

11.4 Eenparige cirkelbewegingen

Opgave 54:

a. periode=5 dus $c = \frac{2\pi}{per} = \frac{2\pi}{5}$

$$y_p = \sin\left(\frac{2\pi}{5}t\right)$$

b. II

Opgave 55:

$rc = -1$ dus $AB = BM$

$amp = 4$ dus $AM = 4$

$$AM^2 = AB^2 + BM^2 = AB^2 + AB^2 = 2AB^2 = 4^2 = 16$$

$$AB^2 = 8$$

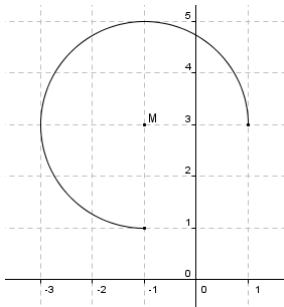
$$AB = \sqrt{8} = 2\sqrt{2}$$

$$x_A = 2 - AB = 2 - 2\sqrt{2}$$

$$y_A = 1 + BM = 1 + 2\sqrt{2}$$

Opgave 56:

a.



b. $x_p = -1 + 2\cos t = 0$

$$2\cos t = 1$$

$$\cos t = \frac{1}{2}$$

$$t = \frac{1}{3}\pi$$

$$y_p = 3 + 2\sin\frac{1}{3}\pi = 3 + 2 \cdot \frac{1}{2}\sqrt{3} = 3 + \sqrt{3}$$

c. $amp = 2$

$$2x^2 = 4$$

$$x^2 = 2$$

$$x = \sqrt{2}$$

$$B(-1 + \sqrt{2}, 3 + \sqrt{2}) \text{ en } C(-1 - \sqrt{2}, 3 - \sqrt{2})$$

d. $x = -1 + 2\cos t = -2$

$$2\cos t = -1$$

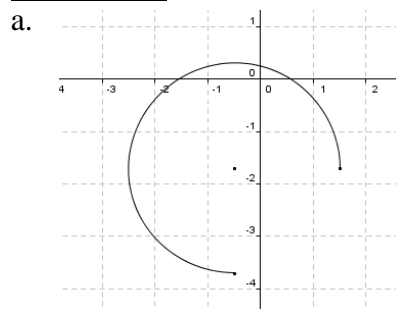
$$\cos t = -\frac{1}{2}$$

$$t = \frac{2}{3}\pi \quad \vee \quad t = \frac{4}{3}\pi$$

$$\text{dus } 2,09 < t < 4,19$$

Opgave 57:

- a.
$$\begin{cases} x_p = 5 + 3 \cos 2t \\ y_p = 2 + 3 \sin 2t \end{cases}$$
- b. $y_p = 2 + 3 \sin 2t = 0$
 $3 \sin 2t = -2$
 $\sin 2t = -\frac{2}{3}$
 $2t = -0,73 = 5,55 \quad \vee \quad 2t = \pi - -0,73 = 3,87$
 $t = 2,78 \quad \vee \quad t = 1,94$
dus $2,78 - 1,94 = 0,84$

Opgave 58:

- b. $y = -\sqrt{3} + 2 \sin 2t = 0$
 $2 \sin 2t = \sqrt{3}$
 $\sin 2t = \frac{1}{2} \sqrt{3}$
 $2t = \frac{1}{3} \pi \quad \vee \quad 2t = \frac{2}{3} \pi$
 $t = \frac{1}{6} \pi \quad \vee \quad t = \frac{1}{3} \pi$
vervalt $(-1\frac{1}{2}, 0)$
- c. $-\frac{1}{2} + 2 \cos 2t = -1\frac{1}{2}$
 $2 \cos 2t = -1$
 $\cos 2t = -\frac{1}{2}$
 $2t = \frac{2}{3} \pi \quad \vee \quad 2t = \frac{4}{3} \pi$
 $t = \frac{1}{3} \pi \quad \vee \quad t = \frac{2}{3} \pi$
 $\frac{1}{3} \pi < t < \frac{2}{3} \pi$
- d. $y_1 = -\sqrt{3} + 2 \sin 2x$ en $y_2 = -2$
intersect geeft $x = 1,64$
dus $1,64 < t \leq \frac{3}{4} \pi$

Opgave 59:

- a. $2 \sin t = 2 \cos t + 1$
 $y_1 = 2 \sin x$ en $y_2 = 2 \cos x + 1$
intersect geeft $x = 1,147 \quad \vee \quad x = 3,566$
 $t = 1,147$ geeft $(0,82; 1,82)$
 $t = 3,566$ geeft $(-1,82; -0,82)$

b. $2 \cos t = 1$

$$\cos t = \frac{1}{2}$$

$$t = \frac{1}{3}\pi \quad \vee \quad t = -\frac{1}{3}\pi$$

$$\frac{\frac{2}{3}\pi}{2\pi} = \frac{1}{3} \text{ dus je hebt } \frac{1}{3} \text{ deel van een cirkel met } r = 2$$

$$L = \frac{1}{3} \cdot 2\pi \cdot 2 = \frac{4}{3}\pi$$

c. $PQ = \sqrt{(x_P - x_Q)^2 + (y_P - y_Q)^2}$

$$= \sqrt{(2 \cos t - \cos 2t)^2 + (2 \sin t - \sin 2t)^2}$$

$$= \sqrt{4 \cos^2 t - 4 \cos t \cdot \cos 2t + \cos^2 2t + 4 \sin^2 t - 4 \sin t \cdot \sin 2t + 4 \sin^2 2t}$$

$$= \sqrt{4 + 1 - 4 \cos t \cdot \cos 2t - 4 \sin t \cdot \sin 2t}$$

$$= \sqrt{5 - 4 \cos t \cdot \cos 2t - 4 \sin t \cdot 2 \sin t \cos t}$$

$$= \sqrt{5 - 4 \cos t \cdot (\cos 2t + 2 \sin^2 t)}$$

$$= \sqrt{5 - 4 \cos t \cdot (2 \cos^2 t - 1 + 2 \sin^2 t)}$$

$$= \sqrt{5 - 4 \cos t \cdot (2 - 1)}$$

$$= \sqrt{5 - 4 \cos t}$$

d. $\sqrt{5 - 4 \cos t} > 1\frac{1}{2}$

$$y_1 = \sqrt{5 - 4 \cos x} \text{ en } y_2 = 1\frac{1}{2}$$

$$\text{intersect geeft } x = 0,813 \quad \vee \quad x = 5,470$$

$$\text{dus } 5,470 - 0,813 = 4,66 \text{ sec}$$

Opgave 60:

a. $T(2,0)$

b. $\begin{cases} x_Q = 3 \cos(\frac{1}{2}(t-2)) \\ y_Q = 3 \sin(\frac{1}{2}(t-2)) \end{cases}$

Opgave 61:

a. $\begin{cases} x_P = 4 + 3 \sin 2\frac{1}{2}t \\ y_P = -2 - 3 \cos 2\frac{1}{2}t \end{cases}$

b. $P(1,12; -2,85)$

c. $per = \frac{2\pi}{2\frac{1}{2}} = \frac{4}{5}\pi$

$$\text{dus na } \frac{3}{4} \text{ van de periode, dus } \frac{3}{4} \cdot \frac{4}{5}\pi = \frac{3}{5}\pi$$

d. $y = -2 - 3 \cos 2\frac{1}{2}t = 0$

$$y_1 = -2 - 3 \cos 2\frac{1}{2}x \text{ de optie zero geeft } x = 0,92 \quad \vee \quad x = 1,59$$

$$(6,24; 0) \text{ en } (1,76; 0)$$

Opgave 62:

a. $T = \frac{1}{2}$ dus $\omega = \frac{2\pi}{\frac{1}{2}} = 4\pi$

- $$\begin{cases} x_P = 15 + 6 \cos(4\pi(t - \frac{1}{10})) \\ y_P = 23 + 6 \sin(4\pi(t - \frac{1}{10})) \end{cases}$$
- b. $\begin{cases} x_Q = 15 + 6 \cos(4\pi(t + \frac{1}{10})) \\ y_Q = 23 + 6 \sin(4\pi(t + \frac{1}{10})) \end{cases}$
- c. $x_R = 15 + 6 \cos(4\pi(t - \frac{1}{4}))$
 punt R is in het punt $(21,23)$ op $t = \frac{1}{4}$
 dus punt R loopt $\frac{1}{4} - \frac{1}{10} = \frac{3}{20}$ achter op punt P , ofwel $\frac{1}{2} - \frac{3}{20} = \frac{7}{20}$ voor op punt P

Opgave 63:

- a. $\begin{cases} x_P = -1 + 3 \cos 4t \\ y_P = 2 + 3 \sin 4t \end{cases}$
 $per = \frac{2\pi}{4} = \frac{1}{2}\pi$
 $\begin{cases} x_Q = -1 + 3 \cos(4(t - \frac{1}{6}\pi)) \\ y_Q = 2 + 3 \sin(4(t - \frac{1}{6}\pi)) \end{cases}$
- b. $\begin{cases} x_P = -1 - 3 \sin 4t \\ y_P = 2 + 3 \cos 4t \end{cases}$
 $\begin{cases} x_Q = -1 - 3 \sin(4(t + \frac{1}{12}\pi)) \\ y_Q = 2 + 3 \cos(4(t + \frac{1}{12}\pi)) \end{cases}$
- c. $\begin{cases} x_P = -1 - 3 \cos 4t \\ y_P = 2 - 3 \sin 4t \end{cases}$
 $\begin{cases} x_Q = -1 - 3 \cos(4(t - \frac{1}{8}\pi)) \\ y_Q = 2 - 3 \sin(4(t - \frac{1}{8}\pi)) \end{cases}$

Opgave 64:

- a. $\frac{1}{3}\pi t + \frac{2}{3}\pi = \frac{1}{3}\pi(t + 2)$
 $per = \frac{2\pi}{\frac{1}{3}\pi} = 6$
 dus $\frac{2}{6} = \frac{1}{3}$
- b. $\frac{1}{3}\pi t - \frac{1}{2}\pi = \frac{1}{3}\pi(t - 1\frac{1}{2})$
 $\frac{1\frac{1}{2}}{6} = \frac{1}{4}$
- c. de fasevoorsprong van Q op R is: $\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$
 dus het faseverschil is: $1 - \frac{7}{12} = \frac{5}{12}$

Opgave 65:

- a. $per = \frac{1}{15}$ dus $\omega = \frac{2\pi}{\frac{1}{15}} = 30\pi$
 $\begin{cases} x_P = 20 \cos(30\pi t) \\ y_P = 20 \sin(30\pi t) \end{cases}$

$$\begin{cases} x_Q = 20 \cos(30\pi(t - \frac{1}{45})) \\ y_Q = 20 \sin(30\pi(t - \frac{1}{45})) \\ x_R = 20 \cos(30\pi(t + \frac{1}{45})) \\ y_R = 20 \sin(30\pi(t + \frac{1}{45})) \end{cases}$$

b. $108 \frac{\text{km}}{\text{uur}} = 30 \frac{\text{m}}{\text{s}}$
 $\text{omtrek} = 2\pi \cdot 0,2 = 0,4\pi \text{ m}$

$$\text{per} = \frac{0,4\pi}{30} = \frac{1}{75}\pi$$

$$\omega = \frac{2\pi}{\frac{1}{75}\pi} = 150$$

$$\begin{cases} x_P = 20 \cos 150t \\ y_P = 20 \sin 150t \end{cases}$$

Opgave 66:

a. de periode van punt P is 2 sec, dus $\omega = \frac{2\pi}{2} = \pi$

$$\begin{cases} x_P = 10 \cos(\pi t) \\ y_P = -10 \sin(\pi t) \end{cases}$$

de periode van punt Q is 1 sec, dus $\omega = \frac{2\pi}{1} = 2\pi$

$$\begin{cases} x_Q = 15 - 5 \cos(2\pi t) \\ y_Q = -5 \sin(2\pi t) \end{cases}$$

b. rol II heeft een periode van 1 sec, dus in 1 sec gaat er $2\pi r = 2\pi \cdot 5 = 10\pi$ cm papier doorheen
dus per uur: $3600 \cdot 10\pi = 113097 \text{ cm} = 1131 \text{ m}$

Opgave 67:

a.
$$\begin{cases} x_P = -2 + 4 \cos(\pi t) \\ y_P = 1 - 4 \sin(\pi t) \end{cases}$$

b. $P(-5,24; 3,35)$

c. $\text{per} = \frac{2\pi}{\pi} = 2$

dus op $t = \frac{1}{2} \vee t = 2\frac{1}{2} \vee t = 4\frac{1}{2}$

d. $x = -2 + 4 \cos(\pi t) = 0$

$$4 \cos(\pi t) = 2$$

$$\cos(\pi t) = \frac{1}{2}$$

$$\pi t = \frac{1}{3}\pi \vee \pi t = -\frac{1}{3}\pi$$

$$t = \frac{1}{3} \vee t = -\frac{1}{3}$$

$$(0, 1 - 2\sqrt{3}) \vee (0, 1 + 2\sqrt{3})$$

e. $y_1 = 1 - 4 \sin(\pi x)$

de optie zero geeft: $t = 0,0804 \vee t = 0,9196$

$$0,9196 - 0,0804 = 0,839$$

$$L = \frac{0,839}{2} \cdot 2\pi \cdot 4 = 10,54$$

Opgave 68:

a. $\omega = \frac{2\pi}{30} = \frac{1}{15}\pi$

$$\text{amp} = \frac{135}{2} = 67,5$$

$$\begin{cases} x_F = -67,5 \sin\left(\frac{1}{15}\pi t\right) \\ y_F = 67,5 + 67,5 \cos\left(\frac{1}{15}\pi t\right) \end{cases}$$

b. Saskia heeft een faseachterstand van $\frac{4}{32} = \frac{1}{8}$ dus $\frac{1}{8} \cdot 30 = 3\frac{3}{4}$

$$\begin{cases} x_S = -67,5 \sin\left(\frac{1}{15}\pi(t - 3\frac{3}{4})\right) \\ y_S = 67,5 + 67,5 \cos\left(\frac{1}{15}\pi(t - 3\frac{3}{4})\right) \end{cases}$$

c. $\text{omtrek} = 2\pi \cdot 67,5 = 135\pi$ m

dus in 1 uur: $2 \cdot 135\pi = 848$ m = 0,85 km

d. $y_1 = 67,5 + 67,5 \cos\frac{1}{15}\pi x$ en $y_2 = 100$

intersect geeft: $t = 5,101$ \vee $t = -5,101$

$$\Delta t = 2 \cdot 5,101 = 10,203$$
 min = 612 sec