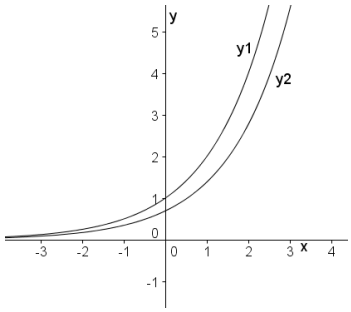
9.3 Het grondtal *e*

Opgave 40:

a.



b. $c = 0{,}6931$

c. $c = 1{,}0986$

Opgave 41:

a. $f'(x) = \lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0}\dfrac{2^{x+h}-2^x}{h} = \lim\limits_{h\to 0}\dfrac{2^x\cdot 2^h-2^x}{h} = \lim\limits_{h\to 0}\dfrac{2^x\cdot(2^h-1)}{h}$

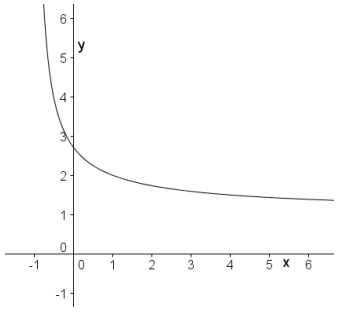
$= \lim\limits_{h\to 0}\dfrac{2^h-1}{h}\cdot 2^x$

b. $f'(0) = \lim\limits_{h\to 0}\dfrac{2^h-1}{h}\cdot 2^0 = \lim\limits_{h\to 0}\dfrac{2^h-1}{h}\cdot 1 = \lim\limits_{h\to 0}\dfrac{2^h-1}{h}$

c. $f'(x) = \lim\limits_{h\to 0}\dfrac{2^h-1}{h}\cdot 2^x = f'(0)\cdot 2^x$

Opgave 42:

a.



b. $\dfrac{1}{x}$ bestaat niet voor $x = 0$

c. $x = 0{,}01 \qquad y_1 = 2{,}7048$

$x = 0{,}001 \qquad y_1 = 2{,}7169$

$x = 0{,}0001 \qquad y_1 = 2{,}7181$

$x = 0{,}00001 \qquad y_1 = 2{,}7181$

d. $a = 2{,}718$

Opgave 43:

a. $2e^2 - e^2 = e^2$

b. $4\sqrt{e} - \sqrt{e} = 3\sqrt{e}$

c. $5e^2 \cdot 3e^3 = 15e^5$

d. $\dfrac{12e^6}{4e^2} = 3e^4$

e. $e^{5x} \cdot e^x = e^{6x}$

f. $e^x \cdot e^2 = e^{x+2}$

g. $5e^x - 3e^x = 2e^x$

h. $e^x \cdot (e^2 + 1) = e^{x+2} + e^x$

i. $e^x \cdot (e^x + 1) = e^{2x} + e^x$

j. $(e^x + 1)^2 = e^{2x} + 2e^x + 1$

k. $(e^{3x} + 3)^2 = e^{6x} + 6e^{3x} + 9$

l. $\dfrac{6e^{2x} - e^x}{e^x} = \dfrac{6e^{2x}}{e^x} - \dfrac{e^x}{e^x} = 6e^x - 1$

Opgave 44:

a. $(2 + 3e^{\frac{1}{2}x})^2 = 4 + 12e^{\frac{1}{2}x} + 9e^x$

b. $(e^x + e^{-x})^2 = e^{2x} + 2 + e^{-2x}$

c. $\dfrac{e^{2x} - 4}{e^x - 2} = \dfrac{(e^x - 2)(e^x + 2)}{e^x - 2} = e^x + 2$

Opgave 45:

a. $(2x + 4)e^x = 0$

$2x + 4 = 0 \quad \vee \quad e^x = 0$

$2x = -4$ k.n.

$x = -2$

b. $x^2 e^x = 3xe^x$

$x^2 e^x - 3xe^x = 0$

$x(x - 3)e^x = 0$

$x = 0 \quad \vee \quad x = 3 \quad \vee \quad e^x = 0$

$x = 0 \quad \vee \quad x = 3$ k.n.

c. $x^2 e^x = e^x$

$x^2 e^x - e^x = 0$

$(x^2 - 1)e^x = 0$

$x^2 = 1 \quad \vee \quad e^x = 0$ (k.n.)

$x = 1 \quad \vee \quad x = -1$

d. $e^{3x} - e^x = 0$

$e^{3x} = e^x$

$3x = x$

$2x = 0$

$x = 0$

e. $e^{4x} - 1 = 0$

$e^{4x} = 1$

$e^{4x} = e^0$

$4x = 0$

$x = 0$

f. $e^x \cdot e^x = e^6$

$e^{2x} = e^6$

$2x = 6$

$x = 3$

Opgave 46:

a. $e^x + e^x = 2e^6$

$2e^x = 2e^6$

$e^x = e^6$

$x = 6$

b. $\frac{e^{5x}}{e^x} = e$

$e^{4x} = e^1$

$4x = 1$

$x = \frac{1}{4}$

c. $2xe^x + e^x = 0$

$(2x+1)e^x = 0$

$2x = -1 \quad \vee \quad e^x = 0$

$x = -\frac{1}{2}$ k.n.

d. $e^{x+2} - \sqrt{e} = 0$

$e^{x+2} = \sqrt{e}$

$e^{x+2} = e^{\frac{1}{2}}$

$x + 2 = \frac{1}{2}$

$x = -1\frac{1}{2}$

e. $e^{2x} + e^x = 2$

$(e^x)^2 + e^x - 2 = 0$

neem $p = e^x$ dan $p^2 + p - 2 = 0$

$(p+2)(p-1) = 0$

$p = -2 \quad \vee \quad p = 1$

$e^x = -2 \quad \vee \quad e^x = 1$

k.n. $x = 0$

f. $e^{6x} + 1 = 2e^{3x}$

$(e^{3x})^2 - 2e^{3x} + 1 = 0$

neem $p = e^{3x}$ dan $p^2 - 2p + 1 = 0$

$(p-1)^2 = 0$

$p = 1$

$e^{3x} = 1$

$e^{3x} = e^0$

$3x = 0$

$x = 0$

Opgave 47:

a. $f(x) = xe^x$

$f'(x) = 1 \cdot e^x + x \cdot e^x = (1+x)e^x$

b. $g(x) = \dfrac{e^x}{x+1}$ `

$g'(x) = \dfrac{(x+1) \cdot e^x - e^x \cdot 1}{(x+1)^2} = \dfrac{xe^x + e^x - e^x}{(x+1)^2} = \dfrac{xe^x}{(x+1)^2}$

Opgave 48:

a. $f(x) = e^x + 2$

$f'(x) = e^x$

b. $f(x) = 2e^x + \dfrac{1}{x} = 2e^x + x^{-1}$

$f'(x) = 2e^x - x^{-2} = 2e^x - \dfrac{1}{x^2}$

c. $f(x) = xe^x + 4$

$f'(x) = 1 \cdot e^x + x \cdot e^x = (1+x)e^x$

d. $f(x) = \dfrac{x}{e^x}$

$f'(x) = \dfrac{e^x \cdot 1 - x \cdot e^x}{(e^x)^2} = \dfrac{1-x}{e^x}$

e. $f(x) = \dfrac{2e^x}{x-1}$

$f'(x) = \dfrac{(x-1) \cdot 2e^x - 2e^x \cdot 1}{(x-1)^2} = \dfrac{2xe^x - 2e^x - 2e^x}{(x-1)^2} = \dfrac{(2x-4)e^x}{(x-1)^2}$

f. $f(x) = (2x-4)e^x$

$f'(x) = 2 \cdot e^x + (2x-4) \cdot e^x = 2e^x + 2xe^x - 4e^x = (2x-2)e^x$

Opgave 49:

a. 5,718

b. – 0,135

c. 20,086

d. 0,366

e. 9,852

f. – 26,229

Opgave 50:

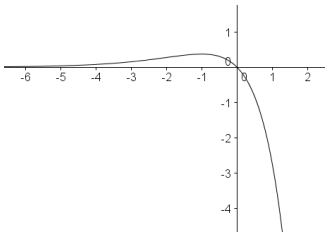
a. $f'(x) = -1 \cdot e^x + -x \cdot e^x = (-1-x)e^x = 0$

$-1-x = 0 \quad \vee \quad e^x = 0$

$-x = 1$ k.n.

$x = -1$

$y = e^{-1} = \frac{1}{e}$



max $f(-1) = \frac{1}{e}$

b. $f'(0) = -1$

$y = -x + b$ door (0,0)

$0 = b$

$k: \quad y = -x$

Opgave 51:

a. $(x^2 - 3)e^x = 0$

$x^2 - 3 = 0 \quad \vee \quad e^x = 0$

$x^2 = 3$ k.n.

$x = \sqrt{3} \quad \vee \quad x = -\sqrt{3}$

b. $f'(x) = 2x \cdot e^x + (x^2 - 3) \cdot e^x = (x^2 + 2x - 3)e^x = 0$

$x^2 + 2x - 3 = 0 \quad \vee \quad e^x = 0$

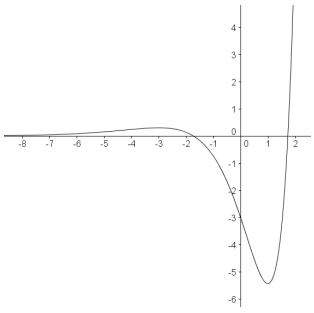
$(x+3)(x-1) = 0$ k.n.

$x = -3 \quad \vee \quad x = 1$

$y = 6e^{-3} = \frac{6}{e^3} \quad \vee \quad y = -2e$

max $f(-3) = \frac{6}{e^3}$

min $f(1) = -2e$



c. als $x \to -\infty$ dan $e^x \to 0$

e^x wint het van $x^2 - 3$ dus voor $x \to -\infty$ geldt $f(x) \to 0$

d. $p = \frac{6}{e^3} \quad \vee \quad -2e < p \leq 0$

Opgave 52:

$$f(x) = \frac{2e^x}{e^x + 1}$$

$$f'(x) = \frac{(e^x + 1) \cdot 2e^x - 2e^x \cdot e^x}{(e^x + 1)^2} = \frac{2e^{2x} + 2e^x - 2e^{2x}}{(e^x + 1)^2} = \frac{2e^x}{(e^x + 1)^2}$$

$$f'(1) = \frac{2e}{(e+1)^2}$$

$$y_P = f(1) = \frac{2e}{e+1}$$

$$k: \quad y = \frac{2e}{(e+1)^2} \cdot x + b \text{ door } (1, \frac{2e}{e+1})$$

$$\frac{2e}{e+1} = \frac{2e}{(e+1)^2} + b$$

$$b = \frac{2e}{e+1} - \frac{2e}{(e+1)^2} = \frac{2e(e+1)}{(e+1)^2} - \frac{2e}{(e+1)^2} = \frac{2e^2 + 2e - 2e}{(e+1)^2} = \frac{2e^2}{(e+1)^2}$$

$$k: \quad y = \frac{2e}{(e+1)^2} \cdot x + \frac{2e^2}{(e+1)^2}$$

k snijden met de lijn $y = 2$ geeft:

$$\frac{2e}{(e+1)^2}\cdot x+\frac{2e^2}{(e+1)^2}=2$$

$$\frac{2e}{(e+1)^2}\cdot x=2-\frac{2e^2}{(e+1)^2}$$

$$\frac{2e}{(e+1)^2}\cdot x=\frac{2(e+1)^2}{(e+1)^2}-\frac{2e^2}{(e+1)^2}$$

$$\frac{2e}{(e+1)^2}\cdot x=\frac{2e^2+4e+2-2e^2}{(e+1)^2}$$

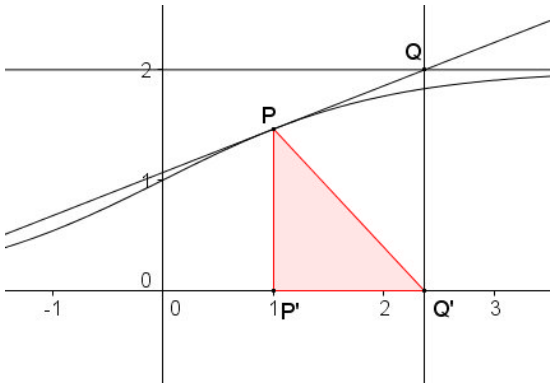
$$\frac{2e}{(e+1)^2}\cdot x=\frac{4e+2}{(e+1)^2}$$

$$x=\frac{4e+2}{(e+1)^2}\cdot\frac{(e+1)^2}{2e}=\frac{4e+2}{2e}=\frac{4e}{2e}+\frac{2}{2e}=2+\frac{1}{e}$$

$$x_{Q'}=2+\frac{1}{e}$$

$$P'Q'=x_{Q'}-x_{P'}=2+\frac{1}{e}-1=1+\frac{1}{e}=\frac{e}{e}+\frac{1}{e}=\frac{e+1}{e}$$

$$Opp(\Delta PP'Q')=\tfrac{1}{2}\cdot PP'\cdot P'Q'=\tfrac{1}{2}\cdot\frac{2e}{e+1}\cdot\frac{e+1}{e}=1$$



Opgave 53:

$f(x)=e^{ax+b}=e^u$ met $u=ax+b$ dus $u'=a$

$f'(x)=e^u\cdot u'=e^{ax+b}\cdot a=a\cdot e^{ax+b}$

Opgave 54:

a. $f(x)=e^{x^2+x}=e^u$ met $u=x^2+x$ dus $u'=2x+1$

$f'(x)=e^u\cdot u'=e^{x^2+x}\cdot(2x+1)=(2x+1)e^{x^2+x}$

b. $g(x)=x^2+2e^{3x}=x^2+2e^u$ met $u=3x$ dus $u'=3$

$g'(x)=2x+2e^u\cdot u'=2x+2e^{3x}\cdot 3=2x+6e^{3x}$

c. $h(x)=xe^{x^2}=x\cdot e^u$ met $u=x^2$ dus $u'=2x$

$h'(x)=1\cdot e^u+x\cdot e^u\cdot u'=e^{x^2}+xe^{x^2}\cdot 2x=e^{x^2}+2x^2e^{x^2}=(1+2x^2)e^{x^2}$

d. $j(x)=3x\cdot e^{2x-1}=3x\cdot e^u$ met $u=2x-1$ dus $u'=2$

$j'(x)=3\cdot e^u+3x\cdot e^u\cdot u'=3e^{2x-1}+3xe^{2x-1}\cdot 2=3e^{2x-1}+6xe^{2x-1}=(3+6x)e^{2x-1}$

e. $k(x)=\dfrac{2e^{-x-1}}{x^2}$

$T(x)=2e^{-x-1}=2e^u$ met $u=-x-1$ dus $u'=-1$

$T'(x)=2e^u\cdot u'=2e^{-x-1}\cdot -1=-2e^{-x-1}$

$$k'(x)=\frac{x^2\cdot -2e^{-x-1}-2e^{-x-1}\cdot 2x}{x^4}$$

$$=\frac{-2x^2e^{-x-1}-4xe^{-x-1}}{x^4}$$

$$= \frac{-2xe^{-x-1} - 4e^{-x-1}}{x^3}$$
$$= \frac{(-2x-4)e^{-x-1}}{x^3}$$

f. $l(x) = \dfrac{e^{2x}}{e^{2x}+1}$

$T(x) = e^{2x} = e^u$ met $u = 2x$ dus $u' = 2$

$T'(x) = e^u \cdot u' = e^{2x} \cdot 2 = 2e^{2x}$

$$l'(x) = \frac{(e^{2x}+1)\cdot 2e^{2x} - e^{2x}\cdot 2e^{2x}}{(e^{2x}+1)^2}$$
$$= \frac{2e^{4x} + 2e^{2x} - 2e^{4x}}{(e^{2x}+1)^2}$$
$$= \frac{2e^{2x}}{(e^{2x}+1)^2}$$

Opgave 55:

a. $f'(x) = \frac{1}{2} \cdot 2e^{2x} = e^{2x}$

$$f'(-1) = e^{-2} = \frac{1}{e^2}$$

$$y_A = f(-1) = \tfrac{1}{2}e^{-2} = \frac{1}{2e^2}$$

k: $y = \dfrac{1}{e^2} \cdot x + b$ door $(-1, \dfrac{1}{2e^2})$

$$\frac{1}{2e^2} = -\frac{1}{e^2} + b$$

$$b = \frac{1}{2e^2} + \frac{1}{e^2} = \frac{1}{2e^2} + \frac{2}{2e^2} = \frac{3}{2e^2}$$

k: $y = \dfrac{1}{e^2} \cdot x + \dfrac{3}{2e^2}$

$$g(x) = \frac{1}{e^{x+3}} = e^{-x-3}$$

$$g'(x) = -e^{-x-3}$$

$$g'(-1) = -e^{-2} = -\frac{1}{e^2}$$

$$y_B = g(-1) = e^{-2} = \frac{1}{e^2}$$

l: $y = -\dfrac{1}{e^2} \cdot x + b$ door $(-1, \dfrac{1}{e^2})$

$$\frac{1}{e^2} = \frac{1}{e^2} + b$$

$$b = 0$$

l: $y = -\dfrac{1}{e^2} \cdot x$

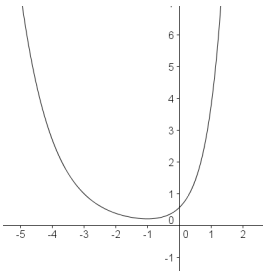
snijpunt van k en l:

$\frac{1}{e^2}\cdot x+\frac{3}{2e^2}=-\frac{1}{e^2}\cdot x$

$\frac{2}{e^2}\cdot x=-\frac{3}{2e^2}$

$x=-\frac{3}{4}$

b. $h(x)=f(x)+g(x)$



$h'(x)=f'(x)+g'(x)=e^{2x}-e^{-x-3}=0$

$e^{2x}=e^{-x-3}$

$2x=-x-3$

$3x=-3$

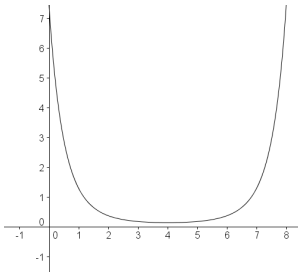
$x=-1$

$\min\ h(-1)=\frac{1}{2}e^{-2}+\frac{1}{e^2}=\frac{1}{2e^2}+\frac{1}{e^2}=\frac{1}{2e^2}+\frac{2}{2e^2}=\frac{3}{2e^2}$

$B_h=[\frac{3}{2e^2},\rightarrow\rangle$

Opgave 56:

a. $f(x)=e^{\frac{1}{4}x^2-2x+2}$



$f'(x)=e^{\frac{1}{4}x^2-2x+2}\cdot(\frac{1}{2}x-2)=0$

$e^{\frac{1}{4}x^2-2x+2}=0 \quad \vee \quad \frac{1}{2}x-2=0$

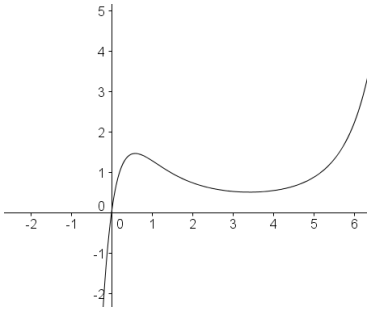
k.n. $\frac{1}{2}x=2$

$x=4$

$\min\ f(4)=e^{-2}=\frac{1}{e^2}$

$B_f=\left[\frac{1}{e^2},\rightarrow\right\rangle$

b. $Opp(OPQR)=OP\cdot PQ=p\cdot e^{\frac{1}{4}p^2-2p+2}$



$Opp'=1\cdot e^{\frac{1}{4}p^2-2p+2}+p\cdot e^{\frac{1}{4}p^2-2p+2}\cdot(\frac{1}{2}p-2)=0$

$e^{\frac{1}{4}p^2-2p+2}+\frac{1}{2}p^2\cdot e^{\frac{1}{4}p^2-2p+2}-2p\cdot e^{\frac{1}{4}p^2-2p+2}=0$

$e^{\frac{1}{4}p^2-2p+2}\cdot(1+\frac{1}{2}p^2-2p)=0$

$e^{\frac{1}{4}p^2-2p+2}=0 \quad \vee \quad \frac{1}{2}p^2-2p+1=0$

k.n. $p^2-4p+2=0$

$p=\frac{4\pm\sqrt{8}}{2}=2\pm\sqrt{2}$

de oppervlakte is minimaal voor $p=2-\sqrt{2}$

Opgave 57:

a. $f_1(x)=(x-1)^2\cdot e^{2x}$

$f_1'(x)=2(x-1)\cdot e^{2x}+(x-1)^2\cdot 2e^{2x}$

$=(2x-2)\cdot e^{2x}+2(x-1)^2\cdot e^{2x}$

$= (2x^2 - 2x) \cdot e^{2x} = 0$

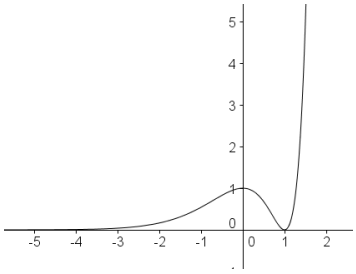
$2x^2 - 2x = 0 \quad \vee \quad e^{2x} = 0$

$2x(x-1) = 0$ k.n.

$x = 0 \quad \vee \quad x = 1$

max $f_1(0) = 1$

min $f_1(1) = 0$



b. $f_a'(x) = 2(x-a) \cdot e^{2x} + (x-a)^2 \cdot 2e^{2x}$

$= (2x - 2a) \cdot e^{2x} + 2(x-a)^2 \cdot e^{2x}$

$= (2x^2 + 2x - 4ax + 2a^2 - 2a) \cdot e^{2x} = 0$

$2x^2 + 2x - 4ax + 2a^2 - 2a = 0 \quad \vee \quad e^{2x} = 0$

$2x^2 + (2-4a)x + 2a^2 - 2a = 0$ k.n.

$x^2 + (1-2a)x + a^2 - a = 0$

$x = \dfrac{-(1-2a) \pm \sqrt{(1-2a)^2 - 4(a^2-a)}}{2} = \dfrac{-1+2a \pm \sqrt{1-4a+4a^2-4a^2+4a}}{2}$

$= \dfrac{-1+2a \pm 1}{2}$

$x = \dfrac{-1+2a+1}{2} = \dfrac{2a}{2} = a \quad \vee \quad x = \dfrac{-1+2a-1}{2} = \dfrac{2a-2}{2} = a-1$

$x_A = a-1$ en $x_B = a$

c. $f(x_B) = f(a) = 0$ dus $y = 0$

d. $y = f(x_A) = f(a-1) = (a-1-a)^2 \cdot e^{2(a-1)} = e^{2(a-1)} = e^{2x}$

dus $y = e^{2x}$

e. $f_a'(0) = 2a^2 - 2a < 0$

$2a(a-1) = 0$

$a = 0 \quad \vee \quad a = 1$

$0 < a < 1$