## 9.4 De natuurlijke logaritme

**Opgave 58:**

a. $e^{{}^e\log 2\cdot x} = (e^{{}^e\log 2})^x = 2^x$

b. $[2^x]' = [e^{{}^e\log 2\cdot x}]' = [e^u]'$ met $u={}^e\log 2\cdot x$ dus $u'={}^e\log 2$

$[e^u]' = e^u \cdot u' = e^{{}^e\log 2\cdot x}\cdot {}^e\log 2 = 2^x \cdot {}^e\log 2$

**Opgave 59:**

a. $\ln e = \ln e^1 = 1$

b. $\ln e\sqrt{e} = \ln e^{1\frac{1}{2}} = 1\frac{1}{2}$

c. $\ln \frac{1}{e} = \ln e^{-1} = -1$

d. $1 = \ln e^0 = 0$

e. $3\ln e\sqrt[3]{e} = 3\cdot \ln e^{1\frac{1}{3}} = 3\cdot 1\frac{1}{3} = 4$

f. $\ln^2 e^3 = (\ln e^3)^2 = 3^2 = 9$

g. $\ln^3 e^2 = (\ln e^2)^3 = 2^3 = 8$

h. $e^{\ln 7} + e^{2\ln 7} = e^{\ln 7} + e^{\ln 7^2} = 7 + 7^2 = 7 + 49 = 56$

i. $e^{\frac{1}{2}\ln 5} = e^{\ln 5^{\frac{1}{2}}} = 5^{\frac{1}{2}} = \sqrt{5}$

j. $e^{\ln 10}\cdot e^{\ln 3} = 10\cdot 3 = 30$

**Opgave 60:**

a. $e^{3x} = 12$
$3x = \ln 12$
$x = \frac{1}{3}\ln 12$

b. $5e^{2x} = 60$
$e^{2x} = 12$
$2x = \ln 12$
$x = \frac{1}{2}\ln 12$

c. $6 + e^{0,5x} = 10$
$e^{0,5x} = 4$
$0,5x = \ln 4$
$x = 2\ln 4$

d. $\dfrac{3}{e^{2x}} = 10$
$e^{2x} = \frac{3}{10}$
$2x = \ln \frac{3}{10}$
$x = \frac{1}{2}\ln \frac{3}{10}$

**Opgave 61:**

a. $2\ln 3 + \ln 4 = \ln 3^2 + \ln 4 = \ln 9 + \ln 4 = \ln 36$

b. $\ln 20 - 3\ln 2 = \ln 20 - \ln 2^3 = \ln 20 - \ln 8 = \ln \frac{20}{8} = \ln 2\frac{1}{2}$

c. $4 + \ln 3 = \ln e^4 + \ln 3 = \ln 3e^4$

d. $1 + \ln 10 = \ln e + \ln 10 = \ln 10e$

e. $\frac{1}{2} + 2\ln 6 = \ln\sqrt{e} + \ln 6^2 = \ln\sqrt{e} + \ln 36 = \ln 36\sqrt{e}$

f. $e + \ln 2 = \ln e^e + \ln 2 = \ln 2e^e$

**Opgave 62:**

a. $\ln x = -1$
$x = e^{-1} = \frac{1}{e}$

b. $4\ln x = 2$
$\ln x = \frac{1}{2}$
$x = e^{\frac{1}{2}} = \sqrt{e}$

c. $\ln 3x = 3$
$3x = e^3$
$x = \frac{1}{3}e^3$

d. $\ln(-x+2) = -2$
$-x + 2 = e^{-2} = \frac{1}{e^2}$
$-x = \frac{1}{e^2} - 2$
$x = 2 - \frac{1}{e^2}$

e. $\ln^2 x = \frac{1}{4}$
$\ln x = \frac{1}{2} \quad \vee \quad \ln x = -\frac{1}{2}$
$x = e^{\frac{1}{2}} = \sqrt{e} \quad \vee \quad x = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$

f. $\ln x = 1 + \ln 5$
$\ln x = \ln e + \ln 5$
$\ln x = \ln 5e$
$x = 5e$

**Opgave 63:**

a. $4e^{1-3x} = 20$
$e^{1-3x} = 5$
$1 - 3x = \ln 5$
$-3x = -1 + \ln 5$
$x = \frac{1}{3} - \frac{1}{3}\ln 5 = -0{,}203$

b. $e^{x^2} = 100$
$x^2 = \ln 100$
$x = \sqrt{\ln 100} = 2{,}146 \quad \vee \quad x = -\sqrt{\ln 100} = -2{,}146$

**Opgave 64:**

a. $3x\ln x = 2\ln x$
$3x\ln x - 2\ln x = 0$
$\ln x \cdot (3x - 2) = 0$
$\ln x = 0 \quad \vee \quad 3x = 2$
$x = e^0 = 1 \quad \vee \quad x = \frac{2}{3}$

b. $\ln^2 x - \ln x = 0$
$\ln x \cdot (\ln x - 1) = 0$

$\ln x = 0 \quad \vee \quad \ln x = 1$

$x = e^0 = 1 \quad \vee \quad x = e$

c. $x^2 \ln(x+1) = 4\ln(x+1)$

$x^2 \ln(x+1) - 4\ln(x+1) = 0$

$\ln(x+1) \cdot (x^2 - 4) = 0$

$\ln(x+1) = 0 \quad \vee \quad x^2 - 4 = 0$

$x + 1 = e^0 = 1 \quad \vee \quad x^2 = 4$

$x = 0 \quad \vee \quad x = 2 \quad \vee \quad x = -2$ (vervalt)

$x = 0 \quad \vee \quad x = 2$

d. $\ln^2 x - 2\ln x - 3 = 0$

neem $p = \ln x$ dan $p^2 - 2p - 3 = 0$

$(p-3)(p+1) = 0$

$p = 3 \quad \vee \quad p = -1$

$\ln x = 3 \quad \vee \quad \ln x = -1$

$x = e^3 \quad \vee \quad x = e^{-1} = \frac{1}{e}$

e. $\ln(x+3) - \ln(x-1) = \ln 2$

$\ln \frac{x+3}{x-1} = \ln 2$

$\frac{x+3}{x-1} = 2$

$2(x-1) = x + 3$

$2x - 2 = x + 3$

$x = 5$

f. $2\ln x = \ln 2 + \ln(x+4)$

$\ln x^2 = \ln 2(x+4)$

$x^2 = 2(x+4)$

$x^2 = 2x + 8$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4 \quad \vee \quad x = -2$ (vervalt)

$x = 4$

**Opgave 65:**

a. $f(x) = 3^{4x-3} = 3^u$ met $u = 4x - 3$ dus $u' = 4$

$f'(x) = 3^u \cdot \ln 3 \cdot u' = 3^{4x-3} \cdot \ln 3 \cdot 4 = 4\ln 3 \cdot 3^{4x-3}$

b. $g(x) = (2x-1) \cdot 2^x$

$g'(x) = 2 \cdot 2^x + (2x-1) \cdot 2^x \cdot \ln 2$

c. $h(x) = \frac{2^x + 1}{2^x - 1}$

$$h'(x) = \frac{(2^x - 1) \cdot 2^x \cdot \ln 2 - (2^x + 1) \cdot 2^x \cdot \ln 2}{(2^x - 1)^2} = \frac{2^{2x} \cdot \ln 2 - 2^x \cdot \ln 2 - 2^{2x} \cdot \ln 2 - 2^x \cdot \ln 2}{(2^x - 1)^2}$$

$$= \frac{-2 \cdot \ln 2 \cdot 2^x}{(2^x - 1)^2}$$

**Opgave 66:**

a. $f(x) = 2^{2x} - 2^x$

$f'(x) = 2^{2x} \cdot \ln 2 \cdot 2 - 2^x \cdot \ln 2 = (2 \cdot 2^{2x} - 2^x) \cdot \ln 2 = 0$

$2 \cdot 2^{2x} - 2^x = 0$

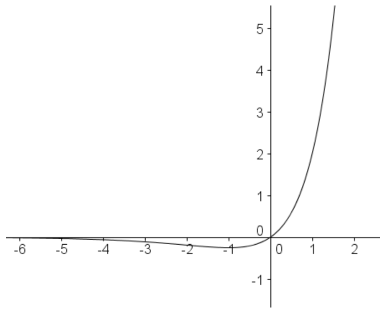
$2^{2x+1} = 2^x$

$2x + 1 = x$

$x = -1$

$y = 2^{-2} - 2^{-1} = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$

$B_g = [-\frac{1}{4}, \rightarrow\rangle$



b. $f'(0) = \ln 2$

dus $0 < a < \ln 2 \quad \vee \quad a > \ln 2$

**Opgave 67:**

a. $f(x) = 2^{x-1} + 2^{-x-2}$

$f'(x) = 2^{x-1} \cdot \ln 2 + 2^{-x-2} \cdot \ln 2 \cdot -1 = (2^{x-1} - 2^{-x-2}) \cdot \ln 2 = 0$

$2^{x-1} - 2^{-x-2} = 0$

$2^{x-1} = 2^{-x-2}$

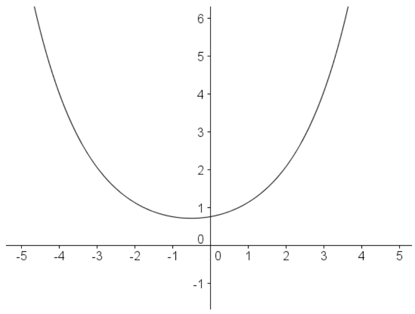
$x - 1 = -x - 2$

$2x = -1$

$x = -\frac{1}{2}$

$y = 2^{-1\frac{1}{2}} + 2^{-1\frac{1}{2}} = 2 \cdot 2^{-1\frac{1}{2}} = 2^{-\frac{1}{2}} = \frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{2}$

$B_f = [\frac{1}{2}\sqrt{2}, \rightarrow\rangle$



b. $f'(x) = (2^{x-1} - 2^{-x-2}) \cdot \ln 2 = -\frac{1}{4}\ln 2$

$2^{x-1} - 2^{-x-2} = -\frac{1}{4}$

$2^{-1} \cdot 2^x - 2^{-2} \cdot 2^x + \frac{1}{4} = 0$

$\frac{1}{2} \cdot 2^x - \frac{1}{4} \cdot \frac{1}{2^x} + \frac{1}{4} = 0$

neem $p = 2^x$ dan $\frac{1}{2}p - \frac{1}{4} \cdot \frac{1}{p} + \frac{1}{4} = 0$

$2p - \frac{1}{p} + 1 = 0$

$2p^2 - 1 + p = 0$

$p = \frac{-1 \pm \sqrt{9}}{4} = \frac{-1 \pm 3}{4}$

$p = -1 \quad \vee \quad p = \frac{1}{2}$

$2^x = -1 \quad \vee \quad 2^x = \frac{1}{2}$

k.n. $\qquad 2^x = 2^{-1}$

$x = -1$ dan $y = \frac{3}{4}$ dus $(-1, \frac{3}{4})$

c. $f'(x) = (2^{x-1} - 2^{-x-2}) \cdot \ln 2 = -3$

$y_1 = (2^{x-1} - 2^{-x-2}) \cdot \ln 2$ en $y_2 = -3$

intersect geeft $x = -4{,}1233$

$y = 4{,}3855$

$b = y + 3x = 4{,}3855 + 3 \cdot -4{,}1233 = -7{,}984$

**Opgave 68:**

a. $[x]' = [e^{\ln x}]' = [e^u]'$ met $u = \ln x$ en $u' = [\ln x]'$

$[e^u]' = e^u \cdot u' = e^{\ln x} \cdot [\ln x]' = x \cdot [\ln x]'$

ook geldt: $[x]' = 1$

b. $x \cdot [\ln x]' = 1$

dus $[\ln x]' = \frac{1}{x}$

c. $g(x) = \frac{{}^e\log x}{{}^e\log 2} = \frac{\ln x}{\ln 2} = \frac{1}{\ln 2} \cdot \ln x$

$g'(x) = \frac{1}{\ln 2} \cdot \frac{1}{x} = \frac{1}{x \ln 2}$

**Opgave 69:**

a. $[\ln 6x]' = [\ln 6 + \ln x]' = [\ln 6]' + [\ln x]' = 0 + \frac{1}{x} = \frac{1}{x}$

b. $f'(x) = \frac{1}{x}$

$g'(x) = \frac{1}{x}$

$h'(x) = \frac{1}{x \ln 2}$

**Opgave 70:**

a. $[\ln x^6]' = [6 \cdot \ln x]' = 6 \cdot [\ln x]' = 6 \cdot \frac{1}{x} = \frac{6}{x}$

b. $f'(x) = \frac{2}{x}$

$g'(x) = \frac{-3}{x}$

$h'(x) = \frac{-1}{x}$

**Opgave 71:**

a. $f(x) = \frac{1 - \ln x}{x}$

$$f'(x) = \frac{x \cdot \frac{-1}{x} - (1 - \ln x) \cdot 1}{x^2} = \frac{-1 - 1 + \ln x}{x^2} = \frac{-2 + \ln x}{x^2}$$

b. $g(x) = x\ln x$

$$g'(x) = 1\cdot \ln x + x\cdot \frac{1}{x} = \ln x + 1$$

c. $f(x) = {}^2\log(4x-1)$

$$f'(x) = \frac{1}{4x-1}\cdot 4\cdot \tfrac{1}{\ln 2} = \frac{4}{(4x-1)\cdot \ln 2}$$

d. $f(x) = \dfrac{\ln 3x}{x}$

$$f'(x) = \frac{x\cdot \frac{1}{x} - \ln 3x\cdot 1}{x^2} = \frac{1-\ln 3x}{x^2}$$

e. $f(x) = x\cdot \ln x^3$

$$f'(x) = 1\cdot \ln x^3 + x\cdot \frac{3}{x} = \ln x^3 + 3$$

f. $f(x) = {}^3\log x^2$

$$f'(x) = \frac{1}{x^2}\cdot 2x\cdot \tfrac{1}{\ln 3} = \frac{2}{x\ln 3}$$

**Opgave 72:**

a. $f(x) = \ln(x^2+2x)$

$$f'(x) = \frac{1}{x^2+2x}\cdot (2x+1) = \frac{2x+1}{x^2+2x}$$

b. $g(x) = \ln 2^x = x\cdot \ln 2$

$g'(x) = \ln 2$

c. $h(x) = {}^2\log(x^2+1)$

$$h'(x) = \frac{1}{x^2+1}\cdot 2x\cdot \frac{1}{\ln 2} = \frac{2x}{(x^2+1)\cdot \ln 2}$$

d. $j(x) = \log 4x^2$

$$j'(x) = \frac{1}{4x^2}\cdot 8x\cdot \frac{1}{\ln 10} = \frac{2}{x\ln 10}$$

**Opgave 73:**

a. $f(x) = x\cdot \ln^2 x$

$$f'(x) = 1\cdot \ln^2 x + x\cdot 2\ln x\cdot \frac{1}{x} = \ln^2 x + 2\ln x$$

b. $g(x) = x^2\cdot {}^3\log 4x$

$$g'(x) = 2x\cdot {}^3\log 4x + x^2\cdot \frac{1}{4x}\cdot 4\cdot \frac{1}{\ln 3} = 2x\cdot {}^3\log 4x + \frac{x}{\ln 3}$$

c. $h(x) = \log^2(4x)$

$$h'(x) = 2\cdot \log 4x\cdot \frac{1}{4x}\cdot 4\cdot \frac{1}{\ln 10} = \frac{2\cdot \log 4x}{x\ln 10}$$

d. $j(x) = \ln^2(4x^2+1)$

$$j'(x) = 2\ln(4x^2+1) \cdot \frac{1}{4x^2+1} \cdot 8x = \frac{16x\ln(4x^2+1)}{4x^2+1}$$

**Opgave 74:**

a. $e^{n\cdot\ln x} = e^{\ln x^n} = x^n$

b. $y = e^{n\cdot\ln x} = e^u$ met $u = n\cdot\ln x$ dus $u' = n\cdot\frac{1}{x}$

c. $y' = [e^{n\cdot\ln x}]' = e^u \cdot u' = e^{n\cdot\ln x}\cdot n\cdot\frac{1}{x} = e^{n\cdot\ln x}\cdot\frac{n}{x}$

d. $[x^n]' = [(e^{\ln x})^n]' = [e^{n\cdot\ln x}]' = e^{n\cdot\ln x}\cdot\frac{n}{x} = (e^{\ln x})^n\cdot\frac{n}{x} = x^n\cdot\frac{n}{x} = n\cdot x^{n-1}$

er is geen beperking voor de waarde van $n$

**Opgave 75:**

a. $f(x) = \frac{10\ln x}{x} = 0$

$10\ln x = 0$

$\ln x = 0$

$x = e^0 = 1$

$$f'(x) = \frac{x\cdot\frac{10}{x} - 10\ln x}{x^2} = \frac{10-10\ln x}{x^2}$$

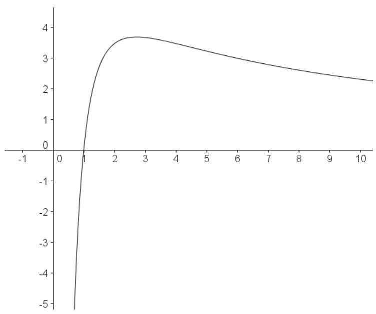
$f'(1) = 10$

$y = 10x + b$ door $(1,0)$

$0 = 10 + b$

$b = -10$

$y = 10x - 10$




b. $f'(x) = \frac{10-10\ln x}{x^2} = 0$

$10 - 10\ln x = 0$

$-10\ln x = -10$

$\ln x = 1$

$x = e$

max $f(e) = \frac{10}{e}$

c. stel $x_B = p$ dan $x_C = 2p$

$y_B = y_C$

$f(p) = f(2p)$

$\frac{10\ln p}{p} = \frac{10\ln 2p}{2p}$

$\frac{20\ln p}{2p} = \frac{10\ln 2p}{2p}$

$20\ln p = 10\ln 2p$

$2\ln p = \ln 2p$

$\ln p^2 = \ln 2p$

$p^2 = 2p$

$p^2 - 2p = 0$

$p(p-2) = 0$

$p = 0 \quad \vee \quad p = 2$

vervalt $\qquad q = f(2) = \frac{10\ln 2}{2} = 5\ln 2$

**Opgave 76:**

a. $\quad f(x) = \frac{x}{\ln x}$

$y_A = f(\frac{1}{e}) = -\frac{1}{e}$

$f'(x) = \frac{\ln x \cdot 1 - x \cdot \frac{1}{x}}{\ln^2 x} = \frac{\ln x - 1}{\ln^2 x}$

$f'(\frac{1}{e}) = \frac{-1-1}{1} = -2$

$y = -2x + b$ door $(\frac{1}{e}, -\frac{1}{e})$

$-\frac{1}{e} = -\frac{2}{e} + b$

$b = \frac{1}{e}$

$y = -2x + \frac{1}{e}$

b. $\quad f'(x) = \frac{\ln x - 1}{\ln^2 x} = -6$

$\ln x - 1 = -6\ln^2 x$

$6\ln^2 x + \ln x - 1 = 0$

neem $p = \ln x$ dan $6p^2 + p - 1 = 0$

$p = \frac{-1 \pm \sqrt{25}}{12}$

$p = \frac{-1-5}{12} = -\frac{1}{2} \quad \vee \quad p = \frac{-1+5}{12} = \frac{1}{3}$

$\ln x = -\frac{1}{2} \quad \vee \quad \ln x = \frac{1}{3}$

$x = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}} \quad \vee \quad x = e^{\frac{1}{3}} = \sqrt[3]{e}$

$y = \frac{-2}{\sqrt{e}} \qquad\qquad y = 3 \cdot \sqrt[3]{e}$

$(\frac{1}{\sqrt{e}}, \frac{-2}{\sqrt{e}})$ en $(\sqrt[3]{e}, 3 \cdot \sqrt[3]{e})$

**Opgave 77:**

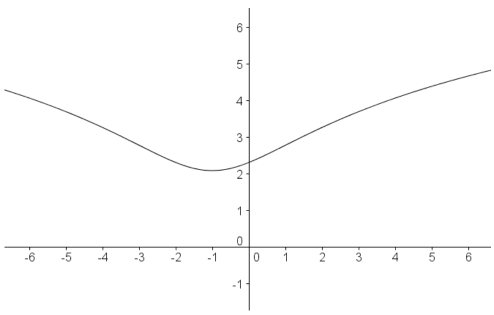
a. $\quad f(x) = \ln(2x^2 + 4x + 10)$

$f'(x) = \frac{1}{2x^2 + 4x + 10} \cdot (4x + 4) = \frac{4x+4}{2x^2 + 4x + 10} = 0$

$4x + 4 = 0$

$4x = -4$

$x = -1$



$y=\ln 8$
$B_f=[\ln 8,\rightarrow\rangle$

b. $f'(x)=\dfrac{4x+4}{2x^2+4x+10}=\frac{2}{5}$
$2(2x^2+4x+10)=5(4x+4)$
$4x^2+8x+20=20x+20$
$4x^2-12x=0$
$4x(x-3)=0$
$x=0 \quad \vee \quad x=3$
$y=\ln 10 \qquad y=\ln 40$
$(0,\ln 10) \qquad (3,\ln 40)$

c. $f'(x)=\dfrac{4x+4}{2x^2+4x+10}=1$
$2x^2+4x+10=4x+4$
$2x^2=-6$
kan niet, dus geen oplossingen

**Opgave 78:**

a. $\ln 2x=\ln\dfrac{4}{x}$
$2x=\dfrac{4}{x}$
$2x^2=4$
$x^2=2$
$x=\sqrt{2} \quad \vee \quad x=-\sqrt{2}$ (vervalt)
$y=\ln 2\sqrt{2}$
$(\sqrt{2},\ln 2\sqrt{2})$

b. $\ln\dfrac{4}{x}=0$
$\dfrac{4}{x}=e^0=1$
$x=4$
$g(x)=\ln\dfrac{4}{x}=\ln 4-\ln x$
$g'(x)=-\dfrac{1}{x}$
$g'(4)=-\frac{1}{4}$
$y=-\frac{1}{4}x+b$ door $(4,0)$
$0=-1+b$
$b=1$
$y=-\frac{1}{4}x+1$

c. $f(x)-g(x)=2 \quad \vee \quad g(x)-f(x)=2$

$\ln 2x - \ln \frac{4}{x} = 2 \quad \vee \quad \ln \frac{4}{x} - \ln 2x = 2$

$\ln \frac{2x}{\frac{4}{x}} = 2 \quad \vee \quad \ln \frac{\frac{4}{x}}{2x} = 2$

$\ln \tfrac{1}{2} x^2 = 2 \quad \vee \quad \ln \frac{2}{x^2} = 2$

$\tfrac{1}{2} x^2 = e^2 \quad \vee \quad \frac{2}{x^2} = e^2$

$x^2 = 2e^2 \quad \vee \quad x^2 = \tfrac{2}{e^2}$

$x = e\sqrt{2} \quad \vee \quad x = -e\sqrt{2}$ (vervalt) $\quad \vee \quad x = \tfrac{1}{e}\sqrt{2} \quad \vee \quad x = -\tfrac{1}{e}\sqrt{2}$ (vervalt)

dus $x = e\sqrt{2} \quad \vee \quad x = \tfrac{1}{e}\sqrt{2}$

d. $y_B = \ln 2x$ en $y_C = \ln \frac{4}{x}$

$y_M = \frac{y_B + y_C}{2} = \tfrac{1}{2}(y_B + y_C) = \tfrac{1}{2}(\ln 2x + \ln \frac{4}{x}) = \tfrac{1}{2}(\ln 8) = \tfrac{1}{2}\ln 8$ dus onafhankelijk van $p$