

K1 De substitutiemethode.

Opgave 1:

- a. $f(x) = \sin(x^2 + x) = \sin u$ met $u = x^2 + x$ dus $u' = 2x + 1$
 $f'(x) = \cos u \cdot u' = \cos(x^2 + x) \cdot (2x + 1) = (2x + 1) \cdot \cos(x^2 + x)$
- b. $G(x) = \sin(x^2 + x) + 3$
 $G'(x) = (2x + 1) \cdot \cos(x^2 + x) = g(x)$
dus $G(x)$ is een primitieve van $g(x)$

Opgave 2:

- a. neem $u = x^2 + 1$ dan $u' = 2x$ dus $6x = 3u'$
 $f(x) = 6x(x^2 + 1)^5 = 3u' \cdot u^5 = 3u^5 \cdot u'$
 $F(x) = \frac{3}{6}u^6 + c = \frac{1}{2}(x^2 + 1)^6 + c$
neem $u = x^3 + 1$ dan $u' = 3x^2$ maar die heb je niet, want er staat $6x$
- b. neem $u = x^4 + 7$ dan $u' = 4x^3$ dus $10x^3 = 2\frac{1}{2}u'$
 $g(x) = \frac{10x^3}{\sqrt{x^4 + 7}} = \frac{2\frac{1}{2}u'}{\sqrt{u}} = \frac{5}{2\sqrt{u}} \cdot u'$
 $G(x) = 5\sqrt{u} + c = 5\sqrt{x^4 + 7} + c$
neem $u = x^4 + x$ dan $u' = 4x^3 + 1$ maar die heb je niet want je mist de 1
- c. neem $u = x^4 + x$ dan $u' = 4x^3 + 1$ dus $10x^3 + 2\frac{1}{2} = 2\frac{1}{2}u'$
dus $a = 2\frac{1}{2}$

Opgave 3:

- a. $f(x) = \sqrt{4x - 1}$
neem $u = 4x - 1$ dan $u' = 4$ dus $1 = \frac{1}{4}u'$
 $f(x) = \sqrt{4x - 1} = \frac{1}{4}u' \cdot \sqrt{u} = \frac{1}{4}\sqrt{u} \cdot u'$
 $F(x) = \frac{1}{4} \cdot \frac{2}{3}u^{\frac{3}{2}} + c = \frac{1}{6}u\sqrt{u} + c = \frac{1}{6}(4x - 1)\sqrt{4x - 1} + c$
- b. manier II

Opgave 4:

- a. $f(x) = 2x(x^2 + 4)^3$
neem $u = x^2 + 4$ dan $u' = 2x$
 $f(x) = u' \cdot u^3 = u^3 \cdot u'$
 $F(x) = \frac{1}{4}u^4 + c = \frac{1}{4}(x^2 + 4)^4 + c$
- b. $g(x) = 6x \cdot \sqrt{x^2 + 1}$
neem $u = x^2 + 1$ dan $u' = 2x$
dus $6x = 3u'$
 $g(x) = 3u' \cdot \sqrt{u} = 3\sqrt{u} \cdot u'$
 $G(x) = 3 \cdot \frac{2}{3}u\sqrt{u} + c = 2u\sqrt{u} + c = 2(x^2 + 1)\sqrt{x^2 + 1} + c$
- c. $h(x) = 6x^2 \cdot (x^3 - 1)^4$

neem $u = x^3 - 1$ dan $u' = 3x^2$

dus $6x^2 = 2u'$

$$h(x) = 2u' \cdot u^4 = 2u^4 \cdot u'$$

$$H(x) = \frac{2}{5}u^5 + c = \frac{2}{5}(x^3 - 1)^5 + c$$

d. $j(x) = 3x^2 \cdot \sin(x^3 - 1)$

neem $u = x^3 - 1$ dan $u' = 3x^2$

$$j(x) = u' \cdot \sin u$$

$$J(x) = -\cos u + c = -\cos(x^3 - 1) + c$$

Opgave 5:

a. $f(x) = (3x - 4)^3$

neem $u = 3x - 4$ dan $u' = 3$

dus $1 = \frac{1}{3}u'$

$$F(x) = \frac{1}{12}u^4 + c = \frac{1}{12}(3x - 4)^4 + c$$

b. $f(x) = (2x - 3)\sqrt{2x - 3} = (2x - 3)^{\frac{1}{2}}$

neem $u = 2x - 3$ dan $u' = 2$

dus $1 = \frac{1}{2}u'$

$$f(x) = \frac{1}{2}u' \cdot u^{\frac{1}{2}} = \frac{1}{2}u^{\frac{1}{2}} \cdot u'$$

$$F(x) = \frac{1}{2} \cdot \frac{2}{5}u^{\frac{2}{2}} + c = \frac{1}{5}u^2 \sqrt{u} + c = \frac{1}{5}(2x - 3)^2 \sqrt{2x - 3} + c$$

c. $f(x) = \frac{2}{\sqrt{1-x}}$

neem $u = 1 - x$ dan $u' = -1$

dus $2 = -2u'$

$$f(x) = \frac{-2u'}{\sqrt{u}}$$

$$F(x) = -4\sqrt{u} + c = -4\sqrt{1-x} + c$$

d. $f(x) = \frac{2x}{2 - 3x^2}$

neem $u = 2 - 3x^2$ dan $u' = -6x$

dus $2x = -\frac{1}{3}u'$

$$f(x) = \frac{-\frac{1}{3}u'}{u} = \frac{-1}{3u} \cdot u'$$

$$F(x) = -\frac{1}{3} \ln|u| + c = -\frac{1}{3} \ln|2 - 3x^2| + c$$

e. $f(x) = \ln(4x + 1)$

neem $u = 4x + 1$ dan $u' = 4$

dus $1 = \frac{1}{4}u'$

$$f(x) = \ln u \cdot \frac{1}{4}u' = \frac{1}{4} \ln u \cdot u'$$

$$F(x) = \frac{1}{4}(u \ln|u| - u) + c = \frac{1}{4}((4x + 1) \ln|4x + 1| - (4x + 1)) + c$$

f. $f(x) = x \ln(x^2 + 1)$

neem $u = x^2 + 1$ dan $u' = 2x$

$$\text{dus } x = \frac{1}{2}u'$$

$$f(x) = \frac{1}{2}u' \cdot \ln u = \frac{1}{2} \ln u \cdot u'$$

$$F(x) = \frac{1}{2}(u \ln u - u) + c = \frac{1}{2} \cdot ((x^2 + 1) \ln(x^2 + 1) - (x^2 + 1)) + c$$

Opgave 6:

a. $f(x) = \frac{\ln x}{x}$

neem $u = \ln x$ dan $u' = \frac{1}{x}$

$$f(x) = \frac{\ln x}{x} = u \cdot u'$$

$$F(x) = \frac{1}{2}u^2 + c = \frac{1}{2} \ln^2 x + c$$

b. $g(x) = x \cdot e^{-x^2}$

neem $u = -x^2$ dan $u' = -2x$

dus $x = -\frac{1}{2}u'$

$$g(x) = -\frac{1}{2}u' \cdot e^u = -\frac{1}{2}e^u \cdot u'$$

$$G(x) = -\frac{1}{2}e^u + c = -\frac{1}{2}e^{-x^2} + c$$

c. $h(x) = x \cdot \sqrt{5 - x^2}$

neem $u = 5 - x^2$ dan $u' = -2x$

$x = -\frac{1}{2}u'$

$$h(x) = -\frac{1}{2}u' \cdot \sqrt{u} = -\frac{1}{2}\sqrt{u} \cdot u'$$

$$H(x) = -\frac{1}{2} \cdot \frac{2}{3}u\sqrt{u} + c = -\frac{1}{3}u\sqrt{u} + c = -\frac{1}{3}(5 - x^2)\sqrt{5 - x^2} + c$$

d. $j(x) = \frac{x}{\sqrt{x^2 + 1}}$

neem $u = x^2 + 1$ dan $u' = 2x$

$x = \frac{1}{2}u'$

$$j(x) = \frac{\frac{1}{2}u'}{\sqrt{u}} = \frac{1}{2\sqrt{u}} \cdot u'$$

$$J(x) = \sqrt{u} + c = \sqrt{x^2 + 1} + c$$

Opgave 7:

$$f(x) = \ln(\cos x)$$

neem $u = \cos x$ dan $u' = -\sin x$

$$f(x) = \ln u$$

$$f'(x) = \frac{1}{u} \cdot u' = \frac{1}{\cos x} \cdot -\sin x = -\frac{\sin x}{\cos x} = -\tan x$$

Opgave 8:

a. $f(x) = \tan(2x) = \frac{\sin(2x)}{\cos(2x)}$

neem $u = \cos(2x)$ dan $u' = -2 \sin(2x)$

$$\sin(2x) = -\frac{1}{2}u'$$

$$f(x) = \frac{-\frac{1}{2}u'}{u} = -\frac{1}{2} \cdot \frac{1}{u} \cdot u'$$

$$F(x) = -\frac{1}{2} \ln|u| + c = -\frac{1}{2} \ln|\cos(2x)| + c$$

$$\int_0^{\frac{1}{6}\pi} \tan(2x) dx = \left[-\frac{1}{2} \ln|\cos(2x)| \right]_0^{\frac{1}{6}\pi} = -\frac{1}{2} \ln \frac{1}{2} - -\frac{1}{2} \ln 1 = -\frac{1}{2} \ln \frac{1}{2} = \ln \sqrt{2}$$

b. $f(x) = \sin^3 x = \sin x \cdot \sin^2 x = \sin x \cdot (1 - \cos^2 x) = \sin x - \sin x \cdot \cos^2 x$
neem $u = \cos x$ dan $u' = -\sin x$

$$f(x) = \sin x + u' \cdot u^2$$

$$F(x) = -\cos x + \frac{1}{3}u^3 + c = -\cos x + \frac{1}{3}\cos^3 x + c$$

$$\int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} \sin^3 x dx = \left[-\cos x + \frac{1}{3}\cos^3 x \right]_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} = -\frac{1}{2} + \frac{1}{24} - \left(-\frac{1}{2}\sqrt{2} + \frac{1}{12}\sqrt{2} \right) = \frac{5}{12}\sqrt{2} - \frac{11}{24}$$

c. $f(x) = \sin^2 x \cdot \cos x$

neem $u = \sin x$ dan $u' = \cos x$

$$f(x) = u^2 \cdot u'$$

$$F(x) = \frac{1}{3}u^3 + c = \frac{1}{3}\sin^3 x + c$$

$$\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} \sin^2 x \cdot \cos x dx = \left[\frac{1}{3}\sin^3 x \right]_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} = \frac{1}{3} - \frac{1}{24} = \frac{7}{24}$$

d. $f(x) = \sin(2x) \cdot \cos x = 2 \sin x \cdot \cos x \cdot \cos x = 2 \sin x \cdot \cos^2 x$
neem $u = \cos x$ dan $u' = -\sin x$

$$2 \sin x = -2u'$$

$$f(x) = -2u' \cdot u^2 = -2u^2 \cdot u'$$

$$F(x) = -\frac{2}{3}u^3 + c = -\frac{2}{3}\cos^3 x + c$$

$$\int_0^{\frac{1}{6}\pi} \sin(2x) \cdot \cos x dx = \left[-\frac{2}{3}\cos^3 x \right]_0^{\frac{1}{6}\pi} = -\frac{1}{4}\sqrt{3} - -\frac{2}{3} = \frac{2}{3} - \frac{1}{4}\sqrt{3}$$

e. $f(x) = \frac{2}{\sqrt{3x+1}}$

neem $u = 3x+1$ dan $u' = 3$

$$2 = \frac{2}{3}u'$$

$$f(x) = \frac{\frac{2}{3}u'}{\sqrt{u}} = \frac{2}{3\sqrt{u}} \cdot u' = \frac{\frac{4}{3}}{2\sqrt{u}} \cdot u'$$

$$F(x) = \frac{4}{3}\sqrt{u} + c$$

$$\int_0^1 \frac{2}{\sqrt{3x+1}} dx = \left[\frac{4}{3}\sqrt{3x+1} \right]_0^1 = \frac{8}{3} - \frac{4}{3} = \frac{4}{3}$$

f. $f(x) = \frac{x}{x^2+1}$

neem $u = x^2+1$ dan $u' = 2x$

$$x = \frac{1}{2}u'$$

$$f(x) = \frac{\frac{1}{2}u'}{u} = \frac{1}{2} \cdot \frac{1}{u} \cdot u'$$

$$F(x) = \frac{1}{2} \ln|u| + c = \frac{1}{2} \ln(x^2 + 1) + c$$

$$\int_0^1 \frac{x}{x^2 + 1} dx = \left[\frac{1}{2} \ln(x^2 + 1) \right]_0^1 = \frac{1}{2} \ln 2 - \frac{1}{2} \ln 1 = \frac{1}{2} \ln 2$$

g. $f(x) = x\sqrt{x^2 + 1}$

neem $u = x^2 + 1$ dan $u' = 2x$

$$x = \frac{1}{2}u'$$

$$f(x) = \frac{1}{2}u' \cdot \sqrt{u} = \frac{1}{2}\sqrt{u} \cdot u'$$

$$F(x) = \frac{1}{3}u\sqrt{u} = \frac{1}{3}(x^2 + 1)\sqrt{x^2 + 1}$$

$$\int_0^{2\sqrt{2}} x\sqrt{x^2 + 1} dx = \left[\frac{1}{3}(x^2 + 1)\sqrt{x^2 + 1} \right]_0^{2\sqrt{2}} = 9 - \frac{1}{3} = 8\frac{2}{3}$$

h. $f(x) = \frac{e^{\sqrt{x}}}{\sqrt{x}}$

neem $u = \sqrt{x}$ dan $u' = \frac{1}{2\sqrt{x}}$

$$\frac{1}{\sqrt{x}} = 2u'$$

$$f(x) = 2u' \cdot e^u = 2e^u \cdot u'$$

$$F(x) = 2e^u = 2e^{\sqrt{x}}$$

$$\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = \left[2e^{\sqrt{x}} \right]_1^4 = 2e^2 - 2e$$

i. $f(x) = x \ln(x^2 + 1)$

neem $u = x^2 + 1$ dan $u' = 2x$

$$x = \frac{1}{2}u'$$

$$f(x) = \frac{1}{2}u' \cdot \ln u = \frac{1}{2} \ln u \cdot u'$$

$$F(x) = \frac{1}{2}(u \ln u - u) = \frac{1}{2}u \ln u - \frac{1}{2}u = \frac{1}{2}(x^2 + 1) \ln(x^2 + 1) - \frac{1}{2}(x^2 + 1)$$

$$\int_1^2 x \ln(x^2 + 1) dx = \left[\frac{1}{2}(x^2 + 1) \ln(x^2 + 1) - \frac{1}{2}(x^2 + 1) \right]_1^2 = 2\frac{1}{2} \ln 5 - 2\frac{1}{2} - (\ln 2 - 1)$$

$$= 2\frac{1}{2} \ln 5 - \ln 2 - 1\frac{1}{2}$$

Opgave 9:

a. $f(x) = \frac{\ln^2 x}{x} = \ln^2 x \cdot \frac{1}{x}$

neem $u = \ln x$ dan $u' = \frac{1}{x}$

$$f(x) = u^2 \cdot u'$$

$$F(x) = \frac{1}{3}u^3 = \frac{1}{3} \ln^3 x$$

$$\int_1^2 \frac{\ln^2 x}{x} dx = \left[\frac{1}{3} \ln^3 x \right]_1^2 = \frac{1}{3} \ln^3 2 - \frac{1}{3} \ln^3 1 = \frac{1}{3} \ln^3 2$$

b. $f(x) = \frac{1}{x \ln x} = \frac{1}{x} \cdot \frac{1}{\ln x}$

neem $u = \ln x$ dan $u' = \frac{1}{x}$

$$f(x) = u' \cdot \frac{1}{u} = \frac{1}{u} \cdot u'$$

$$F(x) = \ln|u| = \ln|\ln|x||$$

$$\int_e^{e^2} \frac{1}{x \ln x} dx = [\ln(\ln x)]_e^{e^2} = \ln(\ln e^2) - \ln(\ln e) = \ln 2 - \ln 1 = \ln 2$$

c. $f(x) = \frac{x^2}{e^{2x^3}} = x^2 \cdot e^{-2x^3}$

neem $u = -2x^3$ dan $u' = -6x^2$

$$x^2 = -\frac{1}{6} u'$$

$$f(x) = -\frac{1}{6} u' \cdot e^u = -\frac{1}{6} e^u \cdot u'$$

$$F(x) = -\frac{1}{6} e^u = -\frac{1}{6} e^{-2x^3}$$

$$\int_0^1 \frac{x^2}{e^{-2x^3}} dx = \left[-\frac{1}{6} e^{-2x^3} \right]_0^1 = -\frac{1}{6} e^{-2} - \left(-\frac{1}{6} \right) = \frac{1}{6} - \frac{1}{6e^2}$$

Opgave 10:

a. $f(x) = \cos 2x \cdot \cos x = (1 - 2 \sin^2 x) \cdot \cos x = \cos x - 2 \cos x \cdot \sin^2 x$

$$g(x) = 2 \cos x \cdot \sin^2 x$$

neem $u = \sin x$ dan $u' = \cos x$

$$2 \cos x = 2u'$$

$$g(x) = 2u' \cdot u^2 = 2u^2 \cdot u'$$

$$G(x) = \frac{2}{3} u^3 = \frac{2}{3} \sin^3 x$$

$$F(x) = \sin x - \frac{2}{3} \sin^3 x + c$$

b. $g(x) = \sin^5 x = \sin^2 x \cdot \sin^2 x \cdot \sin x = (1 - \cos^2 x)(1 - \cos^2 x) \sin x$

$$= (1 - 2 \cos^2 x + \cos^4 x) \sin x = \sin x - 2 \sin x \cdot \cos^2 x + \sin x \cdot \cos^4 x$$

$$h(x) = -2 \sin x \cdot \cos^2 x$$

neem $u = \cos x$ dan $u' = -\sin x$

$$h(x) = 2u' \cdot u^2 = 2u^2 \cdot u'$$

$$H(x) = \frac{2}{3} u^3 = \frac{2}{3} \cos^3 x$$

$$j(x) = \sin x \cdot \cos^4 x$$

neem $u = \cos x$ dan $u' = -\sin x$

$$\sin x = -u'$$

$$j(x) = -u' \cdot u^4 = -u^4 \cdot u'$$

$$J(x) = -\frac{1}{5} u^5 = -\frac{1}{5} \cos^5 x$$

$$G(x) = -\cos x + \frac{2}{3} \cos^3 x - \frac{1}{5} \cos^5 x + c$$

Opgave 11:

$$a. \quad f(x) = \frac{2 + \ln x}{x} = \frac{2}{x} + \frac{\ln x}{x}$$

$$g(x) = \frac{\ln x}{x} = \ln x \cdot \frac{1}{x}$$

$$\text{neem } u = \ln x \text{ dan } u' = \frac{1}{x}$$

$$g(x) = u \cdot u'$$

$$G(x) = \frac{1}{2} u^2 = \frac{1}{2} \ln^2 x$$

$$f(x) = \frac{2 + \ln x}{x} = 0$$

$$2 + \ln x = 0$$

$$\ln x = -2$$

$$x = e^{-2} = \frac{1}{e^2}$$

$$\text{OppV} = \int_{\frac{1}{e^2}}^e \frac{2 + \ln x}{x} dx = \left[2 \ln x + \frac{1}{2} \ln^2 x \right]_{\frac{1}{e^2}}^e = 2 + \frac{1}{2} - (-4 + 2) = 4 \frac{1}{2}$$

$$b. \quad \int_1^p \frac{2 + \ln x}{x} dx = \left[2 \ln x + \frac{1}{2} \ln^2 x \right]_1^p = 2 \ln p + \frac{1}{2} \ln^2 p = 6$$

$$\frac{1}{2} \ln^2 p + 2 \ln p - 6 = 0$$

$$\ln^2 p + 4 \ln p - 12 = 0$$

$$(\ln p + 6)(\ln p - 2) = 0$$

$$\ln p = -6 \quad \vee \quad \ln p = 2$$

$$p = \frac{1}{e^6} \quad \vee \quad p = e^2$$

$$p > 1 \text{ dus } p = e^2$$

Opgave 12:

$$a. \quad \frac{4 \ln^2 x}{x} = \frac{1}{x}$$

$$4 \ln^2 x = 1$$

$$\ln^2 x = \frac{1}{4}$$

$$\ln x = \frac{1}{2} \quad \vee \quad \ln x = -\frac{1}{2}$$

$$x = e^{\frac{1}{2}} = \sqrt{e} \quad \vee \quad x = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$$

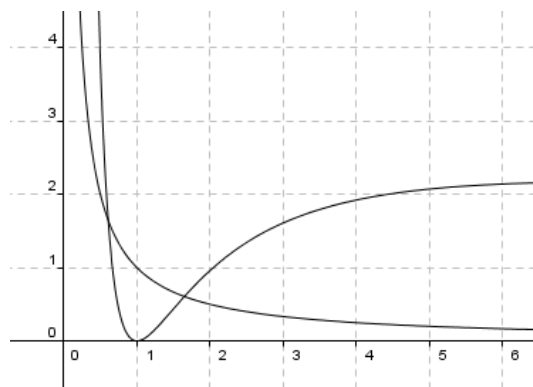
$$\frac{1}{\sqrt{e}} \leq x \leq \sqrt{e}$$

$$b. \quad f(x) = \frac{4 \ln^2 x}{x} = 4 \ln^2 x \cdot \frac{1}{x}$$

$$\text{neem } u = \ln x \text{ dan } u' = \frac{1}{x}$$

$$f(x) = 4u^2 \cdot u'$$

$$F(x) = \frac{4}{3} u^3 = \frac{4}{3} \ln^3 x$$



$$OppV = \int_{\frac{1}{\sqrt{e}}}^{\sqrt{e}} \left(\frac{1}{x} - \frac{4 \ln^2 x}{x} \right) dx = \left[\ln x - \frac{4}{3} \ln^3 x \right]_{\frac{1}{\sqrt{e}}}^{\sqrt{e}} = \frac{1}{2} - \frac{1}{6} - \left(-\frac{1}{2} + \frac{1}{6} \right) = \frac{2}{3}$$

Opgave 13:

a. $f(x) = \frac{3x^2}{x^3 + 4}$

$$f'(x) = \frac{(x^3 + 4) \cdot 6x - 3x^2 \cdot 3x^2}{(x^3 + 4)^2} = \frac{6x^4 + 24x - 9x^4}{(x^3 + 4)^2} = \frac{-3x^4 + 24x}{(x^3 + 4)^2} = 0$$

$$-3x^4 + 24x = 0$$

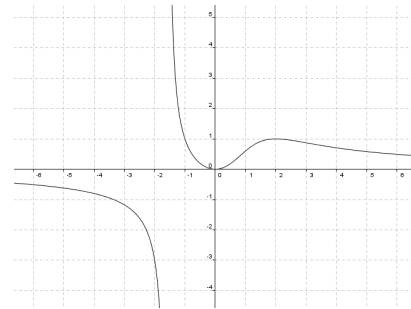
$$-3x(x^3 - 8) = 0$$

$$x = 0 \quad \vee \quad x^3 = 8$$

$$x = 0 \quad \vee \quad x = 2$$

$$y = 0 \quad y = 1$$

dus $0 < p < 1$



b. $f(x) = \frac{3x^2}{x^3 + 4}$

neem $u = x^3 + 4$ dan $u' = 3x^2$

$$f(x) = \frac{u'}{u} = \frac{1}{u} \cdot u'$$

$$F(x) = \ln u = \ln(x^3 + 4)$$

$$OppV = \int_0^p \frac{3x^2}{x^3 + 4} dx = \left[\ln(x^3 + 4) \right]_0^p = \ln(p^3 + 4) - \ln 4 = 2$$

$$\ln\left(\frac{p^3 + 4}{4}\right) = 2$$

$$\ln\left(\frac{1}{4}p^3 + 1\right) = 2$$

$$\frac{1}{4}p^3 + 1 = e^2$$

$$\frac{1}{4}p^3 = e^2 - 1$$

$$p^3 = 4e^2 - 4$$

$$p = \sqrt[3]{4e^2 - 4}$$