9.5 Diagnostische toets

Opgave 1:

a. $^{3}\log 5+2\cdot{}^{3}\log 2={}^{3}\log 5+{}^{3}\log 2^{2}={}^{3}\log 5+{}^{3}\log 4={}^{3}\log 20$

b. $3-{}^{2}\log 5={}^{2}\log 8-{}^{2}\log 5={}^{2}\log\frac{8}{5}$

c. $^{2}\log 8000+3\cdot{}^{2}\log\frac{1}{5}={}^{2}\log 8000+{}^{2}\log(\frac{1}{5})^{3}={}^{2}\log 8000+{}^{2}\log\frac{1}{125}={}^{2}\log 64=6$

Opgave 2:

a. $2\cdot{}^{2}\log(x-1)=1+{}^{2}\log 18$

$^{2}\log(x-1)^{2}={}^{2}\log 2+{}^{2}\log 18$

$^{2}\log(x-1)^{2}={}^{2}\log 36$

$(x-1)^{2}=36$

$x-1=6 \quad \vee \quad x-1=-6$

$x=7 \quad \vee \quad x=-5$ (vervalt)

b. $^{2}\log x=3-{}^{2}\log(x+2)$

$^{2}\log x={}^{2}\log 8-{}^{2}\log(x+2)$

$^{2}\log x={}^{2}\log\frac{8}{x+2}$

$x=\frac{8}{x+2}$

$x(x+2)=8$

$x^{2}+2x-8=0$

$(x+4)(x-2)=0$

$x=-4$ (vervalt) $\quad \vee \quad x=2$

Opgave 3:

a. $^{2}\log x-{}^{\frac{1}{2}}\log(x-1)=3$

$^{2}\log x-\frac{^{2}\log(x-1)}{^{2}\log\frac{1}{2}}=3$

$^{2}\log x+{}^{2}\log(x-1)=3$

$^{2}\log x(x-1)={}^{2}\log 8$

$x(x-1)=8$

$x^{2}-x-8=0$

$x=\frac{1\pm\sqrt{33}}{2}$

$x=\frac{1}{2}+\frac{1}{2}\sqrt{33} \quad \vee \quad x=\frac{1}{2}-\frac{1}{2}\sqrt{33}$ (vervalt)

b. $\log^{2} x-5\cdot\log x=6$

neem $p=\log x$ dan $p^{2}-5p=6$

$p^{2}-5p-6=0$

$(p-6)(p+1)=0$

$p=6 \quad \vee \quad p=-1$

$\log x = 6 \quad \vee \quad \log x = -1$
$x = 10^6 \quad \vee \quad x = 10^{-1} = \frac{1}{10}$

Opgave 4:

a. $3^x + 6 \cdot (\frac{1}{3})^x = 5$

$3^x + 6 \cdot \frac{1}{3^x} - 5 = 0$

neem $p = 3^x$ dan $p + 6 \cdot \frac{1}{p} - 5 = 0$

$p^2 + 6 - 5p = 0$
$(p-2)(p-3) = 0$
$p = 2 \quad \vee \quad p = 3$
$3^x = 2 \quad \vee \quad 3^x = 3$
$x = {}^3\log 2 \quad \vee \quad x = 1$

b. $9^x = 3^x + 12$
$(3^x)^2 - 3^x - 12 = 0$
neem $p = 3^x$ dan $p^2 - p - 12 = 0$
$(p-4)(p+3) = 0$
$p = 4 \quad \vee \quad p = -3$
$3^x = 4 \quad \vee \quad 3^x = -3$
$x = {}^3\log 4$ k.n.
dus $x = {}^3\log 4$

c. $9^x = 3^{x+1} + 4$
$(3^x)^2 = 3 \cdot 3^x + 4$
neem $p = 3^x$ dan $p^2 = 3p + 4$
$p^2 - 3p - 4 = 0$
$(p-4)(p+1) = 0$
$p = 4 \quad \vee \quad p = -1$
$3^x = 4 \quad \vee \quad 3^x = -1$
$x = {}^3\log 4$ k.n.
dus $x = {}^3\log 4$

d. $3^{x+2} + 3^{2x+1} = 12$
$3^2 \cdot 3^x + 3 \cdot 3^{2x} = 12$
$9 \cdot 3^x + 3 \cdot (3^x)^2 = 12$
neem $p = 3^x$ dan $9p + 3p^2 = 12$
$3p^2 + 9p - 12 = 0$
$p^2 + 3p - 4 = 0$
$(p+4)(p-1) = 0$
$p = -4 \quad \vee \quad p = 1$
$3^x = -4 \quad \vee \quad 3^x = 1$
k.n. $x = 0$

Opgave 5:

a. $y = 3^x \xrightarrow{V_{x-as,\frac{1}{3}}} y = \frac{1}{3} \cdot 3^x = 3^{-1} \cdot 3^x = 3^{x-1}$

dus $T(1,0)$

b. $y = {}^3\log x \xrightarrow{T(0,-2)} y = -2 + {}^3\log x = {}^3\log \frac{1}{9} + {}^3\log x = {}^3\log \frac{1}{9} x$

dus $V_{y-as,9}$

Opgave 6:

a. $f(x) - g(x) = 2 \quad \vee \quad g(x) - f(x) = 2$

$3^{x-1} - 4 - (2 - 3^x) = 2 \quad \vee \quad 2 - 3^x - (3^{x-1} - 4) = 2$

$3^{x-1} - 4 - 2 + 3^x = 2 \quad \vee \quad 2 - 3^x - 3^{x-1} + 4 = 2$

$3^{-1} \cdot 3^x + 3^x = 8 \quad \vee \quad -3^x - 3^{x-1} = -4$

$\frac{1}{3} \cdot 3^x + 3^x = 8 \quad \vee \quad 3^x + 3^{x-1} = 4$

$\frac{4}{3} \cdot 3^x = 8 \quad \vee \quad 3^x + 3^{-1} \cdot 3^x = 4$

$3^x = 6 \quad \vee \quad 3^x + \frac{1}{3} \cdot 3^x = 4$

$x = {}^3\log 6 \quad \vee \quad \frac{4}{3} \cdot 3^x = 4$

$3^x = 3$

$x = 1$

dus $p = {}^3\log 6 \quad \vee \quad p = 1$

b. $f(x) = g(x+1) \quad \vee \quad g(x) = f(x+1)$

$3^{x-1} - 4 = 2 - 3^{x+1} \quad \vee \quad 2 - 3^x = 3^x - 4$

$3^{x-1} + 3^{x+1} = 6 \quad \vee \quad -2 \cdot 3^x = -6$

$3^{-1} \cdot 3^x + 3 \cdot 3^x = 6 \quad \vee \quad 3^x = 3$

$\frac{1}{3} \cdot 3^x + 3 \cdot 3^x = 6 \quad \vee \quad x = 1$

$3\frac{1}{3} \cdot 3^x = 6 \qquad y = 2 - 3 = -1$

$3^x = 1\frac{4}{5}$

$x = {}^3\log 1\frac{4}{5}$

$y = 3^{{}^3\log 1\frac{4}{5} - 1} - 4 = 3^{{}^3\log 1\frac{4}{5}} \cdot 3^{-1} - 4 = 1\frac{4}{5} \cdot \frac{1}{3} - 4 = -3\frac{2}{5}$

$q = -3\frac{2}{5} \quad \vee \quad q = -1$

Opgave 7:

a. $x_B = p$ dan $x_C = 3p$

$g(x_B) = f(x_C)$

${}^2\log 4p = {}^2\log(3p + 3)$

$4p = 3p + 3$

$p = 3$

$q = {}^2\log 12$

b. $f(p) = 2 \cdot g(p)$

${}^2\log(p + 3) = 2 \cdot {}^2\log 4p$

${}^2\log(p + 3) = {}^2\log(4p)^2$

$p+3=(4p)^2$

$p+3=16p^2$

$16p^2-p-3=0$

$p=\frac{1\pm\sqrt{193}}{32}$

$p=\frac{1+\sqrt{193}}{32}=0{,}47 \quad \vee \quad p=\frac{1-\sqrt{193}}{32}=-0{,}40$ (vervalt)

Opgave 8:

a. $\frac{3e^3-e^3}{e^2}=\frac{2e^3}{e^2}=2e$

b. $(e^{3x}-5)^2=e^{6x}-10e^{3x}+25$

Opgave 9:

a. $3xe^x-e^x=0$

$e^x(3x-1)=0$

$e^x=0 \quad \vee \quad 3x=1$

k.n. $\quad x=\frac{1}{3}$

b. $e^{2x-1}-\sqrt[3]{e^2}=0$

$e^{2x-1}=\sqrt[3]{e^2}$

$e^{2x-1}=e^{\frac{2}{3}}$

$2x-1=\frac{2}{3}$

$2x=\frac{5}{3}$

$x=\frac{5}{6}$

c. $e^{4x}-e^{x+1}=0$

$e^{4x}=e^{x+1}$

$4x=x+1$

$3x=1$

$x=\frac{1}{3}$

d. $e^{2x}+2e^x=3$

$(e^x)^2+2e^x-3=0$

neem $p=e^x$ dan $p^2+2p-3=0$

$(p+3)(p-1)=0$

$p=-3 \quad \vee \quad p=1$

$e^x=-3 \quad \vee \quad e^x=1$

k.n. $\quad x=0$

Opgave 10:

a. $f(x)=2e^x-3x^2$

$f'(x)=2e^x-6x$

b. $f(x)=\frac{x^2+1}{e^x}$

$f'(x)=\frac{e^x\cdot 2x-(x^2+1)\cdot e^x}{(e^x)^2}=\frac{2x-(x^2+1)}{e^x}=\frac{-x^2+2x-1}{e^x}$

c. $f(x)=(x^2+1)e^x$

$f'(x)=2x\cdot e^x+(x^2+1)e^x=(x^2+2x+1)e^x$

d. $f(x)=\frac{e^x}{x^2+1}$

$f'(x)=\frac{(x^2+1)e^x-e^x\cdot 2x}{(x^2+1)^2}=\frac{x^2e^x-2xe^x+e^x}{(x^2+1)^2}=\frac{(x^2-2x+1)e^x}{(x^2+1)^2}$

e. $f(x)=x^2e^{2x-1}$

$f'(x)=2xe^{2x-1}+x^2e^{2x-1}\cdot 2=(2x^2+2x)e^{2x-1}$

f. $f(x)=e^{x^2+9}$

$f'(x)=e^{x^2+9}\cdot 2x=2xe^{x^2+9}$

Opgave 11:

a. $f(x)=\frac{e^x}{x}$

$f'(x)=\frac{x\cdot e^x-e^x\cdot 1}{x^2}=\frac{(x-1)e^x}{x^2}=0$

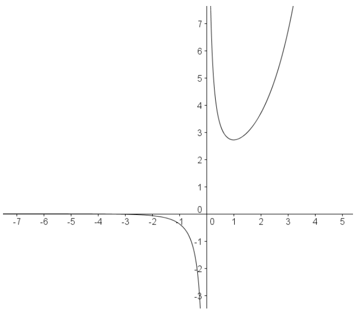
$(x-1)e^x=0$

$x-1=0 \quad \vee \quad e^x=0$

$x=1$ k.n.

$y=e$

min $f(1)=e$



b. $x_A=2 \quad y_A=\frac{1}{2}e^2$

$f'(2)=\frac{1}{4}e^2$

$y=\frac{1}{4}e^2\cdot x+b$ door $(2,\frac{1}{2}e^2)$

$\frac{1}{2}e^2=\frac{1}{2}e^2+b$

$b=0$

$l:\quad y=\frac{1}{2}e^2\cdot x$

Opgave 12:

a. $\ln(e^3\cdot\sqrt{e})=\ln e^{3\frac{1}{2}}=3\frac{1}{2}$

b. $\ln\frac{1}{e^2}=\ln e^{-2}=-2$

Opgave 13:

a. $4+\ln 3=\ln e^4+\ln 3=\ln 3e^4$

b. $\ln 10-4\ln 2=\ln 10-\ln 2^4=\ln 10-\ln 16=\ln\frac{10}{16}=\ln\frac{5}{8}$

Opgave 14:

a. $2\ln 5x = 16$

$\ln 5x = 8$

$5x = e^8$

$x = \frac{1}{5}e^8$

b. $\ln^2 5x = 16$

$\ln 5x = 4 \quad \vee \quad \ln 5x = -4$

$5x = e^4 \quad \vee \quad 5x = e^{-4} = \frac{1}{e^4}$

$x = \frac{1}{5}e^4 \quad \vee \quad x = \frac{1}{5e^4}$

c. $2\ln^2 x - \ln x = 0$

$\ln x \cdot (2\ln x - 1) = 0$

$\ln x = 0 \quad \vee \quad 2\ln x = 1$

$x = 1 \quad \vee \quad \ln x = \frac{1}{2}$

$x = 1 \quad \vee \quad x = e^{\frac{1}{2}} = \sqrt{e}$

d. $\ln(9x+1) - \ln(x+2) = \ln 4$

$\ln \dfrac{9x+1}{x+2} = \ln 4$

$\dfrac{9x+1}{x+2} = 4$

$9x + 1 = 4(x+2)$

$9x + 1 = 4x + 8$

$5x = 7$

$x = 1\frac{2}{5}$

Opgave 15:

a. $f(x) = 2^{3x-4}$

$f'(x) = 2^{3x-4} \cdot \ln 2 \cdot 3 = 3\ln 2 \cdot 2^{3x-4}$

b. $f(x) = x \cdot 3^x$

$f'(x) = 1 \cdot 3^x + x \cdot 3^x \cdot \ln 3 = (1 + x\ln 3) \cdot 3^x$

c. $f(x) = \ln(x \cdot \sqrt[3]{x}) = \ln x^{\frac{4}{3}} = \frac{4}{3}\ln x$

$f'(x) = \dfrac{4}{3} \cdot \dfrac{1}{x} = \dfrac{4}{3x}$

d. $f(x) = {}^2\log 4x$

$f'(x) = \dfrac{1}{4x} \cdot 4 \cdot \dfrac{1}{\ln 2} = \dfrac{1}{x\ln 2}$

e. $f(x) = {}^3\log(5x-6)$

$f'(x) = \dfrac{1}{5x-6} \cdot 5 \cdot \dfrac{1}{\ln 3} = \dfrac{5}{(5x-6) \cdot \ln 3}$

f. $f(x) = \ln(3x^2 + 3)$

$f'(x) = \dfrac{1}{3x^2+3} \cdot 6x = \dfrac{6x}{3x^2+3} = \dfrac{2x}{x^2+1}$

Opgave 16:

a. $f(x)=3^{x-1}+3^{-x+1}$

$f'(x)=3^{x-1}\cdot\ln 3+3^{-x+1}\cdot -1\cdot\ln 3$

$=(3^{x-1}-3^{-x+1})\cdot\ln 3=0$

$3^{x-1}-3^{-x+1}=0$

$3^{x-1}=3^{-x+1}$

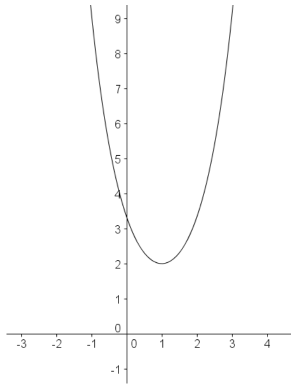
$x-1=-x+1$

$2x=2$

$x=1$

$y=3^0+3^0=2$

$B_f=[2,\rightarrow\rangle$



b. $f'(x)=(3^{x-1}-3^{-x+1})\cdot\ln 3=\frac{8}{3}\ln 3$

$3^{x-1}-3^{-x+1}=\frac{8}{3}$

$3^{-1}\cdot 3^x-3^1\cdot 3^{-x}=\frac{8}{3}$

$\frac{1}{3}\cdot 3^x-3\cdot\frac{1}{3^x}=\frac{8}{3}$

neem $p=3^x$ dan $\frac{1}{3}p-3\cdot\frac{1}{p}=\frac{8}{3}$

$p-9\cdot\frac{1}{p}=8$

$p^2-9=8p$

$p^2-8p-9=0$

$(p-9)(p+1)=0$

$p=9 \quad\vee\quad p=-1$

$3^x=9 \quad\vee\quad 3^x=-1$

$x=2$ k.n.

$y=3+3^{-1}=3+\frac{1}{3}=3\frac{1}{3}$

dus $(2,3\frac{1}{3})$

Opgave 17:

a. $f(x)=\frac{\ln x}{x}=0$

$\ln x=0$

$x=1$

$f'(x)=\frac{x\cdot\frac{1}{x}-\ln x\cdot 1}{x^2}=\frac{1-\ln x}{x^2}$

$f'(1)=1$

$y=x+b$ door $(1,0)$

$0=1+b$

$b=-1$

$y=x-1$

b. $f'(x) = \dfrac{1 - \ln x}{x^2} = 0$

$1 - \ln x = 0$

$\ln x = 1$

$x = e$

$y = \frac{1}{e}$

$B_f = \langle \leftarrow, \frac{1}{e}]$