

Hoofdstuk 11: Goniometrie en beweging.

11.1 Goniometrische formules.

Opgave 1:

$$f(x) = \sin(-x) = -\sin x$$

$$g(x) = \cos(-x) = \cos x$$

$$h(x) = \sin(x + \frac{1}{2}\pi) = \cos x$$

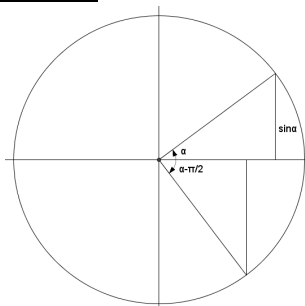
$$j(x) = \cos(x + \frac{1}{2}\pi) = -\sin x$$

$$k(x) = \sin(x + \pi) = -\sin x$$

$$l(x) = \cos(x + \pi) = -\cos x$$

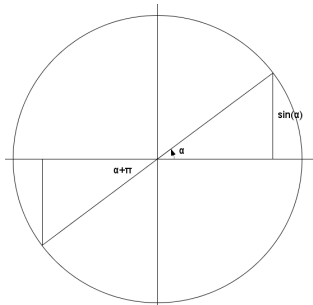
Opgave 2:

a.



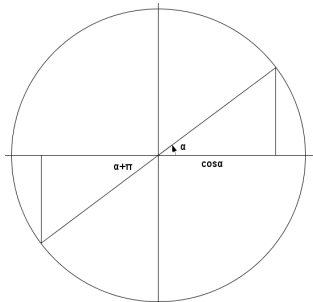
$$\cos(\alpha - \frac{1}{2}\pi) = \sin \alpha$$

b.



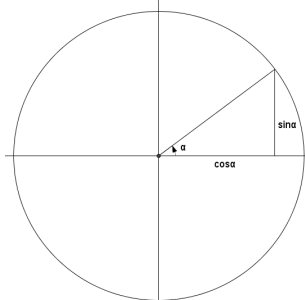
$$\sin(\alpha + \pi) = -\sin \alpha$$

c.



$$\cos(\alpha + \pi) = -\cos \alpha$$

d.



met de stelling van Pythagoras:
 $\sin^2 \alpha + \cos^2 \alpha = 1$

Opgave 3:

- a. $\sin(x + \frac{1}{6}\pi) = \cos(x + \frac{1}{6}\pi - \frac{1}{2}\pi) = \cos(x - \frac{1}{3}\pi)$
 b. $\cos(2x + \frac{1}{3}\pi) = \sin(2x + \frac{1}{3}\pi + \frac{1}{2}\pi) = \sin(2x + \frac{5}{6}\pi)$
 c. $-\sin(3x - \frac{2}{3}\pi) = \sin(3x - \frac{2}{3}\pi + \pi) = \sin(3x + \frac{1}{3}\pi) = \cos(3x + \frac{1}{3}\pi - \frac{1}{2}\pi) = \cos(3x - \frac{1}{6}\pi)$
 d. $-\cos(4x + 1\frac{1}{6}\pi) = \cos(4x + 1\frac{1}{6}\pi + \pi) = \cos(4x + 2\frac{1}{6}\pi) = \sin(4x + 2\frac{1}{6}\pi + \frac{1}{2}\pi) = \sin(4x + 2\frac{2}{3}\pi)$

Opgave 4:

- a. $(\sin x - \cos x)^2 = \sin^2 x - 2\sin x \cos x + \cos^2 x = 1 - 2\sin x \cos x$
 b. $\frac{2\sin^2 x + \cos^2 x}{\cos^2 x} = \frac{2\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = 2 \cdot \left(\frac{\sin x}{\cos x}\right)^2 + 1 = 2\tan^2 x + 1$
 c. $(1 + \tan^2 3x) \cdot \cos^2 3x = (1 + \frac{\sin^2 3x}{\cos^2 3x}) \cdot \cos^2 3x = \cos^2 3x + \sin^2 3x = 1$

Opgave 5:

- a. $\sin^2 x + 4\cos x = 1 - \cos^2 x + 4\cos x$
 b. $2\cos^2 x + \sin x - 2 = 2 \cdot (1 - \sin^2 x) + \sin x - 2$
 $= 2 - 2\sin^2 x + \sin x - 2$
 $= -2\sin^2 x + \sin x$
 c. $2\sin^2 x + \cos^2 x + \cos x = 2 \cdot (1 - \cos^2 x) + \cos^2 x + \cos x$
 $= 2 - 2\cos^2 x + \cos^2 x + \cos x$
 $= 2 - \cos^2 x + \cos x$

Opgave 6:

- $\sin(2x - \frac{1}{3}\pi) = -\cos(x + \frac{1}{3}\pi)$
 $\cos(2x - \frac{1}{3}\pi - \frac{1}{2}\pi) = \cos(x + \frac{1}{3}\pi + \pi)$
 $\cos(2x - \frac{5}{6}\pi) = \cos(x + 1\frac{1}{3}\pi)$
 $2x - \frac{5}{6}\pi = x + 1\frac{1}{3}\pi + k \cdot 2\pi \quad \vee \quad 2x - \frac{5}{6}\pi = -x - 1\frac{1}{3}\pi + k \cdot 2\pi$
 $x = 2\frac{1}{6}\pi + k \cdot 2\pi \quad \vee \quad 3x = -\frac{1}{2}\pi + k \cdot 2\pi$
 $x = -\frac{1}{6}\pi + k \cdot \frac{2}{3}\pi$
 op $[0, 2\pi]$: $x = \frac{1}{6}\pi \quad \vee \quad x = \frac{1}{2}\pi \quad \vee \quad x = 1\frac{1}{6}\pi \quad \vee \quad x = 1\frac{5}{6}\pi$

Opgave 7:

- a. $\sin(x + \frac{1}{2}\pi) = \cos 2x$
 $\cos x = \cos 2x$
 $x = 2x + k \cdot 2\pi \quad \vee \quad x = -2x + k \cdot 2\pi$
 $-x = 0 + k \cdot 2\pi \quad \vee \quad 3x = 0 + k \cdot 2\pi$
 $x = 0 + k \cdot 2\pi \quad \vee \quad x = 0 + k \cdot \frac{2}{3}\pi$
 $x = 0 \quad \vee \quad x = \frac{2}{3}\pi \quad \vee \quad x = 1\frac{1}{3}\pi \quad \vee \quad x = 2\pi$
 b. $\sin 3x = -\cos x$
 $\sin 3x = \sin(x - \frac{1}{2}\pi)$
 $3x = x - \frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad 3x = \pi - (x - \frac{1}{2}\pi) + k \cdot 2\pi$

$$2x = -\frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad 3x = \pi - x + \frac{1}{2}\pi + k \cdot 2\pi$$

$$x = -\frac{1}{4}\pi + k \cdot \pi \quad \vee \quad 4x = 1\frac{1}{2}\pi + k \cdot 2\pi$$

$$x = \frac{3}{8}\pi + k \cdot \frac{1}{2}\pi$$

$$x = \frac{3}{8}\pi \quad \vee \quad x = \frac{3}{4}\pi \quad \vee \quad x = \frac{7}{8}\pi \quad \vee \quad x = 1\frac{3}{8}\pi \quad \vee \quad x = 1\frac{3}{4}\pi \quad \vee \quad x = 1\frac{7}{8}\pi$$

c. $\sin^2 x + \frac{1}{2}\cos x = 1$

$$1 - \cos^2 x + \frac{1}{2}\cos x = 1$$

$$-\cos^2 x + \frac{1}{2}\cos x = 0$$

$$-\cos x(\cos x - \frac{1}{2}) = 0$$

$$\cos x = 0 \quad \vee \quad \cos x = \frac{1}{2}$$

$$x = \frac{1}{2}\pi \quad \vee \quad x = 1\frac{1}{2}\pi \quad \vee \quad x = \frac{1}{3}\pi \quad \vee \quad x = 1\frac{2}{3}\pi$$

d. $\cos(x-1) = -\cos(2x+1)$

$$\cos(x-1) = \cos(2x+1+\pi)$$

$$x-1 = 2x+1+\pi+k \cdot 2\pi \quad \vee \quad x-1 = -2x-1-\pi+k \cdot 2\pi$$

$$-x = 2+\pi+k \cdot 2\pi \quad \vee \quad 3x = -\pi+k \cdot 2\pi$$

$$x = -2-\pi+k \cdot 2\pi \quad \vee \quad x = -\frac{1}{3}\pi+k \cdot \frac{2}{3}\pi$$

$$x = -2+\pi \quad \vee \quad x = \frac{1}{3}\pi \quad \vee \quad x = \pi \quad \vee \quad x = 1\frac{2}{3}\pi$$

e. $\sin(2x+\pi) = 1 - 2\sin 2x$

$$-\sin 2x = 1 - 2\sin 2x$$

$$\sin 2x = 1$$

$$2x = \frac{1}{2}\pi + k \cdot 2\pi$$

$$x = \frac{1}{4}\pi + k \cdot \pi$$

$$x = \frac{1}{4}\pi \quad \vee \quad x = 1\frac{1}{4}\pi$$

f. $2\sin^2 x + \cos^2 x + \cos x = 0$

$$2 \cdot (1 - \cos^2 x) + \cos^2 x + \cos x = 0$$

$$2 - 2\cos^2 x + \cos^2 x + \cos x = 0$$

$$-\cos^2 x + \cos x + 2 = 0$$

$$\cos^2 x - \cos x - 2 = 0$$

$$(\cos x - 2)(\cos x + 1) = 0$$

$$\cos x = 2 \quad \vee \quad \cos x = -1$$

$$\text{k.n.} \quad x = \pi + k \cdot 2\pi$$

$$x = \pi$$

Opgave 8:

a. $\cos(2\pi t) = \sin(\frac{1}{2}\pi t)$

$$\cos(2\pi t) = \cos(\frac{1}{2}\pi t - \frac{1}{2}\pi)$$

$$2\pi t = \frac{1}{2}\pi t - \frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad 2\pi t = -\frac{1}{2}\pi t + \frac{1}{2}\pi + k \cdot 2\pi$$

$$1\frac{1}{2}\pi t = -\frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad 2\frac{1}{2}\pi t = \frac{1}{2}\pi + k \cdot 2\pi$$

$$t = -\frac{1}{3} + k \cdot \frac{4}{3} \quad \vee \quad t = \frac{1}{5} + k \cdot \frac{4}{5}$$

$$t = \frac{1}{5} \quad \vee \quad t = 1 \quad \vee \quad t = 1\frac{4}{5} \quad \vee \quad t = 2\frac{1}{3} \quad \vee \quad t = 2\frac{3}{5}$$

b. $\sin(\frac{\pi t}{6}) = -\cos(\pi t)$

$$\sin(\frac{1}{6}\pi t) = \sin(\pi t - \frac{1}{2}\pi)$$

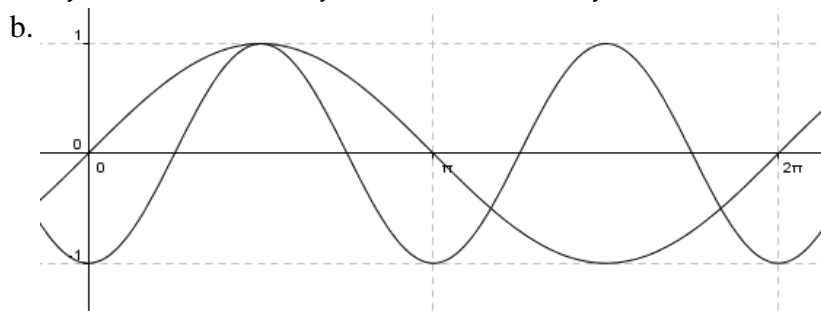
$$\begin{aligned} \frac{1}{6}\pi t &= \pi t - \frac{1}{2}\pi + k \cdot 2\pi & \vee & \quad \frac{1}{6}\pi t = \pi - (\pi t - \frac{1}{2}\pi) + k \cdot 2\pi \\ -\frac{5}{6}\pi t &= -\frac{1}{2}\pi + k \cdot 2\pi & \vee & \quad \frac{1}{6}\pi t = \pi - \pi t + \frac{1}{2}\pi + k \cdot 2\pi \\ t &= \frac{3}{5} + k \cdot \frac{12}{5} & \vee & \quad 1\frac{1}{6}\pi t = 1\frac{1}{2}\pi + k \cdot 2\pi \\ & & & \quad t = \frac{9}{7} + k \cdot \frac{12}{7} \\ t &= \frac{3}{5} & \vee & \quad t = 1\frac{2}{7} & \vee & \quad t = 3 \end{aligned}$$

Opgave 9:

- a. $2\sin x = \sin x$
 $\sin x = 0$
- b. $\sin 2x = \sin x$
 $2x = x + k \cdot 2\pi \quad \vee \quad 2x = \pi - x + k \cdot 2\pi$
- c. niet
- d. niet
- e. $\sin 2x = \sin(x + \frac{1}{3}\pi)$
 $2x = x + \frac{1}{3}\pi + k \cdot 2\pi \quad \vee \quad 2x = \pi - (x + \frac{1}{3}\pi) + k \cdot 2\pi$
- f. niet

Opgave 10:

a. $y = \cos x \xrightarrow{V_{y-as, \frac{1}{2}}} y = \cos 2x \xrightarrow{V_{x-as, -1}} y = -\cos 2x$



- c. $\sin x = -\frac{1}{2}\sqrt{2}$
 $x = 1\frac{1}{4}\pi \quad \vee \quad x = 1\frac{3}{4}\pi$
- d. $-\cos 2x = \frac{1}{2}$
 $\cos 2x = -\frac{1}{2}$
 $2x = \frac{2}{3}\pi + k \cdot 2\pi \quad \vee \quad 2x = \frac{4}{3}\pi + k \cdot 2\pi$
 $x = \frac{1}{3}\pi + k \cdot \pi \quad \vee \quad x = \frac{2}{3}\pi + k \cdot \pi$
 $x = \frac{1}{3}\pi \quad \vee \quad x = \frac{2}{3}\pi \quad \vee \quad x = 1\frac{1}{3}\pi \quad \vee \quad x = 1\frac{2}{3}\pi$
- e. $\sin x = -\cos 2x$
 $\sin x = \sin 2(x - \frac{1}{4}\pi)$
 $x = 2(x - \frac{1}{4}\pi) + k \cdot 2\pi \quad \vee \quad x = \pi - 2(x - \frac{1}{4}\pi) + k \cdot 2\pi$
 $x = 2x - \frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad x = \pi - 2x + \frac{1}{2}\pi + k \cdot 2\pi$
 $-x = -\frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad 3x = 1\frac{1}{2}\pi + k \cdot 2\pi$
 $x = \frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad x = \frac{1}{2}\pi + k \cdot \frac{2}{3}\pi$
snijpunten voor $x = \frac{1}{2}\pi \quad \vee \quad x = 1\frac{1}{6}\pi \quad \vee \quad x = 1\frac{5}{6}\pi$
 $f(x) \leq g(x)$ voor $x = \frac{1}{2}\pi \quad \vee \quad 1\frac{1}{6}\pi \leq x \leq 1\frac{5}{6}\pi$

Opgave 11:

a. $f(x) = \sin(2x - \frac{1}{3}\pi) = \sin(2(x - \frac{1}{6}\pi))$

ev-as: $y = 0$

amp: 1

per: $\frac{2\pi}{2} = \pi$

begin: $(\frac{1}{6}\pi, 0)$

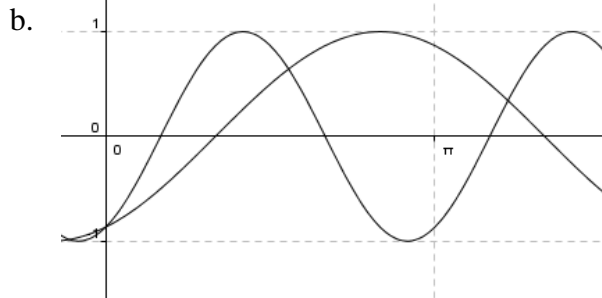
$g(x) = -\cos(x + \frac{1}{6}\pi)$

ev-as: $y = 0$

amp: 1

per: 2π

begin: $(-\frac{1}{6}\pi, -1)$ laagste punt



c. $\sin(2x - \frac{1}{3}\pi) = 0$

$2x - \frac{1}{3}\pi = 0 + k \cdot \pi$

$2x = \frac{1}{3}\pi + k \cdot \pi$

$x = \frac{1}{6}\pi + k \cdot \frac{1}{2}\pi$

$x = \frac{1}{6}\pi \quad \vee \quad x = \frac{2}{3}\pi \quad \vee \quad x = 1\frac{1}{6}\pi$

$-\cos(x + \frac{1}{6}\pi) = 0$

$x + \frac{1}{6}\pi = \frac{1}{2}\pi + k \cdot \pi$

$x = \frac{1}{3}\pi + k \cdot \pi$

$x = \frac{1}{3}\pi \quad \vee \quad x = 1\frac{1}{3}\pi$

d. $\sin(2x - \frac{1}{3}\pi) = \frac{1}{2}$

$2x - \frac{1}{3}\pi = \frac{1}{6}\pi + k \cdot 2\pi \quad \vee \quad 2x - \frac{1}{3}\pi = \frac{5}{6}\pi + k \cdot 2\pi$

$2x = \frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad 2x = 1\frac{1}{6}\pi + k \cdot 2\pi$

$x = \frac{1}{4}\pi + k \cdot \pi \quad \vee \quad x = \frac{7}{12}\pi + k \cdot \pi$

$\frac{1}{4}\pi < x < \frac{7}{12}\pi \quad \vee \quad 1\frac{1}{4}\pi < x \leq 1\frac{1}{2}\pi$

e. $\sin(2x - \frac{1}{3}\pi) = -\cos(x + \frac{1}{6}\pi)$

$\sin(2x - \frac{1}{3}\pi) = \sin(x - \frac{1}{3}\pi)$

$2x - \frac{1}{3}\pi = x - \frac{1}{3}\pi + k \cdot 2\pi \quad \vee \quad 2x - \frac{1}{3}\pi = \pi - (x - \frac{1}{3}\pi) + k \cdot 2\pi$

$x = 0 + k \cdot 2\pi \quad \vee \quad 2x - \frac{1}{3}\pi = \pi - x + \frac{1}{3}\pi + k \cdot 2\pi$

$3x = \frac{5}{3}\pi + k \cdot 2\pi$

$x = \frac{5}{9}\pi + k \cdot \frac{2}{3}\pi$

snijpunten voor $x = 0 \quad \vee \quad x = \frac{5}{9}\pi \quad \vee \quad x = 1\frac{2}{9}\pi$

$f(x) < g(x)$ voor $\frac{5}{9}\pi < x < 1\frac{2}{9}\pi$

Opgave 12:

a. punt C is het snijpunt van AB met de x -as

$\sin \alpha = \frac{AC}{1} = AC$

$AB = 2 \cdot AC = 2 \sin \alpha$

b. $4 \sin^2 \alpha = 1 + 1 - 2 \cdot 1 \cdot 1 \cdot \cos 2\alpha$

$2 \cos 2\alpha = 2 - 4 \sin^2 \alpha$

$\cos 2\alpha = 1 - 2 \sin^2 \alpha$

Opgave 13:

- a. $\cos(t+u) = \cos(t-(-u))$
 $= \cos t \cdot \cos(-u) + \sin t \cdot \sin(-u)$
 $= \cos t \cdot \cos u + \sin t \cdot -\sin u$
 $= \cos t \cdot \cos u - \sin t \cdot \sin u$
- b. $\sin(t+u) = \cos(t+u-\frac{1}{2}\pi)$
 $= \cos t \cdot \cos(u-\frac{1}{2}\pi) - \sin t \cdot \sin(u-\frac{1}{2}\pi)$
 $= \cos t \cdot \sin u - \sin t \cdot -\cos u$
 $= \cos t \cdot \sin u + \sin t \cdot \cos u$
- c. $\sin(t-u) = \sin(t+(-u))$
 $= \sin t \cdot \cos(-u) + \cos t \cdot \sin(-u)$
 $= \sin t \cdot \cos u + \cos t \cdot -\sin u$
 $= \sin t \cdot \cos u - \cos t \cdot \sin u$

Opgave 14:

- a. $\sin 2A = \sin(A+A) = \sin A \cdot \cos A + \cos A \cdot \sin A = 2 \sin A \cdot \cos A$
 $\cos 2A = \cos(A+A) = \cos A \cdot \cos A - \sin A \cdot \sin A = \cos^2 A - \sin^2 A$
- b. $\cos 2A = \cos^2 A - \sin^2 A = \cos^2 A - (1 - \cos^2 A) = \cos^2 A - 1 + \cos^2 A = 2 \cos^2 A - 1$
 $\cos 2A = \cos^2 A - \sin^2 A = 1 - \sin^2 A - \sin^2 A = 1 - 2 \sin^2 A$

Opgave 15:

- a. $\cos 2A = 2 \cos^2 A - 1$
 $-2 \cos^2 A = -\cos 2A - 1$
 $\cos^2 A = \frac{1}{2} \cos 2A + \frac{1}{2}$
- b. $\cos 2A = 1 - 2 \sin^2 A$
 $2 \sin^2 A = 1 - \cos 2A$
 $\sin^2 A = \frac{1}{2} - \frac{1}{2} \cos 2A$

Opgave 16:

- a. $\sin x \cdot \cos x = \frac{1}{2} \sin(x-1)$
 $2 \sin x \cdot \cos x = \sin(x-1)$
 $\sin 2x = \sin(x-1)$
 $2x = x-1 + k \cdot 2\pi \quad \vee \quad 2x = \pi - (x-1) + k \cdot 2\pi$
 $x = -1 + k \cdot 2\pi \quad \vee \quad 2x = \pi - x + 1 + k \cdot 2\pi$
 $3x = \pi + 1 + k \cdot 2\pi$
 $x = \frac{1}{3}\pi + \frac{1}{3} + k \cdot \frac{2}{3}\pi$
- b. $\cos^2 2x = \cos 4x + \frac{1}{2}$
 $\frac{1}{2} + \frac{1}{2} \cos 4x = \cos 4x + \frac{1}{2}$
 $-\frac{1}{2} \cos 4x = 0$
 $\cos 4x = 0$
 $4x = \frac{1}{2}\pi + k \cdot \pi$
 $x = \frac{1}{8}\pi + k \cdot \frac{1}{4}\pi$
- c. $\sin^2(\frac{1}{2}x) = \cos x + 1\frac{1}{4}$

$$\frac{1}{2} - \frac{1}{2} \cos x = \cos x + 1 \frac{1}{4}$$

$$-1 \frac{1}{2} \cos x = \frac{3}{4}$$

$$\cos x = -\frac{1}{2}$$

$$x = \frac{2}{3}\pi + k \cdot 2\pi \quad \vee \quad x = -\frac{2}{3}\pi + k \cdot 2\pi$$

d. $(\sin x + \cos x)^2 = 1 \frac{1}{2}$

$$\sin^2 x + 2 \sin x \cos x + \cos^2 x = 1 \frac{1}{2}$$

$$1 + 2 \sin x \cos x = 1 \frac{1}{2}$$

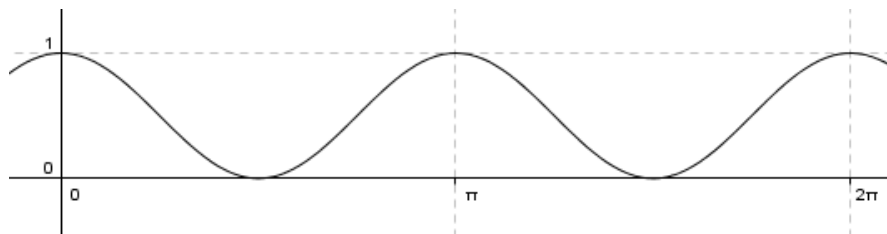
$$\sin 2x = \frac{1}{2}$$

$$2x = \frac{1}{6}\pi + k \cdot 2\pi \quad \vee \quad 2x = \frac{5}{6}\pi + k \cdot 2\pi$$

$$x = \frac{1}{12}\pi + k \cdot \pi \quad \vee \quad x = \frac{5}{12}\pi + k \cdot \pi$$

Opgave 17:

a.



$$y = \frac{1}{2} + \frac{1}{2} \cos 2x$$

b. $y = \sin^2 x + \cos 2x = \frac{1}{2} - \frac{1}{2} \cos 2x + \cos 2x = \frac{1}{2} + \frac{1}{2} \cos 2x$

Opgave 18:

$$\sin 3x = \sin(2x + x)$$

$$= \sin 2x \cdot \cos x + \cos 2x \cdot \sin x$$

$$= 2 \sin x \cdot \cos x \cdot \cos x + (1 - 2 \sin^2 x) \cdot \sin x$$

$$= 2 \sin x \cdot \cos^2 x + \sin x - 2 \sin^3 x$$

$$= 2 \sin x \cdot (1 - \sin^2 x) + \sin x - 2 \sin^3 x$$

$$= 2 \sin x - 2 \sin^3 x + \sin x - 2 \sin^3 x$$

$$= 3 \sin x - 4 \sin^3 x$$

Opgave 19:

a. $y = 1 - \cos x - \sin^2(\frac{1}{2}x)$

$$= 1 - \cos x - (\frac{1}{2} - \frac{1}{2} \cos x)$$

$$= 1 - \cos x - \frac{1}{2} + \frac{1}{2} \cos x$$

$$= \frac{1}{2} - \frac{1}{2} \cos x$$

b. $\cos 3x = \cos(2x + x)$

$$= \cos 2x \cdot \cos x - \sin 2x \cdot \sin x$$

$$= (2 \cos^2 x - 1) \cdot \cos x - 2 \sin x \cdot \cos x \cdot \sin x$$

$$= 2 \cos^3 x - \cos x - 2 \cos x \cdot \sin^2 x$$

$$= 2 \cos^3 x - \cos x - 2 \cos x \cdot (1 - \sin^2 x)$$

$$= 2 \cos^3 x - \cos x - 2 \cos x + 2 \cos^3 x$$

$$= 4 \cos^3 x - 3 \cos x$$

Opgave 20:

- a. $-\cos A = \cos(A + \pi)$
 b. $\cos 2A = 1 - 2\sin^2 A$
 c. $\cos 2A = 1 - 2\sin^2 A$
 d. $\cos 2A = 2\cos^2 A - 1$
 e. $\cos 2A = 1 - 2\sin^2 A$

Opgave 21:

$B(-x_A, y_A)$ en $C(-x_A, -y_A)$

Opgave 22:

- a. $f(-p) + f(p) = -p \cdot \cos(-p) + p \cdot \cos p = -p \cdot \cos p + p \cdot \cos p = 0$
 dus de grafiek van f is symmetrisch in de oorsprong
 b. $g(-p) = -p \cdot \sin(-p) = -p \cdot -\sin p = p \cdot \sin p = g(p)$
 dus de grafiek van g is symmetrisch in de y -as.

Opgave 23:

- a. $f(-p) + f(p) = \cos^2(-p) \cdot \sin(-p) + \cos^2 p \cdot \sin p$
 $= \cos^2 p \cdot -\sin p + \cos^2 p \cdot \sin p = 0$
 dus de grafiek van f is symmetrisch in de oorsprong
 b. $f(\frac{1}{2}\pi - p) = \cos^2(\frac{1}{2}\pi - p) \cdot \sin(\frac{1}{2}\pi - p)$
 $= (-\cos(\frac{1}{2}\pi + p))^2 \cdot \sin(\frac{1}{2}\pi + p)$
 $= \cos^2(\frac{1}{2}\pi + p) \cdot \sin(\frac{1}{2}\pi + p) = f(\frac{1}{2}\pi + p)$
 dus de grafiek van f is symmetrisch in de lijn $x = \frac{1}{2}\pi$.

Opgave 24:

- a. $\sin(-\frac{1}{4}\pi - p) = \sin(-\frac{1}{4}\pi) \cdot \cos(-p) - \cos(-\frac{1}{4}\pi) \cdot \sin(-p)$
 $= -\frac{1}{2}\sqrt{2} \cdot \cos p - \frac{1}{2}\sqrt{2} \cdot -\sin p$
 $= -\frac{1}{2}\sqrt{2} \cdot \cos p + \frac{1}{2}\sqrt{2} \cdot \sin p$
 $\cos(-\frac{1}{4}\pi - p) = \cos(-\frac{1}{4}\pi) \cdot \cos(-p) + \sin(-\frac{1}{4}\pi) \cdot \sin(-p)$
 $= \frac{1}{2}\sqrt{2} \cdot \cos p + -\frac{1}{2}\sqrt{2} \cdot -\sin p$
 $= \frac{1}{2}\sqrt{2} \cdot \cos p + \frac{1}{2}\sqrt{2} \cdot \sin p$
 $f(-\frac{1}{4}\pi - p) = 2\sin(-\frac{1}{4}\pi - p) - 2\cos(-\frac{1}{4}\pi - p)$
 $= -\sqrt{2} \cdot \cos p + \sqrt{2} \cdot \sin p - (\sqrt{2} \cdot \cos p + \sqrt{2} \cdot \sin p)$
 $= -\sqrt{2} \cdot \cos p + \sqrt{2} \cdot \sin p - \sqrt{2} \cdot \cos p - \sqrt{2} \cdot \sin p$
 $= -2\sqrt{2} \cdot \cos p$
 b. $\sin(-\frac{1}{4}\pi + p) = \sin(-\frac{1}{4}\pi) \cdot \cos p + \cos(-\frac{1}{4}\pi) \cdot \sin p$
 $= -\frac{1}{2}\sqrt{2} \cdot \cos p + \frac{1}{2}\sqrt{2} \cdot \sin p$
 $\cos(-\frac{1}{4}\pi + p) = \cos(-\frac{1}{4}\pi) \cdot \cos p - \sin(-\frac{1}{4}\pi) \cdot \sin p$
 $= \frac{1}{2}\sqrt{2} \cdot \cos p - -\frac{1}{2}\sqrt{2} \cdot \sin p$
 $= \frac{1}{2}\sqrt{2} \cdot \cos p + \frac{1}{2}\sqrt{2} \cdot \sin p$

$$\begin{aligned}
f(-\tfrac{1}{4}\pi + p) &= 2\sin(-\tfrac{1}{4}\pi + p) - 2\cos(-\tfrac{1}{4}\pi + p) \\
&= -\sqrt{2} \cdot \cos p + \sqrt{2} \cdot \sin p - (\sqrt{2} \cdot \cos p + \sqrt{2} \cdot \sin p) \\
&= -\sqrt{2} \cdot \cos p + \sqrt{2} \cdot \sin p - \sqrt{2} \cdot \cos p - \sqrt{2} \cdot \sin p \\
&= -2\sqrt{2} \cdot \cos p
\end{aligned}$$

dus $f(-\tfrac{1}{4}\pi - p) = f(-\tfrac{1}{4}\pi + p)$

dus de grafiek van f is symmetrisch in de lijn $x = -\tfrac{1}{4}\pi$

Opgave 25:

a. $\cos(\tfrac{1}{4}\pi - p) = \cos(\tfrac{1}{4}\pi) \cdot \cos p + \sin(\tfrac{1}{4}\pi) \cdot \sin p$
 $= \tfrac{1}{2}\sqrt{2} \cdot \cos p + \tfrac{1}{2}\sqrt{2} \cdot \sin p$

$$\begin{aligned}
\sin(\tfrac{1}{4}\pi - p) &= \sin(\tfrac{1}{4}\pi) \cdot \cos p - \cos(\tfrac{1}{4}\pi) \cdot \sin p \\
&= \tfrac{1}{2}\sqrt{2} \cdot \cos p - \tfrac{1}{2}\sqrt{2} \cdot \sin p
\end{aligned}$$

$$\begin{aligned}
f(\tfrac{1}{4}\pi - p) &= \tfrac{1}{2}\sqrt{2} \cdot \cos p + \tfrac{1}{2}\sqrt{2} \cdot \sin p + \tfrac{1}{2}\sqrt{2} \cdot \cos p - \tfrac{1}{2}\sqrt{2} \cdot \sin p + 1 \\
&= \sqrt{2} \cdot \cos p + 1
\end{aligned}$$

$$\begin{aligned}
\cos(\tfrac{1}{4}\pi + p) &= \cos(\tfrac{1}{4}\pi) \cdot \cos p - \sin(\tfrac{1}{4}\pi) \cdot \sin p \\
&= \tfrac{1}{2}\sqrt{2} \cdot \cos p - \tfrac{1}{2}\sqrt{2} \cdot \sin p
\end{aligned}$$

$$\begin{aligned}
\sin(\tfrac{1}{4}\pi + p) &= \sin(\tfrac{1}{4}\pi) \cdot \cos p + \cos(\tfrac{1}{4}\pi) \cdot \sin p \\
&= \tfrac{1}{2}\sqrt{2} \cdot \cos p + \tfrac{1}{2}\sqrt{2} \cdot \sin p
\end{aligned}$$

$$\begin{aligned}
f(\tfrac{1}{4}\pi + p) &= \tfrac{1}{2}\sqrt{2} \cdot \cos p - \tfrac{1}{2}\sqrt{2} \cdot \sin p + \tfrac{1}{2}\sqrt{2} \cdot \cos p + \tfrac{1}{2}\sqrt{2} \cdot \sin p + 1 \\
&= \sqrt{2} \cdot \cos p + 1
\end{aligned}$$

dus $f(\tfrac{1}{4}\pi - p) = f(\tfrac{1}{4}\pi + p)$

dus de grafiek van f is symmetrisch in de lijn $x = \tfrac{1}{4}\pi$

b. $\cos(\tfrac{3}{4}\pi - p) = \cos(\tfrac{3}{4}\pi) \cdot \cos p + \sin(\tfrac{3}{4}\pi) \cdot \sin p$
 $= -\tfrac{1}{2}\sqrt{2} \cdot \cos p + \tfrac{1}{2}\sqrt{2} \cdot \sin p$

$$\begin{aligned}
\sin(\tfrac{3}{4}\pi - p) &= \sin(\tfrac{3}{4}\pi) \cdot \cos p - \cos(\tfrac{3}{4}\pi) \cdot \sin p \\
&= \tfrac{1}{2}\sqrt{2} \cdot \cos p - -\tfrac{1}{2}\sqrt{2} \cdot \sin p \\
&= \tfrac{1}{2}\sqrt{2} \cdot \cos p + \tfrac{1}{2}\sqrt{2} \cdot \sin p
\end{aligned}$$

$$\begin{aligned}
f(\tfrac{3}{4}\pi - p) &= -\tfrac{1}{2}\sqrt{2} \cos p + \tfrac{1}{2}\sqrt{2} \cdot \sin p + \tfrac{1}{2}\sqrt{2} \cdot \cos p + \tfrac{1}{2}\sqrt{2} \cdot \sin p + 1 \\
&= \sqrt{2} \cdot \sin p + 1
\end{aligned}$$

$$\begin{aligned}
\cos(\tfrac{3}{4}\pi + p) &= \cos(\tfrac{3}{4}\pi) \cdot \cos p - \sin(\tfrac{3}{4}\pi) \cdot \sin p \\
&= -\tfrac{1}{2}\sqrt{2} \cdot \cos p - \tfrac{1}{2}\sqrt{2} \cdot \sin p
\end{aligned}$$

$$\begin{aligned}
\sin(\tfrac{3}{4}\pi + p) &= \sin(\tfrac{3}{4}\pi) \cdot \cos p + \cos(\tfrac{3}{4}\pi) \cdot \sin p \\
&= \tfrac{1}{2}\sqrt{2} \cdot \cos p - \tfrac{1}{2}\sqrt{2} \cdot \sin p
\end{aligned}$$

$$\begin{aligned}
f(\tfrac{3}{4}\pi + p) &= -\tfrac{1}{2}\sqrt{2} \cdot \cos p - \tfrac{1}{2}\sqrt{2} \cdot \sin p + \tfrac{1}{2}\sqrt{2} \cdot \cos p - \tfrac{1}{2}\sqrt{2} \cdot \sin p + 1 \\
&= -\sqrt{2} \cdot \sin p + 1
\end{aligned}$$

$$f(\tfrac{3}{4}\pi - p) + f(\tfrac{3}{4}\pi + p) = \sqrt{2} \cdot \sin p + 1 + -\sqrt{2} \cdot \sin p + 1 = 2$$

dus de grafiek van f is puntsymmetrisch in het punt $(\tfrac{3}{4}\pi, 1)$