## 9.2 Grafieken van exponentiële en logaritmische functies

**Opgave 20:**

a. $T(-3,0)$

b. $f(x) = 2^{x+3} = 2^3 \cdot 2^x = 8 \cdot 2^x$

**Opgave 21:**

a. $V_{y-as,\frac{1}{8}}$

b. $f(x) = {}^2\log 8x = {}^2\log 8 + {}^2\log x = 3 + {}^2\log x$

**Opgave 22:**

a. $y = 2^x \xrightarrow{T(5,0)} y = 2^{x-5}$

$y = 2^{x-5} = 2^{-5} \cdot 2^x = \frac{1}{32} \cdot 2^x$ dus $V_{x-as,\frac{1}{32}}$

b. $y = 4^x \xrightarrow{V_{x-as,2}} y = 2 \cdot 4^x$

$y = 2 \cdot 4^x = 4^{\frac{1}{2}} \cdot 4^x = 4^{x+\frac{1}{2}}$ dus $T(-\frac{1}{2},0)$

c. $y = {}^2\log x \xrightarrow{V_{y-as,\frac{1}{32}}} y = {}^2\log 32x$

$y = {}^2\log 32x = {}^2\log 32 + {}^2\log x = 5 + {}^2\log x$ dus $T(0,5)$

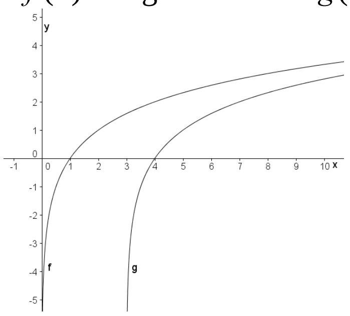
d. $y = {}^4\log x \xrightarrow{T(0,\frac{1}{2})} y = \frac{1}{2} + {}^4\log x$

$y = \frac{1}{2} + {}^4\log x = {}^4\log 4^{\frac{1}{2}} + {}^4\log x = {}^4\log 2 + {}^4\log x = {}^4\log 2x$ dus $V_{y-as,\frac{1}{2}}$

**Opgave 23:**

a. $f(x) = {}^2\log x \xrightarrow{T(3,0)} g(x) = {}^2\log(x-3)$

b.




c. Nee, de verticale asymptoot van de grafiek van *f* is de lijn $x = 0$ dus dit zou na vermenigvuldiging ook de verticale asymptoot van de grafiek van *g* moeten zijn, maar dat is de lijn $x = 3$. Dus er bestaat geen vermenigvuldiging ten opzichte van de *y*-as. Er is ook geen verticale translatie mogelijk want voor $x = 1$ bestaat *f* wel maar *g* niet.

d. $g(x) = {}^2\log(x-3) \xrightarrow{V_{y-as,\frac{1}{4}}} h(x) = {}^2\log(4x-3)$

$h(x) = {}^2\log(4x-3) = {}^2\log 4(x-\frac{3}{4}) = {}^2\log 4 + {}^2\log(x-\frac{3}{4}) = 2 + {}^2\log(x-\frac{3}{4})$

dus $p = -\frac{3}{4}$ en $q = 2$

**Opgave 24:**

a. $V_{x-as,4}$

b. $g(x) = 4 \cdot (\frac{1}{2})^x = 2^2 \cdot (\frac{1}{2})^x = ((2^{-1})^{-1})^2 \cdot (\frac{1}{2})^x = (\frac{1}{2})^{-2} \cdot (\frac{1}{2})^x = (\frac{1}{2})^{x-2}$ dus $T(2,0)$

c. $h(x)=4^x=(2^2)^x=2^{2x}=((\frac{1}{2})^{-1})^{2x}=(\frac{1}{2})^{-2x}$ dus $V_{y-as,-\frac{1}{2}}$

d. $h(x)=4^x=(2^2)^x=2^{2x}=((\frac{1}{2})^{-1})^{2x}=(\frac{1}{2})^{-2x}$ dus $V_{x-as,\frac{1}{4}}$ en $V_{y-as,-\frac{1}{2}}$

e. $g(x)=4\cdot(\frac{1}{2})^x \xrightarrow{T(3,4)} j(x)=4+4\cdot(\frac{1}{2})^{x-3}$

$j(x)=4+4\cdot(\frac{1}{2})^{x-3}=4+4\cdot(\frac{1}{2})^{-3}\cdot(\frac{1}{2})^x=4+4\cdot 8\cdot(\frac{1}{2})^x=4+32\cdot(\frac{1}{2})^x$

dus $a=32$ en $b=4$

**Opgave 25:**

$AB=g(1)-f(1)=6-\frac{1}{2}=5\frac{1}{2}$

**Opgave 26:**

a. $y=g(^2\log 6\frac{2}{5})=8-2^{^2\log 6\frac{2}{5}}=8-6\frac{2}{5}=1\frac{3}{5}$

b. de twee horizontale asymptoten liggen op een afstand 8 van elkaar, dus alleen $f(x)-g(x)=10$ heeft een oplossing

c. $0<a<8$

**Opgave 27:**

a. $3^{x-1}-2=4-3^x$

$3^{x-1}+3^x=6$

$3^{-1}\cdot 3^x+3^x=6$

$\frac{1}{3}\cdot 3^x+3^x=6$

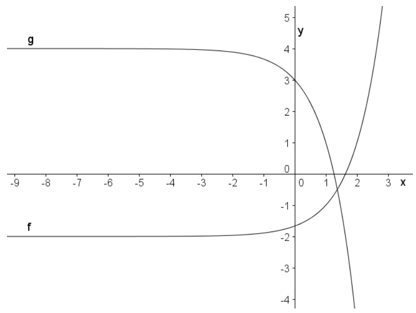
$\frac{4}{3}\cdot 3^x=6$

$3^x=4\frac{1}{2}$

$x={}^3\log 4\frac{1}{2}$

$y=-\frac{1}{2}$

dus $A=({}^3\log 4\frac{1}{2},-\frac{1}{2})$




b. $f(x)-g(x)=6$

$3^{x-1}-2-(4-3^x)=6$

$3^{x-1}-2-4+3^x=6$

$3^{x-1}+3^x=12$

$3^{-1}\cdot 3^x+3^x=12$

$\frac{1}{3}\cdot 3^x+3^x=12$

$\frac{4}{3}\cdot 3^x=12$

$3^x=9$

$x=2$ dus $p=2$

**Opgave 28:**

a. $1-3x>0$

$-3x>-1$

$x<\frac{1}{3}$ dus $D_f=\langle\leftarrow,\frac{1}{3}\rangle$

$x+5>0$

$x>-5$ dus $D_g=\langle -5,\rightarrow\rangle$

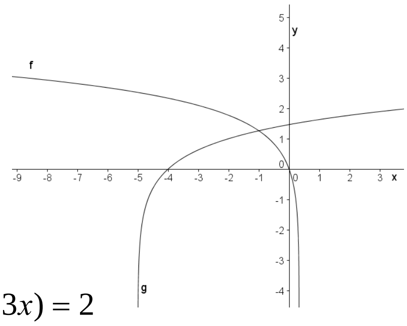
b. $^3\log(1-3x) \leq {}^3\log(x+5)$

$1-3x \leq x+5$

$-4x \leq 4$

$x \geq -1$

dus $-1 \leq x < \frac{1}{3}$




c. $f(x)-g(x)=2 \quad \vee \quad g(x)-f(x)=2$

$^3\log(1-3x) - {}^3\log(x+5) = 2 \quad \vee \quad {}^3\log(x+5) - {}^3\log(1-3x) = 2$

$^3\log\frac{1-3x}{x+5} = {}^3\log 9 \quad \vee \quad {}^3\log\frac{x+5}{1-3x} = {}^3\log 9$

$\frac{1-3x}{x+5} = 9 \quad \vee \quad \frac{x+5}{1-3x} = 9$

$9(x+5) = 1-3x \quad \vee \quad x+5 = 9(1-3x)$

$9x+45 = 1-3x \quad \vee \quad x+5 = 9-27x$

$12x = -44 \quad \vee \quad 28x = 4$

$x = -3\frac{2}{3} \quad \vee \quad x = \frac{1}{7}$

dus $p = -3\frac{2}{3} \quad \vee \quad p = \frac{1}{7}$

**Opgave 29:**

a. $(\frac{3}{2})^{x+2} = 3 \cdot (\frac{2}{3})^x + 3$

$(\frac{3}{2})^{x+2} = 3 \cdot ((\frac{3}{2})^{-1})^x + 3$

$(\frac{3}{2})^{x+2} = 3 \cdot (\frac{3}{2})^{-x} + 3$

$(\frac{3}{2})^2 \cdot (\frac{3}{2})^x = 3 \cdot \frac{1}{(\frac{3}{2})^x} + 3$

stel $p = (\frac{3}{2})^x$ dan $\frac{9}{4}p = 3 \cdot \frac{1}{p} + 3$

$\frac{9}{4}p^2 = 3 + 3p$

$\frac{9}{4}p^2 - 3p - 3 = 0$

$9p^2 - 12p - 12 = 0$

$p = \frac{12 \pm \sqrt{576}}{18} = \frac{12 \pm 24}{18}$

$p = 2 \quad \vee \quad p = -\frac{2}{3}$

$(\frac{3}{2})^x = 2 \quad \vee \quad (\frac{3}{2})^x = -\frac{2}{3}$

$x = {}^{\frac{3}{2}}\log 2$ k.n.

$y = 4\frac{1}{2}$

b. $f(x)-g(x)=4 \quad \vee \quad g(x)-f(x)=4$

$y_1 = (\frac{3}{2})^{x+2} - (3 \cdot (\frac{2}{3})^x + 3)$ $\qquad$ $y_1 = 3 \cdot (\frac{2}{3})^x + 3 - (\frac{3}{2})^{x+2}$

$y_2 = 4$ $\qquad$ $y_2 = 4$

intersect geeft $x = 3{,}085$ $\qquad$ intersect geeft $x = -0{,}117$

dus $p = 3{,}085 \quad \vee \quad p = -0{,}117$

**Opgave 30:**

a. $x_B = x_A + AB = p + 6$

b. $A$ en $B$ liggen op een horizontale lijn dus $y_A = y_B$

$f(x_A) = g(x_B)$

$f(p) = g(p+6)$

c. $q = y_A = f(p)$

**Opgave 31:**

$f(x) = g(x+1\frac{1}{8}) \quad \vee \quad g(x) = f(x+1\frac{1}{8})$

${}^{\frac{1}{2}}\log 2x = 2 + {}^{\frac{1}{2}}\log(x+1\frac{1}{8}+2) \quad \vee \quad 2 + {}^{\frac{1}{2}}\log(x+2) = {}^{\frac{1}{2}}\log 2(x+1\frac{1}{8})$

${}^{\frac{1}{2}}\log 2x = {}^{\frac{1}{2}}\log \frac{1}{4} + {}^{\frac{1}{2}}\log(x+3\frac{1}{8}) \quad \vee \quad {}^{\frac{1}{2}}\log \frac{1}{4} + {}^{\frac{1}{2}}\log(x+2) = {}^{\frac{1}{2}}\log 2(x+1\frac{1}{8})$

${}^{\frac{1}{2}}\log 2x = {}^{\frac{1}{2}}\log \frac{1}{4}(x+3\frac{1}{8}) \quad \vee \quad {}^{\frac{1}{2}}\log \frac{1}{4}(x+2) = {}^{\frac{1}{2}}\log 2(x+1\frac{1}{8})$

$2x = \frac{1}{4}(x+3\frac{1}{8}) \quad \vee \quad \frac{1}{4}(x+2) = 2(x+1\frac{1}{8})$

$8x = x + 3\frac{1}{8} \quad \vee \quad x+2 = 8(x+1\frac{1}{8})$

$7x = 3\frac{1}{8} \quad \vee \quad x+2 = 8x+9$

$x = \frac{25}{56} \quad \vee \quad -7x = 7$

$y = {}^{\frac{1}{2}}\log \frac{25}{28} \quad \vee \quad x = -1$

$y = 2$

dus $q = {}^{\frac{1}{2}}\log \frac{25}{28} \quad \vee \quad q = 2$

**Opgave 32:**

a. de twee verticale asymptoten hebben een afstand van 2, dus alleen rechts van het snijpunt is er een horizontaal lijnstuk met lengte 2

b. $0 < a < 2$

**Opgave 33:**

$f(x) = g(x+2) \quad \vee \quad g(x) = f(x+2)$

$2^{x-2} = 8 - 2^{x+2} \quad \vee \quad 8 - 2^x = 2^{x+2-2}$

$2^{x-2} + 2^{x+2} = 8 \quad \vee \quad 8 = 2^x + 2^x$

$2^{-2} \cdot 2^x + 2^2 \cdot 2^x = 8 \quad \vee \quad 8 = 2 \cdot 2^x$

$\frac{1}{4} \cdot 2^x + 4 \cdot 2^x = 8 \quad \vee \quad 4 = 2^x$

$4\frac{1}{4} \cdot 2^x = 8 \quad \vee \quad x = 2$

$2^x = \frac{32}{17} \qquad y = 4$

$x = {}^{2}\log \frac{32}{17}$

$y = \frac{8}{17}$

dus $q = \frac{8}{17} \quad \vee \quad q = 4$

**Opgave 34:**

a. ${}^{4}\log(x^2 - 1) \leq {}^{2}\log(x+3)$ voorwaarde: $(x < -1 \quad \vee \quad x > 1) \quad \wedge \quad x > -3$

$$\frac{^2\log(x^2-1)}{^2\log 4} = {}^2\log(x+3) \qquad D_f = \langle\leftarrow, -1\rangle \text{ en } \langle 1, \rightarrow\rangle$$

$$\frac{^2\log(x^2-1)}{2} = {}^2\log(x+3) \qquad D_g = \langle -3, \rightarrow\rangle$$

$^2\log(x^2-1) = 2\cdot{}^2\log(x+3)$

$^2\log(x^2-1) = {}^2\log(x+3)^2$

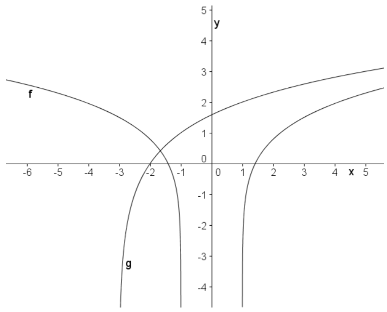
$x^2-1=(x+3)^2$

$x^2-1=x^2+6x+9$

$-6x=10$

$x=-\frac{5}{3}$

$-\frac{5}{3}\le x<-1 \quad \vee \quad x>1$




b. $f(x)-g(x)=\frac{1}{2} \quad \vee \quad g(x)-f(x)=\frac{1}{2}$

$^4\log(x^2-1)-{}^2\log(x+3)=\frac{1}{2} \quad \vee \quad {}^2\log(x+3)-{}^4\log(x^2-1)=\frac{1}{2}$

$\frac{1}{2}\cdot{}^2\log(x^2-1)-{}^2\log(x+3)=\frac{1}{2} \quad \vee \quad {}^2\log(x+3)-\frac{1}{2}\cdot{}^2\log(x^2-1)=\frac{1}{2}$

$^2\log(x^2-1)-2\cdot{}^2\log(x+3)=1 \quad \vee \quad 2\cdot{}^2\log(x+3)-{}^2\log(x^2-1)=1$

$^2\log(x^2-1)-{}^2\log(x+3)^2=1 \quad \vee \quad {}^2\log(x+3)^2-{}^2\log(x^2-1)=1$

$^2\log\frac{x^2-1}{(x+3)^2}={}^2\log 2 \quad \vee \quad {}^2\log\frac{(x+3)^2}{x^2-1}={}^2\log 2$

$\frac{x^2-1}{(x+3)^2}=2 \quad \vee \quad \frac{(x+3)^2}{x^2-1}=2$

$2(x+3)^2=x^2-1 \quad \vee \quad 2(x^2-1)=(x+3)^2$

$2x^2+12x+18=x^2-1 \quad \vee \quad 2x^2-2=x^2+6x+9$

$x^2+12x+19=0 \quad \vee \quad x^2-6x-11=0$

$x=\frac{-12\pm\sqrt{68}}{2}=-6\pm\sqrt{17} \quad \vee \quad x=\frac{6\pm\sqrt{80}}{2}=3\pm2\sqrt{5}$

$x=-6-\sqrt{17}$ (vervalt) $\vee \quad x=-6+\sqrt{17} \quad \vee \quad x=3-2\sqrt{5} \quad \vee \quad x=3+2\sqrt{5}$

dus $p=-6+\sqrt{17} \quad \vee \quad p=3-2\sqrt{5} \quad \vee \quad p=3+2\sqrt{5}$

c. $f(x)=g(x+1) \quad \vee \quad g(x)=f(x+1)$

$^4\log(x^2-1)={}^2\log(x+1+3) \quad \vee \quad {}^2\log(x+3)={}^4\log((x+1)^2-1)$

$\frac{1}{2}\cdot{}^2\log(x^2-1)={}^2\log(x+4) \quad \vee \quad {}^2\log(x+3)=\frac{1}{2}\cdot{}^2\log(x^2+2x)$

$^2\log(x^2-1)^{\frac{1}{2}}={}^2\log(x+4) \quad \vee \quad {}^2\log(x+3)={}^2\log(x^2+2x)^{\frac{1}{2}}$

$(x^2-1)^{\frac{1}{2}}=x+4 \quad \vee \quad x+3=(x^2+2x)^{\frac{1}{2}}$

$x^2-1=(x+4)^2 \quad \vee \quad (x+3)^2=x^2+2x$

$x^2-1=x^2+8x+16 \quad \vee \quad x^2+6x+9=x^2+2x$

$-8x=17 \quad \vee \quad 4x=-9$

$x=-\frac{17}{8} \quad \vee \quad x=-2\frac{1}{4}$

$y={}^2\log\frac{15}{8} \qquad y={}^2\log\frac{3}{4}$

$y={}^2\log 15-{}^2\log 8={}^2\log 15-3 \qquad y={}^2\log 3-{}^2\log 4={}^2\log 3-2$

$y=-3+{}^2\log 15 \qquad y=-2+{}^2\log 3$

**Opgave 35:**

a. $AB = p$

$AB : BC = 1 : 2 = p : 2p$

dus $BC = 2p$

$AC = AB + BC = p + 2p = 3p$

b. $y_B = 2^p$

$y_C = g(3p) = 2^{3p-3}$

$y_B = y_C$ dus $2^p = 2^{3p-3}$

$p = 3p - 3$

$-2p = -3$

$p = 1\frac{1}{2}$

c. $q = y_B = 2^p = 2^{1\frac{1}{2}} = 2\sqrt{2}$

**Opgave 36:**

$x_B = p$ dan $x_C = 2p$

$f(p) = f(2p)$

$6p \cdot 2^{-p} = 6 \cdot 2p \cdot 2^{-2p}$

$6p \cdot 2^{-p} = 12p \cdot 2^{-2p}$

$6p = 0 \quad \vee \quad 2^{-p} = 2 \cdot 2^{-2p}$

$p = 0 \quad \vee \quad 2^{-p} = 2^{1-2p}$

$-p = 1 - 2p$

$p = 1$

$q = f(1) = 6 \cdot 2^{-1} = 6 \cdot \frac{1}{2} = 3$

**Opgave 37:**

a. $x_B = p$ dan $x_C = 3p$

$y_B = y_C$

$f(p) = g(3p)$

${}^2\log p = {}^2\log(3p - 3)$

$p = 3p - 3$

$-2p = -3$

$p = 1\frac{1}{2}$

$q = f(1\frac{1}{2}) = {}^2\log 1\frac{1}{2}$

b. $y_F = 2 \cdot y_E$

$f(r) = 2 \cdot g(r)$

${}^2\log r = 2 \cdot {}^2\log(r - 3)$

${}^2\log r = {}^2\log(r - 3)^2$

$r = (r - 3)^2$

$r = r^2 - 6r + 9$

$r^2 - 7r + 9 = 0$

$r = \dfrac{7 \pm \sqrt{13}}{2}$

$r = \dfrac{7 - \sqrt{13}}{2} = 1{,}697$ (vervalt) $\quad \vee \quad r = \dfrac{7 + \sqrt{13}}{2} = 5{,}303$

dus $r = 5{,}303$

**<u>Opgave 38:</u>**

$x_B = p$ dan $x_C = 3p$

$y_B = y_C$

$f(p) = f(3p)$

$8p \cdot (\frac{1}{3})^p = 8 \cdot 3p \cdot (\frac{1}{3})^{3p}$

$8p \cdot (\frac{1}{3})^p = 24p \cdot (\frac{1}{3})^{3p}$

$8p = 0 \quad \vee \quad (\frac{1}{3})^p = 3 \cdot (\frac{1}{3})^{3p}$

$p = 0 \quad \vee \quad (\frac{1}{3})^p = (\frac{1}{3})^{-1} \cdot (\frac{1}{3})^{3p}$

k.n. $\quad \vee \quad (\frac{1}{3})^p = (\frac{1}{3})^{3p-1}$

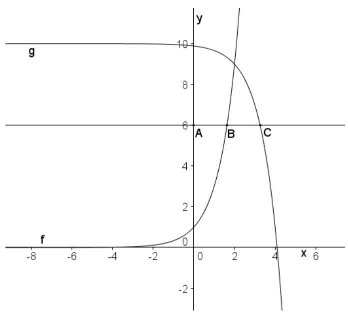
$p = 3p - 1$

$-2p = -1$

$p = \frac{1}{2}$

$y = f(\frac{1}{2}) = 4 \cdot (\frac{1}{3})^{\frac{1}{2}} = 4 \cdot \sqrt{\frac{1}{3}} = 4 \cdot \frac{1}{3}\sqrt{3} = \frac{4}{3}\sqrt{3}$

**<u>Opgave 39:</u>**




a. $x_B = p$ dan $x_C = 2p$

$f(p) = g(2p)$

$3^p = 10 - 3^{2p-2}$

$3^p + 3^{2p-2} = 10$

$3^{-2} \cdot 3^{2p} + 3^p - 10 = 0$

$\frac{1}{9} \cdot (3^p)^2 + 3^p - 10 = 0$

neem $r = 3^p$ dan $\frac{1}{9}r^2 + r - 10 = 0$

$r^2 + 9r - 90 = 0$

$(r + 15)(r - 6) = 0$

$r = -15 \quad \vee \quad r = 6$

$3^p = -15 \quad \vee \quad 3^p = 6$

k.n. $\quad p = {}^3\log 6$

$q = f({}^3\log 6) = 3^{{}^3\log 6} = 6$

b. $y_F = 2 \cdot y_E$

$g(r) = 2 \cdot f(r)$

$10 - 3^{r-2} = 2 \cdot 3^r$

$3^{r-2} + 2 \cdot 3^r - 10 = 0$

$3^{-2} \cdot 3^r + 2 \cdot 3^r - 10 = 0$

$\frac{1}{9} \cdot 3^r + 2 \cdot 3^r - 10 = 0$

$2\frac{1}{9} \cdot 3^r = 10$

$3^r = 4\frac{14}{19}$

$r = {}^3\log 4\frac{14}{19}$