11.3 Goniometrische functies primitiveren

<u>Opgave 42:</u>

a. $F(x) = -\frac{1}{3}\cos 3x$

$F'(x) = -\frac{1}{3} \cdot -\sin 3x \cdot 3 = \sin 3x = f(x)$

b. $G(x) = \frac{1}{5}\sin 5x$

$G'(x) = \frac{1}{5} \cdot \cos 5x \cdot 5 = \cos 5x = g(x)$

<u>Opgave 43:</u>

a. $f(x) = 4\sin\frac{1}{3}x$

$F(x) = 4 \cdot -3\cos\frac{1}{3}x + c = -12\cos\frac{1}{3}x + c$

b. $g(x) = x^2 - 5\cos 2x$

$G(x) = \frac{1}{3}x^3 - 5 \cdot \frac{1}{2}\sin 2x + c = \frac{1}{3}x^3 - 2\frac{1}{2}\sin 2x + c$

c. $h(x) = \sin(2x + \frac{1}{3}\pi)$

$H(x) = -\frac{1}{2}\cos(2x + \frac{1}{3}\pi) + c$

d. $j(x) = 3\cos(\frac{1}{2}x - \frac{1}{6}\pi)$

$J(x) = 3 \cdot 2\sin(\frac{1}{2}x - \frac{1}{6}\pi) + c = 6\sin(\frac{1}{2}x - \frac{1}{6}\pi) + c$

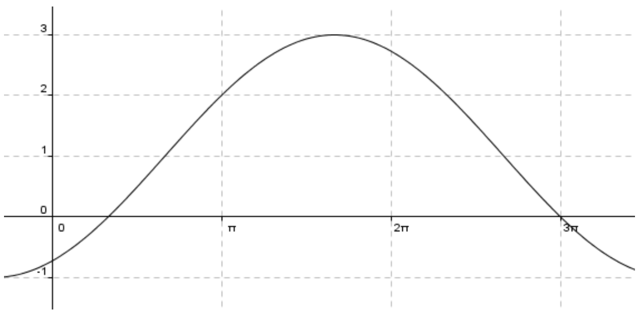
<u>Opgave 44:</u>

a. $\int_0^{\frac{1}{3}\pi} (2x + \cos\frac{1}{2}x)dx = \left[x^2 + 2\sin\frac{1}{2}x\right]_0^{\frac{1}{3}\pi} = \frac{1}{9}\pi^2 + 1 - (0+0) = \frac{1}{9}\pi^2 + 1$

b. $\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} (x^2 - 2\sin(x - \frac{1}{6}\pi))dx = \left[\frac{1}{3}x^3 + 2\cos(x - \frac{1}{6}\pi)\right]_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} =$

$$\frac{1}{81}\pi^3 + \sqrt{3} - (\frac{1}{648}\pi^3 + 2) = \frac{7}{648}\pi^3 + \sqrt{3} - 2$$

<u>Opgave 45:</u>



$1 + 2\cos(\frac{1}{2}x - \frac{5}{6}\pi) = 0$

$2\cos(\frac{1}{2}x - \frac{5}{6}\pi) = -1$

$\cos(\frac{1}{2}x - \frac{5}{6}\pi) = -\frac{1}{2}$

$\frac{1}{2}x - \frac{5}{6}\pi = \frac{2}{3}\pi \quad \vee \quad \frac{1}{2}x - \frac{5}{6}\pi = 1\frac{1}{3}\pi$

$\frac{1}{2}x = 1\frac{1}{2}\pi \quad \vee \quad \frac{1}{2}x = 2\frac{1}{6}\pi = \frac{1}{6}\pi$

$x = 3\pi \quad \vee \quad x = \frac{1}{3}\pi$

$OppV = \int_{\frac{1}{3}\pi}^{3\pi} (1 + 2\cos(\frac{1}{2}x - \frac{5}{6}\pi))dx = \left[x + 4\sin(\frac{1}{2}x - \frac{5}{6}\pi)\right]_{\frac{1}{3}\pi}^{3\pi} =$

$$3\pi + 2\sqrt{3} - (\frac{1}{3}\pi - 2\sqrt{3}) = 2\frac{2}{3}\pi + 4\sqrt{3}$$

<u>Opgave 46:</u>

a. nee, want $y' = \sin^2 x \cdot \cos x$

b. $\cos 2A = 1 - 2\sin^2 A$

$2\sin^2 A = 1 - \cos 2A$

$\sin^2 A = \frac{1}{2} - \frac{1}{2}\cos 2A$

$\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$

c. $f(x) = \sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$

$F(x) = \frac{1}{2}x - \frac{1}{4}\sin 2x + c$

Opgave 47:

a. $\cos 2A = 2\cos^2 A - 1$

$1 + \cos 2A = 2\cos^2 A$

$\frac{1}{2} + \frac{1}{2}\cos 2A = \cos^2 A$

$f(x) = \cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$

$F(x) = \frac{1}{2}x + \frac{1}{4}\sin 2x + c$

b. $\sin^2 A = \frac{1}{2} - \frac{1}{2}\cos 2A$

$g(x) = \sin^2 3x = \frac{1}{2} - \frac{1}{2}\cos 6x$

$G(x) = \frac{1}{2}x - \frac{1}{12}\sin 6x + c$

c. $\sin 2A = 2\sin A\cos A$

$\frac{1}{2}\sin 2a = \sin A\cos A$

$h(x) = \sin\frac{1}{2}x \cdot \cos\frac{1}{2}x = \frac{1}{2}\sin x$

$H(x) = -\frac{1}{2}\cos x + c$

Opgave 48:

a. $f(x) = \tan^2 x = 1 + \tan^2 x - 1$

$F(x) = \tan x - x + c$

b. $g(x) = x + \tan^2 x = x + 1 + \tan^2 x - 1$

$G(x) = \frac{1}{2}x^2 + \tan x - x + c$

Opgave 49:

a. $\int_0^{\frac{1}{6}\pi} \sin 2x\cos 2x dx = \int_{00}^{\frac{1}{6}\pi} \frac{1}{2}\sin 4x dx = \left[-\frac{1}{8}\cos 4x\right]_0^{\frac{1}{6}\pi} = -\frac{1}{8}\cdot -\frac{1}{2} - -\frac{1}{8}\cdot 1 = \frac{1}{16} + \frac{1}{8} = \frac{3}{16}$

b. $\sin^2 A = \frac{1}{2} - \frac{1}{2}\cos 2A$

$2 - \frac{1}{2}\sin^2 x = 2 - \frac{1}{2}\cdot(\frac{1}{2} - \frac{1}{2}\cos 2x) = 2 - \frac{1}{4} + \frac{1}{4}\cos 2x = 1\frac{3}{4} + \frac{1}{4}\cos 2x$

$\int_{\frac{1}{3}\pi}^{\pi} (2 - \frac{1}{2}\sin^2 x)dx = \int_{\frac{1}{3}\pi}^{\pi} (1\frac{3}{4} + \frac{1}{4}\cos 2x)dx = \left[1\frac{3}{4}x + \frac{1}{8}\sin 2x\right]_{\frac{1}{3}\pi}^{\pi} =$

$1\frac{3}{4}\pi + 0 - (\frac{7}{12}\pi + \frac{1}{8}\cdot\frac{1}{2}\sqrt{3}) = 1\frac{1}{6}\pi - \frac{1}{16}\sqrt{3}$

Opgave 50:

$Inh = \pi \cdot \int_0^{\frac{1}{2}\pi} \sin^2 2x dx = \pi \cdot \int_0^{\frac{1}{2}\pi} (\frac{1}{2} - \frac{1}{2}\cos 4x)dx = \pi \cdot \left[\frac{1}{2}x - \frac{1}{8}\sin 4x\right]_0^{\frac{1}{2}\pi} = \pi \cdot (\frac{1}{4}\pi - 0 - (0 - 0)) = \frac{1}{4}\pi^2$

Opgave 51:

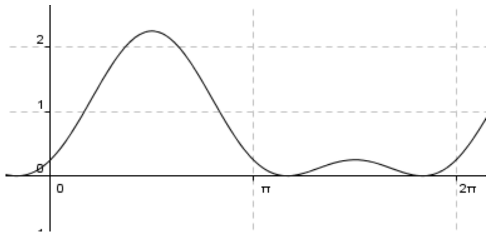
a. $f(x) = 2\sin^2 x + \sin x - 1$

$f'(x) = 4\sin x \cdot \cos x + \cos x = 0$
$\cos x \cdot (4\sin x + 1) = 0$
$\cos x = 0 \quad \vee \quad 4\sin x = -1$
$x = \frac{1}{2}\pi \quad \vee \quad x = 1\frac{1}{2}\pi \quad \vee \quad \sin x = -\frac{1}{4}$
$y = 2 \qquad y = 0 \qquad y = 2\cdot(-\frac{1}{4})^2 - \frac{1}{4} - 1 = -1{,}125$
$B_f = [-1{,}125, 2]$

b. $2\sin^2 x + \sin x - 1 = 0$
$\sin^2 x + \frac{1}{2}\sin x - \frac{1}{2} = 0$
stel $p = \sin x$ dan $p^2 + \frac{1}{2}p - \frac{1}{2} = 0$
$(p+1)(p - \frac{1}{2}) = 0$
$p = -1 \quad \vee \quad p = \frac{1}{2}$
$\sin x = -1 \quad \vee \quad \sin x = \frac{1}{2}$
$x = 1\frac{1}{2}\pi \quad \vee \quad x = \frac{1}{6}\pi \quad \vee \quad x = \frac{5}{6}\pi$
$$OppV = \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} (2\sin^2 x + \sin x - 1)dx = \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} (\sin x - \cos 2x)dx =$$
$$\left[-\cos x - \frac{1}{2}\sin 2x\right]_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} = \frac{1}{2}\sqrt{3} + \frac{1}{4}\sqrt{3} - (-\frac{1}{2}\sqrt{3} - \frac{1}{4}\sqrt{3}) = 1\frac{1}{2}\sqrt{3}$$

Opgave 52:

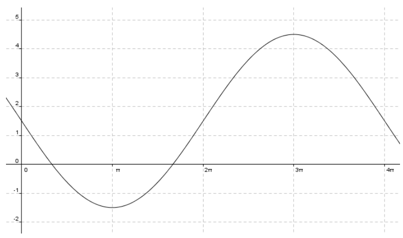
$\sin^2 x + \sin x + \frac{1}{4} = 0$
$(\sin x + \frac{1}{2})^2 = 0$
$\sin x = -\frac{1}{2}$
$x = 1\frac{1}{6}\pi \quad \vee \quad x = 1\frac{5}{6}\pi$



$$OppV = \int_{1\frac{1}{6}\pi}^{1\frac{5}{6}\pi} (\sin^2 x + \sin x + \frac{1}{4})dx = \int_{1\frac{1}{6}\pi}^{1\frac{5}{6}\pi} (\frac{1}{2} - \frac{1}{2}\cos 2x + \sin x + \frac{1}{4})dx =$$
$$\left[\frac{3}{4}x - \frac{1}{4}\sin 2x - \cos x\right]_{1\frac{1}{6}\pi}^{1\frac{5}{6}\pi} = 1\frac{3}{8}\pi + \frac{1}{8}\sqrt{3} - \frac{1}{2}\sqrt{3} - (\frac{7}{8}\pi - \frac{1}{8}\sqrt{3} + \frac{1}{2}\sqrt{3}) = \frac{1}{2}\pi - \frac{3}{4}\sqrt{3}$$

Opgave 53:

a.



ev.as: $y = 1\frac{1}{2}$
amp: 3
per= $\frac{2\pi}{\frac{1}{2}} = 4\pi$
beginpunt: $x = 0$ omlaag

b. $1\frac{1}{2} - 3\sin\frac{1}{2}x = 0$
$-3\sin\frac{1}{2}x = -1\frac{1}{2}$
$\sin\frac{1}{2}x = \frac{1}{2}$
$\frac{1}{2}x = \frac{1}{6}\pi \quad \vee \quad \frac{1}{2}x = \frac{5}{6}\pi$
$x = \frac{1}{3}\pi \quad \vee \quad x = \frac{5}{3}\pi$

$$OppV = -\int_{\frac{1}{3}\pi}^{\frac{5}{3}\pi} (1\tfrac{1}{2} - 3\sin\tfrac{1}{2}x)dx = -\left[1\tfrac{1}{2}x + 6\cos\tfrac{1}{2}x\right]_{\frac{1}{3}\pi}^{\frac{5}{3}\pi} = -(2\tfrac{1}{2}\pi - 3\sqrt{3} - (\tfrac{1}{2}\pi + 3\sqrt{3})) =$$

$$= -(2\pi - 6\sqrt{3}) = 6\sqrt{3} - 2\pi$$

c. $$Inh = \pi \cdot \int_{\frac{1}{3}\pi}^{\frac{5}{3}\pi} (1\tfrac{1}{2} - 3\sin\tfrac{1}{2}x)^2 dx = \pi \cdot \int_{\frac{1}{3}\pi}^{\frac{5}{3}\pi} (2\tfrac{1}{4} - 9\sin\tfrac{1}{2}x + 9\sin^2\tfrac{1}{2}x)dx$$

$$= \pi \cdot \int_{\frac{1}{3}\pi}^{\frac{5}{3}\pi} (2\tfrac{1}{4} - 9\sin\tfrac{1}{2}x + 9\cdot(\tfrac{1}{2} - \tfrac{1}{2}\cos x))dx = \pi \cdot \int_{\frac{1}{3}\pi}^{\frac{5}{3}\pi} (6\tfrac{3}{4} - 9\sin\tfrac{1}{2}x - 4\tfrac{1}{2}\cos x)dx$$

$$= \pi \cdot \left[6\tfrac{3}{4}x + 18\cos\tfrac{1}{2}x - 4\tfrac{1}{2}\sin x\right]_{\frac{1}{3}\pi}^{\frac{5}{3}\pi} =$$

$$= \pi \cdot (11\tfrac{1}{4}\pi - 9\sqrt{3} + 2\tfrac{1}{4}\sqrt{3} - (2\tfrac{1}{4}\pi + 9\sqrt{3} - 2\tfrac{1}{4}\sqrt{3}))$$

$$= \pi \cdot (9\pi - 13\tfrac{1}{2}\sqrt{3})$$

$$= 9\pi^2 - 13\tfrac{1}{2}\pi\sqrt{3}$$