

## Hoofdstuk 11: Goniometrie en beweging.

### 11.1 Goniometrische formules.

#### Opgave 1:

$$f(x) = \sin(-x) = -\sin x$$

$$g(x) = \cos(-x) = \cos x$$

$$h(x) = \sin(x + \frac{1}{2}\pi) = \cos x$$

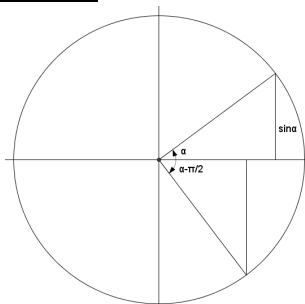
$$j(x) = \cos(x + \frac{1}{2}\pi) = -\sin x$$

$$k(x) = \sin(x + \pi) = -\sin x$$

$$l(x) = \cos(x + \pi) = -\cos x$$

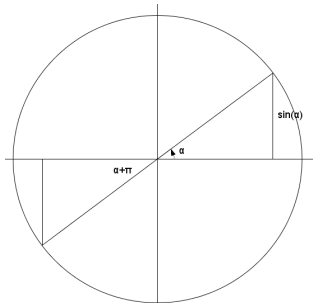
#### Opgave 2:

a.



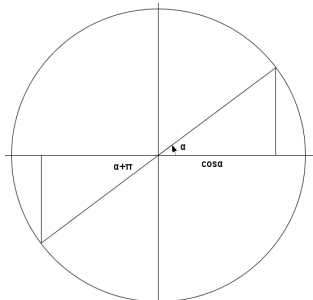
$$\cos(\alpha - \frac{1}{2}\pi) = \sin \alpha$$

b.



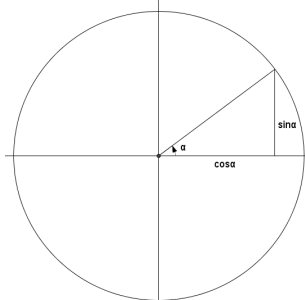
$$\sin(\alpha + \pi) = -\sin \alpha$$

c.



$$\cos(\alpha + \pi) = -\cos \alpha$$

d.



met de stelling van Pythagoras:  
 $\sin^2 \alpha + \cos^2 \alpha = 1$

**Opgave 3:**

- a.  $\sin(x + \frac{1}{6}\pi) = \cos(x + \frac{1}{6}\pi - \frac{1}{2}\pi) = \cos(x - \frac{1}{3}\pi)$   
 b.  $\cos(2x + \frac{1}{3}\pi) = \sin(2x + \frac{1}{3}\pi + \frac{1}{2}\pi) = \sin(2x + \frac{5}{6}\pi)$   
 c.  $-\sin(3x - \frac{2}{3}\pi) = \sin(3x - \frac{2}{3}\pi + \pi) = \sin(3x + \frac{1}{3}\pi) = \cos(3x + \frac{1}{3}\pi - \frac{1}{2}\pi) = \cos(3x - \frac{1}{6}\pi)$   
 d.  $-\cos(4x + 1\frac{1}{6}\pi) = \cos(4x + 1\frac{1}{6}\pi + \pi) = \cos(4x + 2\frac{1}{6}\pi) = \sin(4x + 2\frac{1}{6}\pi + \frac{1}{2}\pi) = \sin(4x + 2\frac{2}{3}\pi)$

**Opgave 4:**

- a.  $(\sin x - \cos x)^2 = \sin^2 x - 2\sin x \cos x + \cos^2 x = 1 - 2\sin x \cos x$   
 b.  $\frac{2\sin^2 x + \cos^2 x}{\cos^2 x} = \frac{2\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = 2 \cdot \left(\frac{\sin x}{\cos x}\right)^2 + 1 = 2\tan^2 x + 1$   
 c.  $(1 + \tan^2 3x) \cdot \cos^2 3x = (1 + \frac{\sin^2 3x}{\cos^2 3x}) \cdot \cos^2 3x = \cos^2 3x + \sin^2 3x = 1$

**Opgave 5:**

- a.  $\sin^2 x + 4\cos x = 1 - \cos^2 x + 4\cos x$   
 b.  $2\cos^2 x + \sin x - 2 = 2 \cdot (1 - \sin^2 x) + \sin x - 2$   
 $= 2 - 2\sin^2 x + \sin x - 2$   
 $= -2\sin^2 x + \sin x$   
 c.  $2\sin^2 x + \cos^2 x + \cos x = 2 \cdot (1 - \cos^2 x) + \cos^2 x + \cos x$   
 $= 2 - 2\cos^2 x + \cos^2 x + \cos x$   
 $= 2 - \cos^2 x + \cos x$

**Opgave 6:**

- $\sin(2x - \frac{1}{3}\pi) = -\cos(x + \frac{1}{3}\pi)$   
 $\cos(2x - \frac{1}{3}\pi - \frac{1}{2}\pi) = \cos(x + \frac{1}{3}\pi + \pi)$   
 $\cos(2x - \frac{5}{6}\pi) = \cos(x + 1\frac{1}{3}\pi)$   
 $2x - \frac{5}{6}\pi = x + 1\frac{1}{3}\pi + k \cdot 2\pi \quad \vee \quad 2x - \frac{5}{6}\pi = -x - 1\frac{1}{3}\pi + k \cdot 2\pi$   
 $x = 2\frac{1}{6}\pi + k \cdot 2\pi \quad \vee \quad 3x = -\frac{1}{2}\pi + k \cdot 2\pi$   
 $x = -\frac{1}{6}\pi + k \cdot \frac{2}{3}\pi$   
 op  $[0, 2\pi]$ :  $x = \frac{1}{6}\pi \quad \vee \quad x = \frac{1}{2}\pi \quad \vee \quad x = 1\frac{1}{6}\pi \quad \vee \quad x = 1\frac{5}{6}\pi$

**Opgave 7:**

- a.  $\sin(x + \frac{1}{2}\pi) = \cos 2x$   
 $\cos x = \cos 2x$   
 $x = 2x + k \cdot 2\pi \quad \vee \quad x = -2x + k \cdot 2\pi$   
 $-x = 0 + k \cdot 2\pi \quad \vee \quad 3x = 0 + k \cdot 2\pi$   
 $x = 0 + k \cdot 2\pi \quad \vee \quad x = 0 + k \cdot \frac{2}{3}\pi$   
 $x = 0 \quad \vee \quad x = \frac{2}{3}\pi \quad \vee \quad x = 1\frac{1}{3}\pi \quad \vee \quad x = 2\pi$   
 b.  $\sin 3x = -\cos x$   
 $\sin 3x = \sin(x - \frac{1}{2}\pi)$   
 $3x = x - \frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad 3x = \pi - (x - \frac{1}{2}\pi) + k \cdot 2\pi$

$$2x = -\frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad 3x = \pi - x + \frac{1}{2}\pi + k \cdot 2\pi$$

$$x = -\frac{1}{4}\pi + k \cdot \pi \quad \vee \quad 4x = 1\frac{1}{2}\pi + k \cdot 2\pi$$

$$x = \frac{3}{8}\pi + k \cdot \frac{1}{2}\pi$$

$$x = \frac{3}{8}\pi \quad \vee \quad x = \frac{3}{4}\pi \quad \vee \quad x = \frac{7}{8}\pi \quad \vee \quad x = 1\frac{3}{8}\pi \quad \vee \quad x = 1\frac{3}{4}\pi \quad \vee \quad x = 1\frac{7}{8}\pi$$

c.  $\sin^2 x + \frac{1}{2}\cos x = 1$

$$1 - \cos^2 x + \frac{1}{2}\cos x = 1$$

$$-\cos^2 x + \frac{1}{2}\cos x = 0$$

$$-\cos x(\cos x - \frac{1}{2}) = 0$$

$$\cos x = 0 \quad \vee \quad \cos x = \frac{1}{2}$$

$$x = \frac{1}{2}\pi \quad \vee \quad x = 1\frac{1}{2}\pi \quad \vee \quad x = \frac{1}{3}\pi \quad \vee \quad x = 1\frac{2}{3}\pi$$

d.  $\cos(x-1) = -\cos(2x+1)$

$$\cos(x-1) = \cos(2x+1+\pi)$$

$$x-1 = 2x+1+\pi+k \cdot 2\pi \quad \vee \quad x-1 = -2x-1-\pi+k \cdot 2\pi$$

$$-x = 2+\pi+k \cdot 2\pi \quad \vee \quad 3x = -\pi+k \cdot 2\pi$$

$$x = -2-\pi+k \cdot 2\pi \quad \vee \quad x = -\frac{1}{3}\pi+k \cdot \frac{2}{3}\pi$$

$$x = -2+\pi \quad \vee \quad x = \frac{1}{3}\pi \quad \vee \quad x = \pi \quad \vee \quad x = 1\frac{2}{3}\pi$$

e.  $\sin(2x+\pi) = 1 - 2\sin 2x$

$$-\sin 2x = 1 - 2\sin 2x$$

$$\sin 2x = 1$$

$$2x = \frac{1}{2}\pi + k \cdot 2\pi$$

$$x = \frac{1}{4}\pi + k \cdot \pi$$

$$x = \frac{1}{4}\pi \quad \vee \quad x = 1\frac{1}{4}\pi$$

f.  $2\sin^2 x + \cos^2 x + \cos x = 0$

$$2 \cdot (1 - \cos^2 x) + \cos^2 x + \cos x = 0$$

$$2 - 2\cos^2 x + \cos^2 x + \cos x = 0$$

$$-\cos^2 x + \cos x + 2 = 0$$

$$\cos^2 x - \cos x - 2 = 0$$

$$(\cos x - 2)(\cos x + 1) = 0$$

$$\cos x = 2 \quad \vee \quad \cos x = -1$$

$$\text{k.n.} \quad x = \pi + k \cdot 2\pi$$

$$x = \pi$$

### **Opgave 8:**

a.  $\cos(2\pi t) = \sin(\frac{1}{2}\pi t)$

$$\cos(2\pi t) = \cos(\frac{1}{2}\pi t - \frac{1}{2}\pi)$$

$$2\pi t = \frac{1}{2}\pi t - \frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad 2\pi t = -\frac{1}{2}\pi t + \frac{1}{2}\pi + k \cdot 2\pi$$

$$1\frac{1}{2}\pi t = -\frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad 2\frac{1}{2}\pi t = \frac{1}{2}\pi + k \cdot 2\pi$$

$$t = -\frac{1}{3} + k \cdot \frac{4}{3} \quad \vee \quad t = \frac{1}{5} + k \cdot \frac{4}{5}$$

$$t = \frac{1}{5} \quad \vee \quad t = 1 \quad \vee \quad t = 1\frac{4}{5} \quad \vee \quad t = 2\frac{1}{3} \quad \vee \quad t = 2\frac{3}{5}$$

b.  $\sin(\frac{\pi t}{6}) = -\cos(\pi t)$

$$\sin(\frac{1}{6}\pi t) = \sin(\pi t - \frac{1}{2}\pi)$$

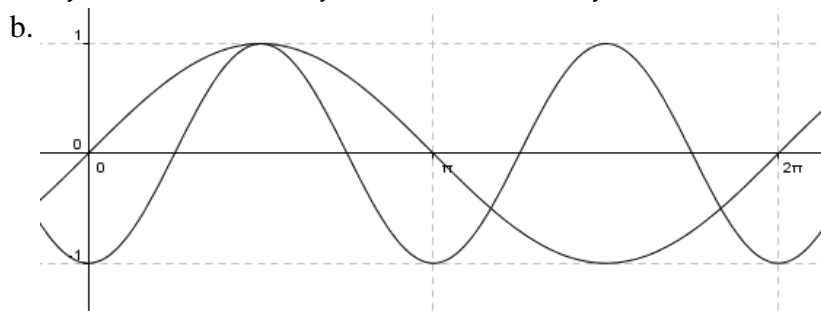
$$\begin{aligned} \frac{1}{6}\pi t &= \pi t - \frac{1}{2}\pi + k \cdot 2\pi & \vee & \frac{1}{6}\pi t = \pi - (\pi t - \frac{1}{2}\pi) + k \cdot 2\pi \\ -\frac{5}{6}\pi t &= -\frac{1}{2}\pi + k \cdot 2\pi & \vee & \frac{1}{6}\pi t = \pi - \pi t + \frac{1}{2}\pi + k \cdot 2\pi \\ t &= \frac{3}{5} + k \cdot \frac{12}{5} & \vee & 1\frac{1}{6}\pi t = 1\frac{1}{2}\pi + k \cdot 2\pi \\ & & & t = \frac{9}{7} + k \cdot \frac{12}{7} \\ t &= \frac{3}{5} & \vee & t = 1\frac{2}{7} & \vee & t = 3 \end{aligned}$$

### Opgave 9:

- a.  $2\sin x = \sin x$   
 $\sin x = 0$
- b.  $\sin 2x = \sin x$   
 $2x = x + k \cdot 2\pi \quad \vee \quad 2x = \pi - x + k \cdot 2\pi$
- c. niet
- d. niet
- e.  $\sin 2x = \sin(x + \frac{1}{3}\pi)$   
 $2x = x + \frac{1}{3}\pi + k \cdot 2\pi \quad \vee \quad 2x = \pi - (x + \frac{1}{3}\pi) + k \cdot 2\pi$
- f. niet

### Opgave 10:

a.  $y = \cos x \xrightarrow{V_{y-as, \frac{1}{2}}} y = \cos 2x \xrightarrow{V_{x-as, -1}} y = -\cos 2x$



- c.  $\sin x = -\frac{1}{2}\sqrt{2}$   
 $x = 1\frac{1}{4}\pi \quad \vee \quad x = 1\frac{3}{4}\pi$
- d.  $-\cos 2x = \frac{1}{2}$   
 $\cos 2x = -\frac{1}{2}$   
 $2x = \frac{2}{3}\pi + k \cdot 2\pi \quad \vee \quad 2x = \frac{4}{3}\pi + k \cdot 2\pi$   
 $x = \frac{1}{3}\pi + k \cdot \pi \quad \vee \quad x = \frac{2}{3}\pi + k \cdot \pi$   
 $x = \frac{1}{3}\pi \quad \vee \quad x = \frac{2}{3}\pi \quad \vee \quad x = 1\frac{1}{3}\pi \quad \vee \quad x = 1\frac{2}{3}\pi$
- e.  $\sin x = -\cos 2x$   
 $\sin x = \sin 2(x - \frac{1}{4}\pi)$   
 $x = 2(x - \frac{1}{4}\pi) + k \cdot 2\pi \quad \vee \quad x = \pi - 2(x - \frac{1}{4}\pi) + k \cdot 2\pi$   
 $x = 2x - \frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad x = \pi - 2x + \frac{1}{2}\pi + k \cdot 2\pi$   
 $-x = -\frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad 3x = 1\frac{1}{2}\pi + k \cdot 2\pi$   
 $x = \frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad x = \frac{1}{2}\pi + k \cdot \frac{2}{3}\pi$   
snijpunten voor  $x = \frac{1}{2}\pi \quad \vee \quad x = 1\frac{1}{6}\pi \quad \vee \quad x = 1\frac{5}{6}\pi$   
 $f(x) \leq g(x)$  voor  $x = \frac{1}{2}\pi \quad \vee \quad 1\frac{1}{6}\pi \leq x \leq 1\frac{5}{6}\pi$

**Opgave 11:**

a.  $f(x) = \sin(2x - \frac{1}{3}\pi) = \sin(2(x - \frac{1}{6}\pi))$

ev-as:  $y = 0$

amp: 1

per:  $\frac{2\pi}{2} = \pi$

begin:  $(\frac{1}{6}\pi, 0)$

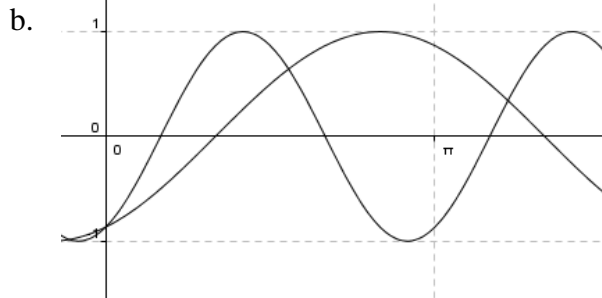
$g(x) = -\cos(x + \frac{1}{6}\pi)$

ev-as:  $y = 0$

amp: 1

per:  $2\pi$

begin:  $(-\frac{1}{6}\pi, -1)$  laagste punt



c.  $\sin(2x - \frac{1}{3}\pi) = 0$

$2x - \frac{1}{3}\pi = 0 + k \cdot \pi$

$2x = \frac{1}{3}\pi + k \cdot \pi$

$x = \frac{1}{6}\pi + k \cdot \frac{1}{2}\pi$

$x = \frac{1}{6}\pi \quad \vee \quad x = \frac{2}{3}\pi \quad \vee \quad x = 1\frac{1}{6}\pi$

$-\cos(x + \frac{1}{6}\pi) = 0$

$x + \frac{1}{6}\pi = \frac{1}{2}\pi + k \cdot \pi$

$x = \frac{1}{3}\pi + k \cdot \pi$

$x = \frac{1}{3}\pi \quad \vee \quad x = 1\frac{1}{3}\pi$

d.  $\sin(2x - \frac{1}{3}\pi) = \frac{1}{2}$

$2x - \frac{1}{3}\pi = \frac{1}{6}\pi + k \cdot 2\pi \quad \vee \quad 2x - \frac{1}{3}\pi = \frac{5}{6}\pi + k \cdot 2\pi$

$2x = \frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad 2x = 1\frac{1}{6}\pi + k \cdot 2\pi$

$x = \frac{1}{4}\pi + k \cdot \pi \quad \vee \quad x = \frac{7}{12}\pi + k \cdot \pi$

$\frac{1}{4}\pi < x < \frac{7}{12}\pi \quad \vee \quad 1\frac{1}{4}\pi < x \leq 1\frac{1}{2}\pi$

e.  $\sin(2x - \frac{1}{3}\pi) = -\cos(x + \frac{1}{6}\pi)$

$\sin(2x - \frac{1}{3}\pi) = \sin(x - \frac{1}{3}\pi)$

$2x - \frac{1}{3}\pi = x - \frac{1}{3}\pi + k \cdot 2\pi \quad \vee \quad 2x - \frac{1}{3}\pi = \pi - (x - \frac{1}{3}\pi) + k \cdot 2\pi$

$x = 0 + k \cdot 2\pi \quad \vee \quad 2x - \frac{1}{3}\pi = \pi - x + \frac{1}{3}\pi + k \cdot 2\pi$

$3x = \frac{5}{3}\pi + k \cdot 2\pi$

$x = \frac{5}{9}\pi + k \cdot \frac{2}{3}\pi$

snijpunten voor  $x = 0 \quad \vee \quad x = \frac{5}{9}\pi \quad \vee \quad x = 1\frac{2}{9}\pi$

$f(x) < g(x)$  voor  $\frac{5}{9}\pi < x < 1\frac{2}{9}\pi$

**Opgave 12:**a. punt  $C$  is het snijpunt van  $AB$  met de  $x$ -as

$\sin \alpha = \frac{AC}{1} = AC$

$AB = 2 \cdot AC = 2 \sin \alpha$

b.  $4 \sin^2 \alpha = 1 + 1 - 2 \cdot 1 \cdot 1 \cdot \cos 2\alpha$ 

$2 \cos 2\alpha = 2 - 4 \sin^2 \alpha$

$\cos 2\alpha = 1 - 2 \sin^2 \alpha$

**Opgave 13:**

- a.  $\cos(t+u) = \cos(t-(-u))$   
 $= \cos t \cdot \cos(-u) + \sin t \cdot \sin(-u)$   
 $= \cos t \cdot \cos u + \sin t \cdot -\sin u$   
 $= \cos t \cdot \cos u - \sin t \cdot \sin u$
- b.  $\sin(t+u) = \cos(t+u-\frac{1}{2}\pi)$   
 $= \cos t \cdot \cos(u-\frac{1}{2}\pi) - \sin t \cdot \sin(u-\frac{1}{2}\pi)$   
 $= \cos t \cdot \sin u - \sin t \cdot -\cos u$   
 $= \cos t \cdot \sin u + \sin t \cdot \cos u$
- c.  $\sin(t-u) = \sin(t+(-u))$   
 $= \sin t \cdot \cos(-u) + \cos t \cdot \sin(-u)$   
 $= \sin t \cdot \cos u + \cos t \cdot -\sin u$   
 $= \sin t \cdot \cos u - \cos t \cdot \sin u$

**Opgave 14:**

- a.  $\sin 2A = \sin(A+A) = \sin A \cdot \cos A + \cos A \cdot \sin A = 2 \sin A \cdot \cos A$   
 $\cos 2A = \cos(A+A) = \cos A \cdot \cos A - \sin A \cdot \sin A = \cos^2 A - \sin^2 A$
- b.  $\cos 2A = \cos^2 A - \sin^2 A = \cos^2 A - (1 - \cos^2 A) = \cos^2 A - 1 + \cos^2 A = 2 \cos^2 A - 1$   
 $\cos 2A = \cos^2 A - \sin^2 A = 1 - \sin^2 A - \sin^2 A = 1 - 2 \sin^2 A$

**Opgave 15:**

- a.  $\cos 2A = 2 \cos^2 A - 1$   
 $-2 \cos^2 A = -\cos 2A - 1$   
 $\cos^2 A = \frac{1}{2} \cos 2A + \frac{1}{2}$
- b.  $\cos 2A = 1 - 2 \sin^2 A$   
 $2 \sin^2 A = 1 - \cos 2A$   
 $\sin^2 A = \frac{1}{2} - \frac{1}{2} \cos 2A$

**Opgave 16:**

- a.  $\sin x \cdot \cos x = \frac{1}{2} \sin(x-1)$   
 $2 \sin x \cdot \cos x = \sin(x-1)$   
 $\sin 2x = \sin(x-1)$   
 $2x = x-1 + k \cdot 2\pi \quad \vee \quad 2x = \pi - (x-1) + k \cdot 2\pi$   
 $x = -1 + k \cdot 2\pi \quad \vee \quad 2x = \pi - x + 1 + k \cdot 2\pi$   
 $3x = \pi + 1 + k \cdot 2\pi$   
 $x = \frac{1}{3}\pi + \frac{1}{3} + k \cdot \frac{2}{3}\pi$
- b.  $\cos^2 2x = \cos 4x + \frac{1}{2}$   
 $\frac{1}{2} + \frac{1}{2} \cos 4x = \cos 4x + \frac{1}{2}$   
 $-\frac{1}{2} \cos 4x = 0$   
 $\cos 4x = 0$   
 $4x = \frac{1}{2}\pi + k \cdot \pi$   
 $x = \frac{1}{8}\pi + k \cdot \frac{1}{4}\pi$
- c.  $\sin^2(\frac{1}{2}x) = \cos x + 1\frac{1}{4}$

$$\frac{1}{2} - \frac{1}{2} \cos x = \cos x + 1 \frac{1}{4}$$

$$-1 \frac{1}{2} \cos x = \frac{3}{4}$$

$$\cos x = -\frac{1}{2}$$

$$x = \frac{2}{3}\pi + k \cdot 2\pi \quad \vee \quad x = -\frac{2}{3}\pi + k \cdot 2\pi$$

d.  $(\sin x + \cos x)^2 = 1 \frac{1}{2}$

$$\sin^2 x + 2 \sin x \cos x + \cos^2 x = 1 \frac{1}{2}$$

$$1 + 2 \sin x \cos x = 1 \frac{1}{2}$$

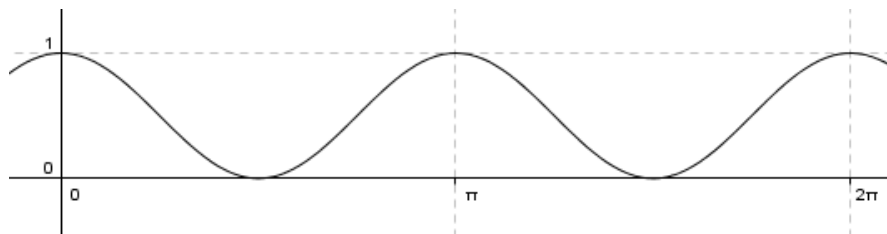
$$\sin 2x = \frac{1}{2}$$

$$2x = \frac{1}{6}\pi + k \cdot 2\pi \quad \vee \quad 2x = \frac{5}{6}\pi + k \cdot 2\pi$$

$$x = \frac{1}{12}\pi + k \cdot \pi \quad \vee \quad x = \frac{5}{12}\pi + k \cdot \pi$$

### Opgave 17:

a.



$$y = \frac{1}{2} + \frac{1}{2} \cos 2x$$

b.  $y = \sin^2 x + \cos 2x = \frac{1}{2} - \frac{1}{2} \cos 2x + \cos 2x = \frac{1}{2} + \frac{1}{2} \cos 2x$

### Opgave 18:

$$\sin 3x = \sin(2x + x)$$

$$= \sin 2x \cdot \cos x + \cos 2x \cdot \sin x$$

$$= 2 \sin x \cdot \cos x \cdot \cos x + (1 - 2 \sin^2 x) \cdot \sin x$$

$$= 2 \sin x \cdot \cos^2 x + \sin x - 2 \sin^3 x$$

$$= 2 \sin x \cdot (1 - \sin^2 x) + \sin x - 2 \sin^3 x$$

$$= 2 \sin x - 2 \sin^3 x + \sin x - 2 \sin^3 x$$

$$= 3 \sin x - 4 \sin^3 x$$

### Opgave 19:

a.  $y = 1 - \cos x - \sin^2(\frac{1}{2}x)$

$$= 1 - \cos x - (\frac{1}{2} - \frac{1}{2} \cos x)$$

$$= 1 - \cos x - \frac{1}{2} + \frac{1}{2} \cos x$$

$$= \frac{1}{2} - \frac{1}{2} \cos x$$

b.  $\cos 3x = \cos(2x + x)$

$$= \cos 2x \cdot \cos x - \sin 2x \cdot \sin x$$

$$= (2 \cos^2 x - 1) \cdot \cos x - 2 \sin x \cdot \cos x \cdot \sin x$$

$$= 2 \cos^3 x - \cos x - 2 \cos x \cdot \sin^2 x$$

$$= 2 \cos^3 x - \cos x - 2 \cos x \cdot (1 - \cos^2 x)$$

$$= 2 \cos^3 x - \cos x - 2 \cos x + 2 \cos^3 x$$

$$= 4 \cos^3 x - 3 \cos x$$

**Opgave 20:**

- a.  $-\cos A = \cos(A + \pi)$   
 b.  $\cos 2A = 1 - 2\sin^2 A$   
 c.  $\cos 2A = 1 - 2\sin^2 A$   
 d.  $\cos 2A = 2\cos^2 A - 1$   
 e.  $\cos 2A = 1 - 2\sin^2 A$

**Opgave 21:**

$B(-x_A, y_A)$  en  $C(-x_A, -y_A)$

**Opgave 22:**

- a.  $f(-p) + f(p) = -p \cdot \cos(-p) + p \cdot \cos p = -p \cdot \cos p + p \cdot \cos p = 0$   
 dus de grafiek van  $f$  is symmetrisch in de oorsprong  
 b.  $g(-p) = -p \cdot \sin(-p) = -p \cdot -\sin p = p \cdot \sin p = g(p)$   
 dus de grafiek van  $g$  is symmetrisch in de  $y$ -as.

**Opgave 23:**

- a.  $f(-p) + f(p) = \cos^2(-p) \cdot \sin(-p) + \cos^2 p \cdot \sin p$   
 $= \cos^2 p \cdot -\sin p + \cos^2 p \cdot \sin p = 0$   
 dus de grafiek van  $f$  is symmetrisch in de oorsprong  
 b.  $f(\frac{1}{2}\pi - p) = \cos^2(\frac{1}{2}\pi - p) \cdot \sin(\frac{1}{2}\pi - p)$   
 $= (-\cos(\frac{1}{2}\pi + p))^2 \cdot \sin(\frac{1}{2}\pi + p)$   
 $= \cos^2(\frac{1}{2}\pi + p) \cdot \sin(\frac{1}{2}\pi + p) = f(\frac{1}{2}\pi + p)$   
 dus de grafiek van  $f$  is symmetrisch in de lijn  $x = \frac{1}{2}\pi$ .

**Opgave 24:**

- a.  $\sin(-\frac{1}{4}\pi - p) = \sin(-\frac{1}{4}\pi) \cdot \cos(p) - \cos(-\frac{1}{4}\pi) \cdot \sin(p)$   
 $= -\frac{1}{2}\sqrt{2} \cdot \cos p - \frac{1}{2}\sqrt{2} \cdot \sin p$   
 $= -\frac{1}{2}\sqrt{2} \cdot \cos p - \frac{1}{2}\sqrt{2} \cdot \sin p$   
 $\cos(-\frac{1}{4}\pi - p) = \cos(-\frac{1}{4}\pi) \cdot \cos(p) + \sin(-\frac{1}{4}\pi) \cdot \sin(p)$   
 $= \frac{1}{2}\sqrt{2} \cdot \cos p + -\frac{1}{2}\sqrt{2} \cdot \sin p$   
 $= \frac{1}{2}\sqrt{2} \cdot \cos p - \frac{1}{2}\sqrt{2} \cdot \sin p$   
 $f(-\frac{1}{4}\pi - p) = 2\sin(-\frac{1}{4}\pi - p) - 2\cos(-\frac{1}{4}\pi - p)$   
 $= -\sqrt{2} \cdot \cos p - \sqrt{2} \cdot \sin p - (\sqrt{2} \cdot \cos p - \sqrt{2} \cdot \sin p)$   
 $= -\sqrt{2} \cdot \cos p - \sqrt{2} \cdot \sin p - \sqrt{2} \cdot \cos p + \sqrt{2} \cdot \sin p$   
 $= -2\sqrt{2} \cdot \cos p$   
 b.  $\sin(-\frac{1}{4}\pi + p) = \sin(-\frac{1}{4}\pi) \cdot \cos p + \cos(-\frac{1}{4}\pi) \cdot \sin p$   
 $= -\frac{1}{2}\sqrt{2} \cdot \cos p + \frac{1}{2}\sqrt{2} \cdot \sin p$   
 $\cos(-\frac{1}{4}\pi + p) = \cos(-\frac{1}{4}\pi) \cdot \cos p - \sin(-\frac{1}{4}\pi) \cdot \sin p$   
 $= \frac{1}{2}\sqrt{2} \cdot \cos p - -\frac{1}{2}\sqrt{2} \cdot \sin p$   
 $= \frac{1}{2}\sqrt{2} \cdot \cos p + \frac{1}{2}\sqrt{2} \cdot \sin p$

$$\begin{aligned}
f(-\tfrac{1}{4}\pi + p) &= 2\sin(-\tfrac{1}{4}\pi + p) - 2\cos(-\tfrac{1}{4}\pi + p) \\
&= -\sqrt{2} \cdot \cos p + \sqrt{2} \cdot \sin p - (\sqrt{2} \cdot \cos p + \sqrt{2} \cdot \sin p) \\
&= -\sqrt{2} \cdot \cos p + \sqrt{2} \cdot \sin p - \sqrt{2} \cdot \cos p - \sqrt{2} \cdot \sin p \\
&= -2\sqrt{2} \cdot \cos p
\end{aligned}$$

dus  $f(-\frac{1}{4}\pi - p) = f(-\frac{1}{4}\pi + p)$

dus de grafiek van  $f$  is symmetrisch in de lijn  $x = -\frac{1}{4}\pi$

**Opgave 25:**

a.  $\cos(\frac{1}{4}\pi - p) = \cos(\frac{1}{4}\pi) \cdot \cos p + \sin(\frac{1}{4}\pi) \cdot \sin p$   
 $= \frac{1}{2}\sqrt{2} \cdot \cos p + \frac{1}{2}\sqrt{2} \cdot \sin p$

$$\begin{aligned}
\sin(\tfrac{1}{4}\pi - p) &= \sin(\tfrac{1}{4}\pi) \cdot \cos p - \cos(\tfrac{1}{4}\pi) \cdot \sin p \\
&= \tfrac{1}{2}\sqrt{2} \cdot \cos p - \tfrac{1}{2}\sqrt{2} \cdot \sin p
\end{aligned}$$

$$\begin{aligned}
f(\tfrac{1}{4}\pi - p) &= \tfrac{1}{2}\sqrt{2} \cdot \cos p + \tfrac{1}{2}\sqrt{2} \cdot \sin p + \tfrac{1}{2}\sqrt{2} \cdot \cos p - \tfrac{1}{2}\sqrt{2} \cdot \sin p + 1 \\
&= \sqrt{2} \cdot \cos p + 1
\end{aligned}$$

$$\begin{aligned}
\cos(\tfrac{1}{4}\pi + p) &= \cos(\tfrac{1}{4}\pi) \cdot \cos p - \sin(\tfrac{1}{4}\pi) \cdot \sin p \\
&= \tfrac{1}{2}\sqrt{2} \cdot \cos p - \tfrac{1}{2}\sqrt{2} \cdot \sin p
\end{aligned}$$

$$\begin{aligned}
\sin(\tfrac{1}{4}\pi + p) &= \sin(\tfrac{1}{4}\pi) \cdot \cos p + \cos(\tfrac{1}{4}\pi) \cdot \sin p \\
&= \tfrac{1}{2}\sqrt{2} \cdot \cos p + \tfrac{1}{2}\sqrt{2} \cdot \sin p
\end{aligned}$$

$$\begin{aligned}
f(\tfrac{1}{4}\pi + p) &= \tfrac{1}{2}\sqrt{2} \cdot \cos p - \tfrac{1}{2}\sqrt{2} \cdot \sin p + \tfrac{1}{2}\sqrt{2} \cdot \cos p + \tfrac{1}{2}\sqrt{2} \cdot \sin p + 1 \\
&= \sqrt{2} \cdot \cos p + 1
\end{aligned}$$

dus  $f(\frac{1}{4}\pi - p) = f(\frac{1}{4}\pi + p)$

dus de grafiek van  $f$  is symmetrisch in de lijn  $x = \frac{1}{4}\pi$

b.  $\cos(\frac{3}{4}\pi - p) = \cos(\frac{3}{4}\pi) \cdot \cos p + \sin(\frac{3}{4}\pi) \cdot \sin p$   
 $= -\frac{1}{2}\sqrt{2} \cdot \cos p + \frac{1}{2}\sqrt{2} \cdot \sin p$

$$\begin{aligned}
\sin(\tfrac{3}{4}\pi - p) &= \sin(\tfrac{3}{4}\pi) \cdot \cos p - \cos(\tfrac{3}{4}\pi) \cdot \sin p \\
&= \tfrac{1}{2}\sqrt{2} \cdot \cos p - -\tfrac{1}{2}\sqrt{2} \cdot \sin p \\
&= \tfrac{1}{2}\sqrt{2} \cdot \cos p + \tfrac{1}{2}\sqrt{2} \cdot \sin p
\end{aligned}$$

$$\begin{aligned}
f(\tfrac{3}{4}\pi - p) &= -\tfrac{1}{2}\sqrt{2} \cos p + \tfrac{1}{2}\sqrt{2} \cdot \sin p + \tfrac{1}{2}\sqrt{2} \cdot \cos p + \tfrac{1}{2}\sqrt{2} \cdot \sin p + 1 \\
&= \sqrt{2} \cdot \sin p + 1
\end{aligned}$$

$$\begin{aligned}
\cos(\tfrac{3}{4}\pi + p) &= \cos(\tfrac{3}{4}\pi) \cdot \cos p - \sin(\tfrac{3}{4}\pi) \cdot \sin p \\
&= -\tfrac{1}{2}\sqrt{2} \cdot \cos p - \tfrac{1}{2}\sqrt{2} \cdot \sin p
\end{aligned}$$

$$\begin{aligned}
\sin(\tfrac{3}{4}\pi + p) &= \sin(\tfrac{3}{4}\pi) \cdot \cos p + \cos(\tfrac{3}{4}\pi) \cdot \sin p \\
&= \tfrac{1}{2}\sqrt{2} \cdot \cos p - \tfrac{1}{2}\sqrt{2} \cdot \sin p
\end{aligned}$$

$$\begin{aligned}
f(\tfrac{3}{4}\pi + p) &= -\tfrac{1}{2}\sqrt{2} \cdot \cos p - \tfrac{1}{2}\sqrt{2} \cdot \sin p + \tfrac{1}{2}\sqrt{2} \cdot \cos p - \tfrac{1}{2}\sqrt{2} \cdot \sin p + 1 \\
&= -\sqrt{2} \cdot \sin p + 1
\end{aligned}$$

$$f(\tfrac{3}{4}\pi - p) + f(\tfrac{3}{4}\pi + p) = \sqrt{2} \cdot \sin p + 1 + -\sqrt{2} \cdot \sin p + 1 = 2$$

dus de grafiek van  $f$  is puntsymmetrisch in het punt  $(\frac{3}{4}\pi, 1)$