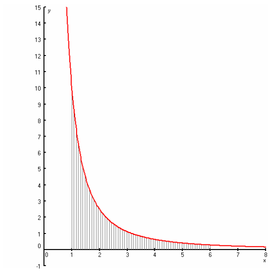
10.5 Diagnostische toets

Opgave 1:

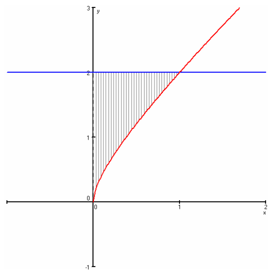


a. $Opp(V) = f(1{,}5) + f(2{,}5) + f(3{,}5) + f(4{,}5) + f(5{,}5) = 7{,}69$

b. $y_1 = \dfrac{10}{x^2}$

$$Opp(V) = \int_1^6 \frac{10}{x^2}\,dx = fnInt(Y_1, X, 1, 6) = 8{,}3333$$

Opgave 2:



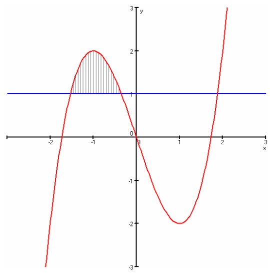
$y_1 = x + \sqrt{x}$ en $y_2 = 2$

intersect geeft $x = 1$

neem $y_3 = y_2 - y_1$

$$Opp(V) = \int_0^1 2 - (x + \sqrt{x})dx = fnInt(Y_3, X, 0, 1) = 0{,}8333$$

Opgave 3:



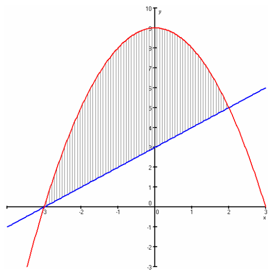
$y_1 = x^3 - 3x$ en $y_2 = 1$

intersect geeft $x = -1{,}53 \quad \vee \quad x = -0{,}35$

neem $y_3 = y_1 - y_2$

$$Opp(V) = \int_{-1{,}53}^{-0{,}35} (x^3 - 3x - 1)dx = fnInt(Y_3, X, -1.53, -0.35) = 0{,}78$$

Opgave 4:



$y_1 = 9 - x^2$ en $y_2 = x + 3$

intersect geeft $x = -3 \quad \vee \quad x = 2$

neem $y_3 = y_1 - y_2$

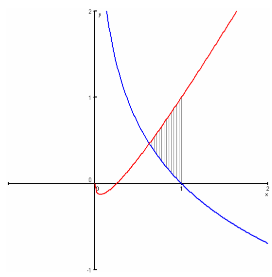
$$Opp(V) = \int_{-3}^{2} (9 - x^2 - (x + 3))dx = fnInt(Y_3, X, -3, 2) = 20{,}83$$

Opgave 5:

$y_1 = \frac{1}{3}x^3 - 2x$ en $y_2 = abs(y_1)$

$$Opp(V) = \int_0^3 \left|\tfrac{1}{3}x^3 - 2x\right|dx = fnInt(Y_2, X, 0, 3) = 3{,}75$$

Opgave 6:



$y_1 = 2x - \sqrt{x}$ en $y_2 = -\ln x$

intersect geeft $x = 0{,}629$

neem $y_3 = \pi \cdot ({y_1}^2 - {y_2}^2)$

a. $$Inh(V) = \int_{0{,}629}^{1} \pi \cdot ({y_1}^2 - {y_2}^2)dx = fnInt(Y_3, X, 0.629, 1) = 0{,}752$$

b. de optie zero geeft bij y_1 dat $x = 0{,}25$

neem $y_4 = \pi \cdot {y_1}^2$ en $y_5 = \pi \cdot {y_2}^2$

$$Inh(W) = \int_{0,25}^{0,629} \pi \cdot {y_1}^2 dx + \int_{0,629}^{1} \pi \cdot {y_2}^2 dx = fnInt(Y_4, X, 0.25, 0.629) + fnInt(Y_5, X, 0.629, 1) =$$

$$= 0{,}0801 + 0{,}0743 = 0{,}154$$

Opgave 7:

a. $F(x) = (x^3 + 5)^4 + 2$

$f(x) = F'(x) = 4(x^3+5)^3 \cdot 3x^2 = 12x^2 \cdot (x^3+5)^3$

b. $G(x) = e^{x} - 3$

$g(x) = G'(x) = e^{x^3} \cdot 3x^2 = 3x^2 \cdot e^{x^3}$

Opgave 8:

a. $f(x) = \frac{6}{x^4} = 6x^{-4}$

$F(x) = -2x^{-3} + c = -\frac{2}{x^3} + c$

b. $f(x) = \frac{2x+6}{x^2} = \frac{2x}{x^2} + \frac{6}{x^2} = \frac{2}{x} + 6x^{-2}$

$F(x) = 2\ln|x| - 6x^{-1} + c = 2\ln|x| - \frac{6}{x} + c$

c. $f(x) = 3 \cdot 2^x$

$F(x) = 3 \cdot \frac{2^x}{\ln 2} + c$

d. $f(x) = 6e^x + x^2$

$F(x) = 6e^x + \frac{1}{3}x^3 + c$

e. $f(x) = \ln x^2 = 2\ln x$

$F(x) = 2 \cdot (x\ln x - x) + c = 2x\ln x - 2x + c$

f. $f(x) = {}^2\log 4x = \frac{\ln 4x}{\ln 2} = \frac{1}{\ln 2} \cdot \ln 4x$

$F(x) = \frac{1}{\ln 2} \cdot \frac{1}{4} \cdot (4x\ln 4x - 4x) + c = \frac{x\ln 4x - x}{\ln 2} + c$

Opgave 9:

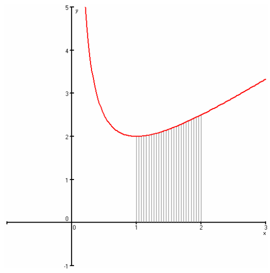
a. $f(x) = \frac{x^2+1}{x} = x + \frac{1}{x}$

$F(x) = \frac{1}{2}x^2 + \ln|x| + c$

$F(1) = \frac{1}{2} + c = e$

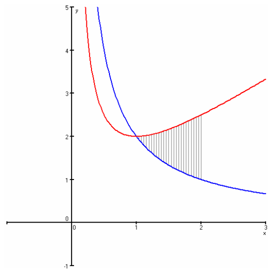
$c = e - \frac{1}{2}$

$F(x) = \frac{1}{2}x^2 + \ln|x| + e - \frac{1}{2}$



b. $Opp(V)=\int_1^2 (x+\frac{1}{x})dx=\left[\frac{1}{2}x^2+\ln x\right]_1^2=$

$=2+\ln 2-(\frac{1}{2}+\ln 1)=1\frac{1}{2}+\ln 2$



c. $x+\frac{1}{x}=\frac{2}{x}$

$x=\frac{1}{x}$

$x^2=1$

$x=1 \quad \vee \quad x=-1$

$Inh=\int_1^2 \pi\cdot(f^2(x)-g^2(x))dx=\int_1^2 \pi\cdot((x+\frac{1}{x})^2-(\frac{2}{x})^2)dx=$

$=\int_1^2 \pi\cdot(x^2+2+\frac{1}{x^2}-\frac{4}{x^2})dx=\int_1^2 \pi\cdot(x^2+2-\frac{3}{x^2})dx=$

$=\pi\cdot\left[\frac{1}{3}x^3+2x+\frac{3}{x}\right]_1^2=\pi\cdot(\frac{8}{3}+4+1\frac{1}{2}-(\frac{1}{3}+2+3))=2\frac{5}{6}\pi$

Opgave 10:

a. $f(x)=(2x+6)^5$

$F(x)=\frac{1}{6}(2x+6)^6\cdot\frac{1}{2}+c=\frac{1}{12}(2x+6)^6+c$

b. $f(x)=\frac{10}{(3x-1)^2}=10(3x-1)^{-2}$

$F(x)=-10(3x-1)^{-1}\cdot\frac{1}{3}+c=-\frac{10}{3(3x-1)}+c$

c. $f(x)=(5x+2)^2\cdot\sqrt{5x+2}=(5x+2)^{2\frac{1}{2}}$

$F(x)=\frac{2}{7}(5x+2)^{3\frac{1}{2}}\cdot\frac{1}{5}+c=\frac{2}{35}(5x+2)^3\cdot\sqrt{5x+2}+c$

d. $f(x)=4e^{2x+3}$

$F(x)=4\cdot e^{2x+3}\cdot\frac{1}{2}+c=2e^{2x+3}+c$

e. $f(x)=8\cdot 2^{2x-1}$

$F(x)=8\cdot 2^{2x-1}\cdot\frac{1}{\ln 2}\cdot\frac{1}{2}=\frac{4\cdot 2^{2x-1}}{\ln 2}+c$

f. $f(x)=\ln(2x+3)$

$F(x)=\frac{1}{2}\cdot((2x+3)\ln(2x+3)-(2x+3))+c=\frac{1}{2}(2x+3)\ln(2x+3)-x-1\frac{1}{2}+c$

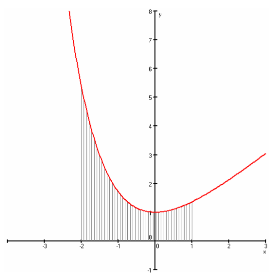
Opgave 11:

neem $y_1=x+e^{-x}$

$Opp(V)=\int_{-2}^{1}(x+e^{-x})dx=fnInt(Y_1,X,-2,1)=5{,}52$

$Opp=\int_{-2}^{p}(x+e^{-x})dx=\left[\frac{1}{2}x^2-e^{-x}\right]_{-2}^{p}=\frac{1}{2}p^2-e^{-p}-(2-e^2)=$

$=\frac{1}{2}p^2-e^{-p}-2+e^2=2{,}76$



$y_2 = \frac{1}{2}x^2 - e^{-x} - 2 + e^2$ en $y_3 = 2{,}76$

intersect geeft $x = -1{,}213$ dus $p = -1{,}213$

Opgave 12:

a. $Inh = \frac{4}{3}\pi \cdot 4^3 = \frac{256}{3}\pi$

b. $Inh = \int_{-4}^{-2} \pi \cdot y^2 dx = \int_{-4}^{-2} \pi \cdot (16 - x^2)dx = \pi \cdot \left[16x - \frac{1}{3}x^3\right]_{-4}^{-2} = \pi \cdot (-32 + \frac{8}{3} - (-64 + \frac{64}{3})) =$

$= 13\frac{1}{3}\pi$

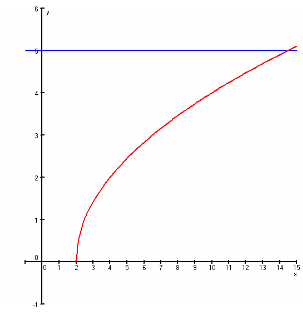
c. $Inh(helft\ bol) = \frac{128}{3}\pi$

$Inh = \int_{-2}^{p} \pi \cdot (16 - x^2)dx = \pi \cdot \left[16x - \frac{1}{3}x^3\right]_{-2}^{p} = \pi \cdot (16p - \frac{1}{3}p^3 - (-32 + \frac{8}{3})) = \frac{128}{3}\pi$

neem $y_1 = 16x - \frac{1}{3}x^3 + 29\frac{1}{3}$ en $y_2 = \frac{128}{3}$

intersect geeft $x = 0{,}85$ dus $p = 0{,}85$

Opgave 13:



a. $\sqrt{2x - 4} = 5$

$2x - 4 = 25$

$2x = 29$

$x = 14\frac{1}{2}$

$f(x) = \sqrt{2x - 4}$

$f'(x) = \dfrac{1}{2\sqrt{2x-4}} \cdot 2 = \dfrac{1}{\sqrt{2x-4}}$

neem $y_1 = \sqrt{2x-4}$ en $y_2 = \sqrt{1 + {y_1}^2}$

$OA = 2 \quad OC = 5 \quad BC = 14\frac{1}{2}$

$boog(AB) = \int_{2}^{14\frac{1}{2}} y_2 dx = fnInt(y_2, x, 2, 14.5) = 13{,}90$

$omtrek = 2 + 5 + 14{,}5 + 13{,}90 = 35{,}40$

b. $Inh = \pi \cdot 5^2 \cdot 14\frac{1}{2} - \int_{2}^{14{,}5} \pi \cdot (\sqrt{2x-4})^2 dx = 362{,}5\pi - \int_{2}^{14{,}5} \pi(2x-4)dx =$

$= 362{,}5\pi - \pi \cdot \left[x^2 - 4x\right]_{2}^{14{,}5} = 362{,}5\pi - \pi \cdot (210\frac{1}{4} - 58 - (4 - 8)) =$

$= 362{,}5\pi - 156{,}25\pi = 206\frac{1}{4}\pi$

c. $y = \sqrt{2x-4}$

$y^2 = 2x - 4$

$2x = y^2 + 4$

$x = \frac{1}{2}y^2 + 2$

$x^2 = (\frac{1}{2}y^2 + 2)^2 = \frac{1}{4}y^4 + 2y^2 + 4$

$Inh = \int_{0}^{5} \pi \cdot x^2 dx = \int_{0}^{5} \pi \cdot (\frac{1}{4}y^4 + 2y^2 + 4)dy = \pi \cdot \left[\frac{1}{20}y^5 + \frac{2}{3}y^3 + 4y\right]_{0}^{5} = 259\frac{7}{12}\pi$