

11.5 Diagnostische toets

Opgave 1:

- a. $-\cos(3x - \frac{1}{4}\pi) = \cos(3x - \frac{1}{4}\pi + \pi)$
 $= \cos(3x + \frac{3}{4}\pi)$
 $= \sin(3x + \frac{3}{4}\pi + \frac{1}{2}\pi)$
 $= \sin(3x + 1\frac{1}{4}\pi)$
- b. $(\sin x + \cos x)^2 = \sin^2 x + 2 \sin x \cos x + \cos^2 x$
 $= 1 + \sin 2x$
- c. $2 + \cos x - 2 \sin^2 x = 2 + \cos x - 2(1 - \cos^2 x)$
 $= 2 + \cos x - 2 + 2 \cos^2 x$
 $= 2 \cos^2 x + \cos x$

Opgave 2:

- a. $\sin(3x - \frac{1}{4}\pi) = \cos 2x$
 $\sin(3x - \frac{1}{4}\pi) = \sin(2x + \frac{1}{2}\pi)$
 $3x - \frac{1}{4}\pi = 2x + \frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad 3x - \frac{1}{4}\pi = \pi - (2x + \frac{1}{2}\pi) + k \cdot 2\pi$
 $x = \frac{3}{4}\pi + k \cdot 2\pi \quad \vee \quad 3x - \frac{1}{4}\pi = \pi - 2x - \frac{1}{2}\pi + k \cdot 2\pi$
 $5x = \frac{3}{4}\pi + k \cdot 2\pi$
 $x = \frac{3}{20}\pi + k \cdot \frac{2}{5}\pi$
 $x = \frac{3}{20}\pi \quad \vee \quad x = \frac{11}{20}\pi \quad \vee \quad x = \frac{3}{4}\pi \quad \vee \quad x = \frac{19}{20}\pi$
- b. $2 \sin^2 2x = \sin 2x + 1$
 $2 \sin^2 2x - \sin 2x - 1 = 0$
 $\sin^2 2x - \frac{1}{2} \sin 2x - \frac{1}{2} = 0$
 $(\sin 2x - 1)(\sin 2x + \frac{1}{2}) = 0$
 $\sin 2x = 1 \quad \vee \quad \sin 2x = -\frac{1}{2}$
 $2x = \frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad 2x = \frac{7}{6}\pi + k \cdot 2\pi \quad \vee \quad 2x = \frac{11}{6}\pi + k \cdot 2\pi$
 $x = \frac{1}{4}\pi + k \cdot \pi \quad \vee \quad x = \frac{7}{12}\pi + k \cdot \pi \quad \vee \quad x = \frac{11}{12}\pi + k \cdot \pi$
 $x = \frac{1}{4}\pi \quad \vee \quad x = \frac{7}{12}\pi \quad \vee \quad x = \frac{11}{12}\pi \quad \vee \quad x = 1\frac{1}{4}\pi \quad \vee \quad x = 1\frac{7}{12}\pi \quad \vee \quad x = 1\frac{11}{12}\pi$
- c. $\cos(\frac{2}{5}\pi t) = -\sin(\frac{1}{6}\pi t)$
 $\cos(\frac{2}{5}\pi t) = \sin(\frac{1}{6}\pi t + \pi)$
 $\cos(\frac{2}{5}\pi t) = \cos(\frac{1}{6}\pi t + \pi - \frac{1}{2}\pi)$
 $\cos(\frac{2}{5}\pi t) = \cos(\frac{1}{6}\pi t + \frac{1}{2}\pi)$
 $\frac{2}{5}\pi t = \frac{1}{6}\pi t + \frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad \frac{2}{5}\pi t = -\frac{1}{6}\pi t - \frac{1}{2}\pi + k \cdot 2\pi$
 $\frac{7}{30}\pi t = \frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad \frac{17}{30}\pi t = -\frac{1}{2}\pi + k \cdot 2\pi$
 $t = \frac{15}{7} + k \cdot \frac{60}{7} \quad \vee \quad t = -\frac{15}{17} + k \cdot \frac{60}{17}$
 $t = \frac{15}{7} \quad \vee \quad t = \frac{45}{7} \quad \vee \quad t = \frac{105}{17} \quad \vee \quad t = \frac{165}{17}$

Opgave 3:

- a. $\sin(x + \frac{1}{3}\pi) = 2 \sin(2x) \cos(2x)$
 $\sin(x + \frac{1}{3}\pi) = \sin(4x)$

$$x + \frac{1}{3}\pi = 4x + k \cdot 2\pi \quad \vee \quad x + \frac{1}{3}\pi = \pi - 4x + k \cdot 2\pi$$

$$-3x = -\frac{1}{3}\pi + k \cdot 2\pi \quad \vee \quad 5x = \frac{2}{3}\pi + k \cdot 2\pi$$

$$x = \frac{1}{9}\pi + k \cdot \frac{2}{3}\pi \quad \vee \quad x = \frac{2}{15}\pi + k \cdot \frac{2}{5}\pi$$

b. $\sin^2 2x + \frac{1}{4} = \cos 4x$

$$\sin^2 2x + \frac{1}{4} = 1 - 2\sin^2 2x$$

$$3\sin^2 2x = \frac{3}{4}$$

$$\sin^2 2x = \frac{1}{4}$$

$$\sin 2x = \frac{1}{2} \quad \vee \quad \sin 2x = -\frac{1}{2}$$

$$2x = \frac{1}{6}\pi + k \cdot 2\pi \quad \vee \quad 2x = \frac{5}{6}\pi + k \cdot 2\pi \quad \vee \quad 2x = \frac{7}{6}\pi + k \cdot 2\pi \quad \vee \quad 2x = \frac{11}{6}\pi + k \cdot 2\pi$$

$$x = \frac{1}{12}\pi + k \cdot \pi \quad \vee \quad x = \frac{5}{12}\pi + k \cdot \pi \quad \vee \quad x = \frac{7}{12}\pi + k \cdot \pi \quad \vee \quad x = \frac{11}{12}\pi + k \cdot \pi$$

Opgave 4:

$$\begin{aligned} \frac{2 \tan x}{1 - \tan^2 x} &= \frac{\frac{2 \sin x}{\cos x}}{1 - \frac{\sin^2 x}{\cos^2 x}} = \frac{\frac{2 \sin x}{\cos x}}{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}} = \frac{\frac{2 \sin x}{\cos x}}{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}} = \frac{\frac{2 \sin x}{\cos x}}{\frac{\cos 2x}{\cos^2 x}} = \frac{2 \sin x}{\cos x} \cdot \frac{\cos^2 x}{\cos 2x} = \\ &= \frac{2 \sin x \cos x}{\cos 2x} = \frac{\sin 2x}{\cos 2x} = \tan 2x \end{aligned}$$

Opgave 5:

$$f\left(1\frac{1}{2}\pi + p\right) = \sin(3\pi + 2p) + \cos\left(1\frac{1}{2}\pi + p\right)$$

$$= \sin(\pi + 2p) + \cos\left(p - \frac{1}{2}\pi\right)$$

$$= -\sin 2p + \sin p$$

$$f\left(1\frac{1}{2}\pi - p\right) = \sin(3\pi - 2p) + \cos\left(1\frac{1}{2}\pi - p\right)$$

$$= \sin(\pi - 2p) + \cos\left(-p - \frac{1}{2}\pi\right)$$

$$= -\sin(-2p) + \sin(-p)$$

$$= \sin 2p - \sin p$$

$$f\left(1\frac{1}{2}\pi + p\right) + f\left(1\frac{1}{2}\pi - p\right) = -\sin 2p + \sin p + \sin 2p - \sin p = 0$$

Opgave 6:

a. $f(x) = \cos 2x + \sin 2x$

$$f'(x) = -2\sin 2x + 2\cos 2x$$

b. $f(x) = 2\cos^3 x$

$$f'(x) = 6\cos^2 x \cdot -\sin x = -6\cos^2 x \cdot \sin x$$

c. $f(x) = \frac{\cos x}{\sin x}$

$$f'(x) = \frac{\sin x \cdot -\sin x - \cos x \cdot \cos x}{\sin^2 x} = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-1}{\sin^2 x}$$

d. $f(x) = x^2 \cdot \sin\left(2x - \frac{1}{2}\pi\right)$

$$f'(x) = 2x \cdot \sin\left(2x - \frac{1}{2}\pi\right) + x^2 \cdot 2\cos\left(2x - \frac{1}{2}\pi\right) = 2x \sin\left(2x - \frac{1}{2}\pi\right) + 2x^2 \cos\left(2x - \frac{1}{2}\pi\right)$$

e. $f(x) = \sin x \cdot \tan 2x$

$$f'(x) = \cos x \cdot \tan 2x + \sin x \cdot \frac{1}{\cos^2 2x} \cdot 2 = \cos x \cdot \tan 2x + \frac{2 \sin x}{\cos^2 2x}$$

f. $f(x) = \frac{\tan 2x}{\sin x}$

$$f'(x) = \frac{\sin x \cdot \frac{1}{\cos^2 2x} \cdot 2 - \tan 2x \cdot \cos x}{\sin^2 x} = \frac{\frac{2 \sin x}{\cos^2 2x} - \tan 2x \cdot \cos x}{\sin^2 x}$$

Opgave 7:

a. $f(x) = 3 - 2 \sin(x - \frac{1}{6} \pi)$

ev.as: $y = 3$

amp: 2

per = 2π

beginpunt: $(\frac{1}{6} \pi, 3)$ dalend door de ev. as

$$x_{top} = \frac{1}{6} \pi + \frac{1}{4} \cdot 2\pi = \frac{2}{3} \pi \text{ dus } (\frac{2}{3} \pi, 1)$$

$$x_{top} = \frac{1}{6} \pi + \frac{3}{4} \cdot 2\pi = 1\frac{2}{3} \pi \text{ dus } (1\frac{2}{3} \pi, 5)$$

b. $g(x) = -4 + 3 \cos(2x - \frac{1}{4} \pi)$
 $= -4 + 3 \cos 2(x - \frac{1}{8} \pi)$

ev.as: $y = -4$

amp: 3

per = π

beginpunt: $(\frac{1}{8} \pi, -1)$

$$x_{top} = \frac{1}{8} \pi \quad \vee \quad x_{top} = \frac{1}{8} \pi + \frac{1}{2} \cdot \pi = \frac{5}{8} \pi$$

$(\frac{1}{8} \pi, -1)$ en $(\frac{5}{8} \pi, -7)$

Opgave 8:

a. $f(x) = \sin 2x - 2 \sin x$

$$y_A = f(\pi) = 0$$

$$f'(x) = 2 \cos 2x - 2 \cos x$$

$$f'(\pi) = 2 - -2 = 4$$

$$y = 4x + b \text{ door } (\pi, 0)$$

$$0 = 4\pi + b$$

$$b = -4\pi$$

$$y = 4x - 4\pi$$

b. $f'(x) = 2 \cos 2x - 2 \cos x = 0$

$$2 \cos 2x = 2 \cos x$$

$$\cos 2x = \cos x$$

$$2x = x + k \cdot 2\pi \quad \vee \quad 2x = -x + k \cdot 2\pi$$

$$x = 0 + k \cdot 2\pi \quad \vee \quad 3x = 0 + k \cdot 2\pi$$

$$x = 0 + k \cdot \frac{2}{3} \pi$$

$$B(\frac{2}{3} \pi, -1\frac{1}{2} \sqrt{3}) \quad C(1\frac{1}{3} \pi, 1\frac{1}{2} \sqrt{3})$$

c. $f'(x) = 2 \cos 2x - 2 \cos x = -2$

$$\begin{aligned}
2 \cos 2x - 2 \cos x + 2 &= 0 \\
\cos 2x - \cos x + 1 &= 0 \\
2 \cos^2 x - 1 - \cos x + 1 &= 0 \\
2 \cos^2 x - \cos x &= 0 \\
\cos x(2 \cos x - 1) &= 0 \\
\cos x = 0 \quad \vee \quad 2 \cos x &= 1 \\
\cos x = 0 \quad \vee \quad \cos x = \frac{1}{2} \\
x = \frac{1}{2}\pi \quad \vee \quad x = 1\frac{1}{2}\pi \quad \vee \quad x = \frac{1}{3}\pi \quad \vee \quad x = 1\frac{2}{3}\pi \\
\left(\frac{1}{3}\pi, -\frac{1}{2}\sqrt{3}\right) \quad \left(\frac{1}{2}\pi, -2\right) \quad \left(1\frac{1}{2}\pi, 2\right) \quad \left(1\frac{2}{3}\pi, \frac{1}{2}\sqrt{3}\right)
\end{aligned}$$

Opgave 9:

- a. $f(x) = -\frac{1}{2}\sin(2x + \frac{1}{2}\pi)$
 $F(x) = -\frac{1}{4}\cos(2x + \frac{1}{2}\pi) + c$
- b. $g(x) = 3x^2 + \cos\frac{1}{3}x$
 $G(x) = x^3 + 3\sin\frac{1}{3}x + c$
- c. $h(x) = x - 2\sin^2 x = x + \cos 2x - 1$
 $H(x) = \frac{1}{2}x^2 - x + \frac{1}{2}\sin 2x + c$
- d. $k(x) = 2 + \tan^2 x = 1 + 1 + \tan^2 x$
 $K(x) = x + \tan x + c$

Opgave 10:

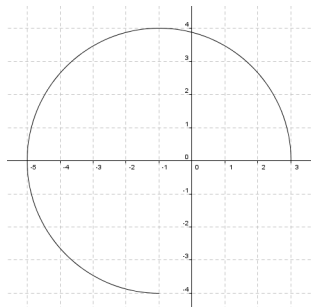
- a. $\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} (\sin 2x + \cos x) dx = \left[-\frac{1}{2}\cos 2x + \sin x\right]_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} = \frac{1}{4} + \frac{1}{2}\sqrt{3} - \left(-\frac{1}{4} + \frac{1}{2}\right) = \frac{1}{2}\sqrt{3}$
- b. $\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \sin^2 x dx = \int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) dx = \left[\frac{1}{2}x - \frac{1}{4}\sin 2x\right]_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} = \frac{1}{6}\pi - \frac{1}{8}\sqrt{3} - \left(\frac{1}{12}\pi - \frac{1}{8}\sqrt{3}\right) = \frac{1}{12}\pi$

Opgave 11:

$$\text{Inh} = \pi \int_{\frac{1}{2}\pi}^{1\frac{1}{2}\pi} \cos^2 x dx = \pi \int_{\frac{1}{2}\pi}^{1\frac{1}{2}\pi} \left(\frac{1}{2}\cos 2x + \frac{1}{2}\right) dx = \pi \cdot \left[\frac{1}{4}\sin 2x + \frac{1}{2}x\right]_{\frac{1}{2}\pi}^{1\frac{1}{2}\pi} = \pi \cdot \left(\frac{3}{4}\pi - \frac{1}{4}\pi\right) = \frac{1}{2}\pi^2$$

Opgave 12:

a.



b. $-1 + 4 \cos 2t = 1$

$$4 \cos 2t = 2$$

$$\cos 2t = \frac{1}{2}$$

$$2t = \frac{1}{3} \pi$$

$$t = \frac{1}{6} \pi$$

$$A(1, 2\sqrt{3})$$

c. $4 \sin 2t = 2$

$$\sin 2t = \frac{1}{2}$$

$$2t = \frac{1}{6} \pi \quad \vee \quad 2t = \frac{5}{6} \pi$$

$$t = \frac{1}{12} \pi \quad \vee \quad t = \frac{5}{12} \pi$$

$\frac{5}{12} \pi - \frac{1}{12} \pi = \frac{1}{3} \pi$, de periode is π dus je hebt $\frac{1}{3}$ deel van de cirkel

$$L = \frac{1}{3} \cdot 2\pi \cdot 4 = \frac{8}{3} \pi$$

Opgave 13:

a. $\omega = \frac{2\pi}{3}$

periode = 3 dus $t = 5$ is hetzelfde als $t = 2$

$$\begin{cases} x_P = 5 + 13 \cos \frac{2}{3} \pi (t - 2) \\ y_P = 12 + 13 \sin \frac{2}{3} \pi (t - 2) \end{cases}$$

b. $\begin{cases} x_Q = 5 + 13 \cos \frac{2}{3} \pi (t - 3) \\ y_Q = 12 + 13 \sin \frac{2}{3} \pi (t - 3) \end{cases}$

c. $\frac{1}{4} \text{ per} = \frac{3}{4}$

$$\begin{cases} x_R = 5 + 13 \cos \frac{2}{3} \pi (t - 1\frac{1}{4}) \\ y_R = 12 + 13 \sin \frac{2}{3} \pi (t - 1\frac{1}{4}) \end{cases}$$