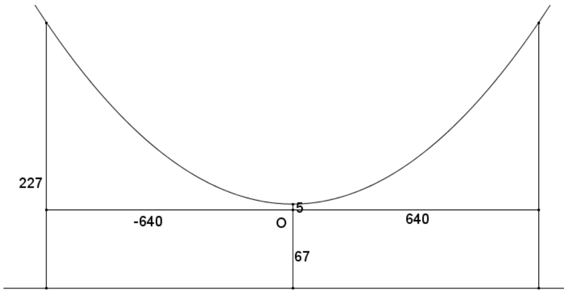
$f(x) = x^3 - 3x^2 + 5$

$f'(x) = 3x^2 - 6x$

neem $y_1 = \sqrt{1 + (3x^2 - 6x)^2}$

$boog\ AB = \int_0^3 \sqrt{1 + (3x^2 - 6x)^2}\, dx = fnInt(Y_1, X, 0{,}3) = 8{,}81$ dus $omtrek = 3 + 8{,}81 = 11{,}81$

Opgave 67:



parabool: $y = ax^2 + b$ door (0,5) en (640,160)

$b = 5$

$160 = a \cdot 640^2 + 5$

$155 = 409600a$

$a = \frac{155}{409600}$

$f(x) = \frac{155}{409600}x^2 + 5$

$f'(x) = \frac{310}{409600}x$

neem $y_1 = \sqrt{1 + (\frac{310}{409600}x)^2}$

$$lengte\ kabel = \int_{-640}^{640} \sqrt{1 + (\frac{310}{409600}x)^2}\,dx = fnInt(Y_1, X, -640, 640) = 1328\text{ m}$$

Opgave 68:

a. $x = 20$ dus $y = 15{,}0$ m

b. $f(x) = 4e^{0{,}062x} + 4e^{-0{,}062x}$

$f'(x) = 4e^{0{,}062x} \cdot 0{,}062 + 4e^{-0{,}062x} \cdot 0{,}062 = 0{,}248e^{0{,}062x} - 0{,}248e^{-0{,}062x}$

neem $y_1 = 0{,}248e^{0{,}062x} - 0{,}248e^{-0{,}062x}$ en $y_2 = \sqrt{1 + y_1^{\ 2}}$

$$lengte\ kabel = \int_{-20}^{20} \sqrt{1 + y_1^{\ 2}}\,dx = fnInt(Y_2, X, -20, 20) = 43{,}2\text{ m}$$

c. $$Opp = \int_{-20}^{20} 4(e^{0{,}062x} + e^{-0{,}062x})\,dx = fnInt(Y_1, X, -20, 20) = 409\text{ m}^2$$

Opgave 69:

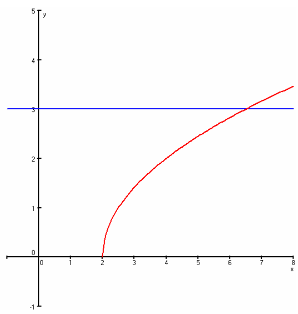
$y = \frac{1}{2}$ dus $\sqrt{x} = \frac{1}{2}$

$x = \frac{1}{4}$ dus $r_1 = \frac{1}{4}$

$y = 1\frac{1}{2}$ dus $\sqrt{x} = 1\frac{1}{2}$

$x = 2\frac{1}{4}$ dus $r_2 = 2\frac{1}{4}$

Opgave 70:



a. $y = \sqrt{2x - 4}$

$y^2 = 2x - 4$

$2x = y^2 + 4$

$x = \frac{1}{2}y^2 + 2$

$$Inh = \int_0^3 \pi \cdot x^2 dy = \int_0^3 \pi \cdot (\tfrac{1}{2}y^2 + 2)^2 dy = \int_0^3 \pi \cdot (\tfrac{1}{4}y^4 + 2y^2 + 4)dy =$$

$$= \pi \cdot \left[\tfrac{1}{20}y^5 + \tfrac{2}{3}y^3 + 4y\right]_0^3 = \pi \cdot (\tfrac{243}{20} + 18 + 12) = 42\tfrac{3}{20}\pi$$

b. $\sqrt{2x - 4} = 3$

$2x - 4 = 9$

$2x = 13$

$x = 6\frac{1}{2}$

$$Inh = \pi \cdot 3^2 \cdot 6\frac{1}{2} - \int_2^{6\frac{1}{2}} \pi \cdot (\sqrt{2x-4})^2\,dx = 58\frac{1}{2}\pi - \int_2^{6\frac{1}{2}} \pi \cdot (2x-4)\,dx =$$

$$= 58\frac{1}{2}\pi - \pi \cdot \left[x^2 - 4x\right]_2^{6\frac{1}{2}} = 58\frac{1}{2}\pi - \pi \cdot (16\frac{1}{4} - -4) = 58\frac{1}{2}\pi - 20\frac{1}{4}\pi = 38\frac{1}{4}\pi$$

Opgave 71:

a. $Inh = \int_0^4 \pi \cdot x^2 dy = \int_0^4 \pi \cdot y\,dy = \pi \cdot [\frac{1}{2}y^2]_0^4 = 8\pi$

b. $q = p^2$

$$Inh(V_P) = \int_0^p \pi \cdot y^2 dx = \int_0^p \pi \cdot x^4 dx = \pi \cdot \left[\frac{1}{5}x^5\right]_0^p = \frac{1}{5}\pi\, p^5$$

$$Inh(W_P) = \int_0^{p^2} \pi \cdot x^2 dy = \int_0^{p^2} \pi \cdot y dy = \pi \cdot \left[\frac{1}{2}y^2\right]_0^{p^2} = \frac{1}{2}\pi\, p^4$$

$\frac{1}{5}\pi\, p^5 = \frac{1}{2}\pi\, p^4$

$2p^5 = 5p^4$

$2p^5 - 5p^4 = 0$

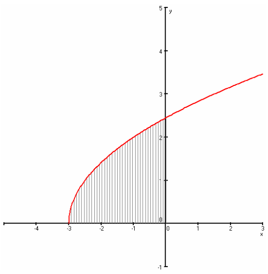
$p^4(2p-5) = 0$

$p = 0 \quad \vee \quad 2p = 5$

$p = 0 \quad \vee \quad p = 2\frac{1}{2}$

$q = 6\frac{1}{4}$

Opgave 72:



a. $Inh(L) = \int_{-3}^0 \pi \cdot (\sqrt{2x+6})^2 dx = \int_{-3}^0 \pi \cdot (2x+6)dx =$

$$= \pi \cdot \left[x^2 + 6x\right]_{-3}^0 = \pi(0 - (9-18)) = 9\pi$$

b. $y = \sqrt{2x+6}$

$y^2 = 2x+6$

$2x = y^2 - 6$

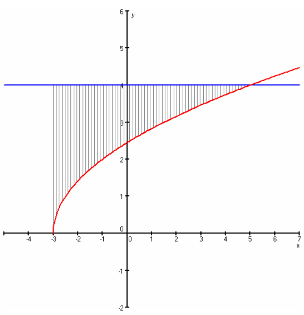
$x = \frac{1}{2}y^2 - 3$

$x^2 = (\frac{1}{2}y^2 - 3)^2 = \frac{1}{4}y^4 - 3y^2 + 9$

$$Inh(M) = \int_0^{\sqrt{6}} \pi \cdot (\frac{1}{4}y^4 - 3y^2 + 9)dy = \pi \cdot \left[\frac{1}{20}y^5 - y^3 + 9y\right]_0^{\sqrt{6}} = \pi \cdot (1\frac{4}{5}\sqrt{6} - 6\sqrt{6} + 9\sqrt{6}) =$$

$$= 4\frac{4}{5}\pi\sqrt{6}$$

c. transleer de grafiek van *f* 3 naar rechts



$g(x) = \sqrt{2(x-3)+6} = \sqrt{2x}$

$y = \sqrt{2x}$

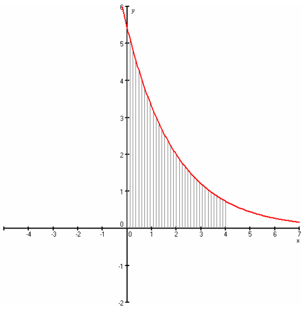
$y^2 = 2x$

$x = \frac{1}{2}y^2$

$x^2 = \frac{1}{4}y^4$

$Inh(N) = \int_0^4 \pi \cdot \frac{1}{4}y^4 dy = \pi \cdot \left[\frac{1}{20}y^5\right]_0^4 = 51\frac{1}{5}\pi$

Opgave 73:



a. $Opp(V) = \int_0^4 2e^{-\frac{1}{2}x+1}dx = \left[-4e^{-\frac{1}{2}x+1}\right]_0^4 = -4e^{-1} - -4e = 4e - \frac{4}{e}$

b. $OA = 4 \quad AB = 2e^{-1} = \frac{2}{e} \quad OC = 2e$

$f(x) = 2e^{-\frac{1}{2}x+1}$

$f'(x) = -e^{-\frac{1}{2}x+1}$

neem $y_1 = -e^{-\frac{1}{2}x+1}$ en $y_2 = \sqrt{1+y_1^{\,2}}$

$boog\ BC = \int_0^4 \sqrt{1+y_1^{\,2}}dx = fnInt(Y_2, X, 0,4) = 6{,}39$

$omtrek = 4 + \frac{2}{e} + 2e + 6{,}39 = 16{,}56$

c. $y = 2e^{-\frac{1}{2}x+1}$

$\frac{1}{2}y = e^{-\frac{1}{2}x+1}$

$-\frac{1}{2}x + 1 = \ln\frac{1}{2}y$

$-\frac{1}{2}x = -1 + \ln\frac{1}{2}y$

$x = 2 - 2\ln\frac{1}{2}y$

$Inh(L) = \pi \cdot 4^2 \cdot \frac{2}{e} + \int_{\frac{2}{e}}^{2e} \pi \cdot (2 - 2\ln\frac{1}{2}y)^2 dy = \frac{32\pi}{e} + fnInt(Y_1, X, \frac{2}{e}, 2e) = \frac{32\pi}{e} + 44{,}18 = 81{,}16$