K.2 Partieel integreren

Opgave 14:

a. $\left[f(x)\cdot g(x)\right]' = f'(x)\cdot g(x) + f(x)\cdot g'(x)$

$d(f(x)\cdot g(x)) = d(f(x))\cdot g(x) + f(x)\cdot d(g(x)) = g(x)\cdot df(x) + f(x)\cdot dg(x)$

b. $d(f(x)\cdot g(x)) = g(x)\cdot df(x) + f(x)\cdot dg(x)$

$-g(x)\cdot df(x) = -d(f(x)\cdot g(x)) + f(x)\cdot dg(x)$

$g(x)\cdot df(x) = d(f(x)\cdot g(x)) - f(x)\cdot dg(x)$

c. $f(x) = \frac{1}{2}x^2$ en $g(x) = \ln x$ invullen in $g(x)\cdot df(x) = d(f(x)\cdot g(x)) - f(x)\cdot dg(x)$ geeft:

$\ln x \cdot d\,\frac{1}{2}x^2 = d(\frac{1}{2}x^2\cdot \ln x) - \frac{1}{2}x^2\cdot d\ln x$

d. herleiden van het rechterlid geeft:

$\ln x \cdot d\,\frac{1}{2}x^2 = d(\frac{1}{2}x^2\cdot \ln x) - \frac{1}{2}x^2\cdot \frac{1}{x}dx$

$\ln x \cdot d\,\frac{1}{2}x^2 = d(\frac{1}{2}x^2\cdot \ln x) - \frac{1}{2}xdx$

e. dit kun je verder herleiden tot:

$\ln x \cdot d\,\frac{1}{2}x^2 = d(\frac{1}{2}x^2\cdot \ln x) - d\,\frac{1}{4}x^2$

$\ln x \cdot d\,\frac{1}{2}x^2 = d(\frac{1}{2}x^2\cdot \ln x - \frac{1}{4}x^2)$

f. $H(x) = \frac{1}{2}x^2\cdot \ln x - \frac{1}{4}x^2$ is een primitieve van $h(x) = x\ln x$

Opgave 15:

$\int x\cdot \sin x dx = \frac{1}{2}x^2\cdot \sin x - \int \frac{1}{2}x^2\cdot \cos x dx$

maar nu moet je $\frac{1}{2}x^2\cdot \cos x$ primitiveren, wat je niet lukt

Opgave 16:

a. $\int x\cdot e^{2x}dx = \frac{1}{2}e^{2x}\cdot x - \int \frac{1}{2}e^{2x}\cdot 1dx = \frac{1}{2}x\cdot e^{2z} - \frac{1}{4}e^{2x} + c$

b. $\int 2x\cdot \cos x dx = \sin x\cdot 2x - \int \sin x\cdot 2dx = 2x\cdot \sin x + 2\cos x + c$

c. $\int x\cdot \ln^3 x dx = \frac{1}{2}x^x\cdot \ln^3 x - \int \frac{1}{2}x^2\cdot 3\ln^2 x\cdot \frac{1}{x}dx$

$= \frac{1}{2}x^2\cdot \ln^3 x - \int 1\frac{1}{2}x\cdot \ln^2 x dx$

$= \frac{1}{2}x^2\cdot \ln^3 x - \left(\frac{3}{4}x^2\cdot \ln^2 x - \int \frac{3}{4}x^2\cdot 2\ln x\cdot \frac{1}{x}dx\right)$

$= \frac{1}{2}x^2\cdot \ln^3 x - \frac{3}{4}x^2\cdot \ln^2 x + \int 1\frac{1}{2}x\cdot \ln x dx$

$= \frac{1}{2}x^2\cdot \ln^3 x - \frac{3}{4}x^2\cdot \ln^2 x + \frac{3}{4}x^2\cdot \ln x - \int \frac{3}{4}x^2\cdot \frac{1}{x}dx$

$= \frac{1}{2}x^2\cdot \ln^3 x - \frac{3}{4}x^2\cdot \ln^2 x + \frac{3}{4}x^2\cdot \ln x - \int \frac{3}{4}x dx$

$= \frac{1}{2}x^2\cdot \ln^3 x - \frac{3}{4}x^2\cdot \ln^2 x + \frac{3}{4}x^2\cdot \ln x - \frac{3}{8}x^2 + c$

d. $\int x^3\cdot \ln x dx = \frac{1}{4}x^4\cdot \ln x - \int \frac{1}{4}x^4\cdot \frac{1}{x}dx$

$= \frac{1}{4}x^4\cdot \ln x - \int \frac{1}{4}x^3 dx$

$= \frac{1}{4}x^4\cdot \ln x - \frac{1}{16}x^4 + c$

Opgave 17:

a. $\int 2x(e^x+1)dx = (e^x+x)\cdot 2x - \int (e^x+x)\cdot 2dx$

$= 2xe^x + 2x^2 - \int (2e^x+2x)dx$

$= 2xe^x + 2x^2 - (2e^x + x^2)$

$= 2xe^x + 2x^2 - 2e^x - x^2$

$= 2xe^x - 2e^x + x^2$

$\int_0^1 2x(e^x+1)dx = \left[2xe^x - 2e^x + x^2\right]_0^1 = 2e - 2e + 1 - (0-2-0) = 3$

b. $\int (3x+1)\sin x dx = -\cos x\cdot(3x+1) - \int -\cos x \cdot 3dx$

$= -(3x+1)\cdot\cos x + \int 3\cos x dx$

$= -(3x+1)\cdot \cos x + 3\sin x$

$\int_0^{\pi} (3x+1)\sin x dx = \left[-(3x+1)\cdot\cos x + 3\sin x\right]_0^{\pi} = -(3\pi+1)\cdot -1 + 0 - (-1+0) = 3\pi + 2$

Opgave 18:

$\int 1\cdot \ln x dx = x\cdot \ln x - \int x \cdot \frac{1}{x}dx = x\cdot\ln x - \int 1dx = x\cdot \ln x - x$

Opgave 19:

a. $f(x) = x^2\cdot \ln x$

$f'(x) = 2x\cdot\ln x + x^2\cdot\frac{1}{x} = 2x\cdot \ln x + x = 0$

$x(2\ln x + 1) = 0$

$x = 0 \quad \vee \quad 2\ln x = -1$

k.n. $\quad \ln x = -\frac{1}{2}$

$x = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$

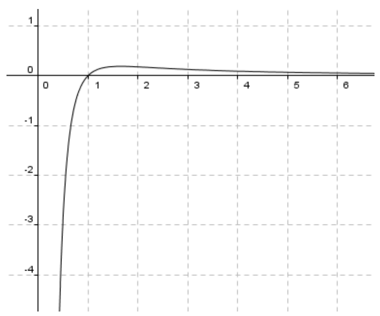
$y = -\frac{1}{2e}$

b. $\int x^2\cdot \ln x dx = \frac{1}{3}x^3\cdot\ln x - \int \frac{1}{3}x^3\cdot\frac{1}{x}dx = \frac{1}{3}x^3\cdot \ln x - \int \frac{1}{3}x^2 dx = \frac{1}{3}x^3\cdot\ln x - \frac{1}{9}x^3$

$OppV = \int_1^e x^2\cdot\ln x dx = \left[\frac{1}{3}x^3\cdot \ln x - \frac{1}{9}x^3\right]_1^e = \frac{1}{3}e^3 - \frac{1}{9}e^3 - (0-\frac{1}{9}) = \frac{2}{9}e^3 + \frac{1}{9}$

Opgave 20:

a. $f(x) = \frac{\ln x}{x^2}$



$f'(x) = \frac{x^2\cdot\frac{1}{x} - \ln x\cdot 2x}{x^4} = \frac{x - 2x\ln x}{x^4} = 0$

$x - 2x\ln x = 0$

$x(1-2\ln x) = 0$

$x = 0 \quad \vee \quad 2\ln x = 1$

k.n. $\ln x = \frac{1}{2}$

$x = e^{\frac{1}{2}} = \sqrt{e}$

$y = \frac{\frac{1}{2}}{e} = \frac{1}{2e}$

$B_f = \langle \leftarrow, \frac{1}{2e}]$

b. $\int \frac{\ln x}{x^2} dx = \int \ln x \cdot \frac{1}{x^2} dx = -\frac{1}{x} \cdot \ln x - \int -\frac{1}{x} \cdot \frac{1}{x} dx = -\frac{\ln x}{x} + \int \frac{1}{x^2} dx = -\frac{\ln x}{x} - \frac{1}{x}$

$OppV = \int_1^e \frac{\ln x}{x^2} dx = \left[-\frac{\ln x}{x} - \frac{1}{x} \right]_1^e = -\frac{1}{e} - \frac{1}{e} - (0-1) = 1 - \frac{2}{e}$

Opgave 21:

a. $f(x) = x^2 e^x$

$dx^2 e^x - 2xe^x dx = 2xe^x dx + x^2 e^x dx - 2xe^x dx = x^2 e^x dx$

b. $d(2xe^x - 2e^x) = (2e^x + 2xe^x - 2e^x)dx = 2xe^x dx$

c. $x^2 e^x dx = dx^2 e^x - 2xe^x dx$

$= dx^2 e^x - d(2xe^x - 2e^x)$

$= dx^2 e^x - d2xe^x + d2e^x$

$= d(x^2 e^x - 2xe^x + 2e^x)$

$= d((x^2 - 2x + 2)e^x)$

dus als $F(x) = (x^2 - 2x + 2)e^x + c$ dan $F'(x) = x^2 e^x = f(x)$

Opgave 22:

$\int e^x \sin x dx = e^x \sin x - \int e^x \cos x dx$

$= e^x \sin x - \left(e^x \cdot \cos x - \int e^x \cdot -\sin x dx\right)$

$= e^x \sin x - e^x \cos x - \int e^x \sin x dx$

$2 \cdot \int e^x \sin x dx = e^x \sin x - e^x \cos x$

$\int e^x \sin x dx = \frac{1}{2} e^x \sin x - \frac{1}{2} e^x \cos x + c$

Opgave 23:

a. $\int \frac{1}{4} x^2 \cos x dx = \sin x \cdot \frac{1}{4} x^2 - \int \sin x \cdot \frac{1}{2} x dx$

$= \frac{1}{4} x^2 \cdot \sin x - \left(-\cos x \cdot \frac{1}{2} x - \int -\cos x \cdot \frac{1}{2} dx\right)$

$= \frac{1}{4} x^2 \cdot \sin x + \frac{1}{2} x \cdot \cos x - \int \frac{1}{2} \cos x dx$

$= \frac{1}{4} x^2 \cdot \sin x + \frac{1}{2} x \cdot \cos x - \frac{1}{2} \sin x + c$

b. $\int e^{-x} \cdot \cos x dx = \sin x \cdot e^{-x} - \int \sin x \cdot -e^{-x} dx$

$= \sin x \cdot e^{-x} + \int \sin x \cdot e^{-x} dx$

$= \sin x \cdot e^{-x} + -\cos x \cdot e^{-x} - \int -\cos x \cdot -e^{-x} dx$

$= \sin x \cdot e^{-x} - \cos x \cdot e^{-x} - \int \cos x \cdot e^{-x} dx$

$2 \cdot \int e^{-x} \cdot \cos x = \sin x \cdot e^{-x} - \cos x \cdot e^{-x}$

$$\int e^{-x} \cdot \cos x dx = \tfrac{1}{2}\sin x \cdot e^{-x} - \tfrac{1}{2}\cos x \cdot e^{-x} + c$$

c. $\int e^{2x} \cdot \sin x dx = -\cos x \cdot e^{2x} - \int -\cos x \cdot 2e^{2x} dx$

$$= -\cos x \cdot e^{2x} + \int 2\cos x \cdot e^{2x} dx$$

$$= -\cos x \cdot e^{2x} + 2\sin x \cdot e^{2x} - \int 2\sin x \cdot 2e^{2x} dx$$

$$= -\cos x \cdot e^{2x} + 2\sin x \cdot e^{2x} - 4 \cdot \int \sin x \cdot e^{2x} dx$$

$$5 \cdot \int \sin x \cdot e^{2x} dx = -\cos x \cdot e^{2x} + 2\sin x \cdot e^{2x}$$

$$\int \sin x \cdot e^{2x} dx = -\tfrac{1}{5}\cos x \cdot e^{2x} + \tfrac{2}{5}\sin x \cdot e^{2x} + c$$

Opgave 24:

a. $\int (x^2 - x)e^x = e^x \cdot (x^2 - x) - \int e^x \cdot (2x-1) dx$

$$= (x^2 - x)e^x - \left(e^x \cdot (2x-1) - \int e^x \cdot 2dx\right)$$

$$= x^2 e^x - xe^x - 2xe^x + e^x + 2e^x$$

$$= x^2 e^x - 3xe^x + 3e^x$$

$$= (x^2 - 3x + 3)e^x$$

$$\int_1^3 (x^2 - x)e^x dx = \left[(x^2 - 3x + 3)e^x\right]_1^3 = 3e^3 - e$$

b. $\int x \ln^2 x dx = \tfrac{1}{2}x^2 \cdot \ln^2 x - \int \tfrac{1}{2}x^2 \cdot 2\ln x \cdot \frac{1}{x} dx$

$$= \tfrac{1}{2}x^2 \cdot \ln^2 x - \int x \ln x dx$$

$$= \tfrac{1}{2}x^2 \cdot \ln^2 x - \left(\tfrac{1}{2}x^2 \cdot \ln x - \int \tfrac{1}{2}x^2 \cdot \frac{1}{x} dx\right)$$

$$= \tfrac{1}{2}x^2 \cdot \ln^2 x - \tfrac{1}{2}x^2 \cdot \ln x + \int \tfrac{1}{2}x dx$$

$$= \tfrac{1}{2}x^2 \cdot \ln^2 x - \tfrac{1}{2}x^2 \cdot \ln x + \tfrac{1}{4}x^2$$

$$\int_1^e x \ln^2 x dx = \left[\tfrac{1}{2}x^2 \cdot \ln^2 x - \tfrac{1}{2}x^2 \cdot \ln x + \tfrac{1}{4}x^2\right]_1^e = \tfrac{1}{2}e^2 - \tfrac{1}{2}e^2 + \tfrac{1}{4}e^2 - \tfrac{1}{4} = \tfrac{1}{4}e^2 - \tfrac{1}{4}$$

Opgave 25:

a. $f(x) = (2x^2 + x - 1)e^x$

$f'(x) = (4x+1)e^x + (2x^2 + x - 1)e^x = (2x^2 + 5x)e^x = 0$

$2x^2 + 5x = 0 \quad \vee \quad e^x = 0$

$2x(x + 2\tfrac{1}{2}) = 0$ k.n.

$x = 0 \quad \vee \quad x = -2\tfrac{1}{2}$

max $f(-2\tfrac{1}{2}) = 9e^{-2\frac{1}{2}} = \frac{9}{e^2\sqrt{e}}$

min $f(0) = -1$

b. $f(x) = (2x^2 + x - 1)e^x = 0$

$2x^2 + x - 1 = 0 \quad \vee \quad e^x = 0$

$x^2 + \tfrac{1}{2}x - \tfrac{1}{2} = 0$ k.n.

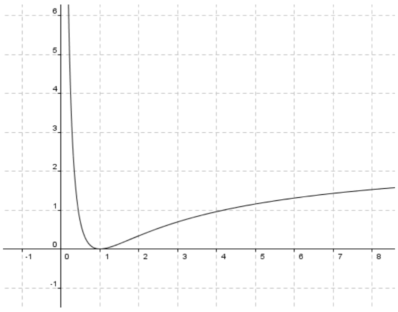
$(x+1)(x-\frac{1}{2})=0$

$x=-1 \quad \vee \quad x=\frac{1}{2}$

$$\int(2x^2+x-1)e^x dx = e^x\cdot(2x^2+x-1)-\int e^x\cdot(4x+1)dx$$
$$= 2x^2e^x+xe^x-e^x-\left(e^x\cdot(4x+1)-\int e^x\cdot 4dx\right)$$
$$= 2x^2e^x+xe^x-e^x-4xe^x-e^x+4e^x$$
$$= 2x^2e^x-3xe^x+2e^x$$

$$OppV=-\int_{-1}^{\frac{1}{2}}(2x^2+x-1)e^x dx=-\left[(2x^2-3x+2)e^x\right]_{-1}^{\frac{1}{2}}=-\left(e^{\frac{1}{2}}-7e^{-1}\right)=\tfrac{7}{e}-\sqrt{e}$$

Opgave 26:



a. $f(x)=\dfrac{\ln^2 x}{\sqrt{x}}=0$

$\ln^2 x=0$

$\ln x=0$

$x=1$

$$\int\frac{\ln^2 x}{\sqrt{x}}dx=2\sqrt{x}\cdot\ln^2 x-\int 2\sqrt{x}\cdot 2\ln x\cdot\frac{1}{x}dx$$
$$=2\sqrt{x}\cdot\ln^2 x-\int\frac{4}{\sqrt{x}}\cdot\ln x dx$$
$$=2\sqrt{x}\cdot\ln^2 x-\left(8\sqrt{x}\cdot\ln x-\int 8\sqrt{x}\cdot\frac{1}{x}dx\right)$$
$$=2\sqrt{x}\cdot\ln^2 x-8\sqrt{x}\cdot\ln x+\int\frac{8}{\sqrt{x}}dx$$
$$=2\sqrt{x}\cdot\ln^2 x-8\sqrt{x}\cdot\ln x+16\sqrt{x}$$

$$OppV=\int_1^e\frac{\ln^2 x}{\sqrt{x}}dx=\left[2\sqrt{x}\cdot\ln^2 x-8\sqrt{x}\cdot\ln x+16\sqrt{x}\right]_1^e=2\sqrt{e}-8\sqrt{e}+16\sqrt{e}-16$$
$$=10\sqrt{e}-16$$

b. $f^2(x)=\dfrac{\ln^4 x}{x}=\ln^4 x\cdot\dfrac{1}{x}$

neem $u=\ln x$ dan $u'=\dfrac{1}{x}$

$g(x)=\dfrac{\ln^4 x}{x}=u^4\cdot u'$

$G(x)=\frac{1}{5}u^5=\frac{1}{5}\ln^5 x$

$$\pi\cdot\int_1^e\frac{\ln^4 x}{x}dx=\pi\cdot\left[\tfrac{1}{5}\ln^5 x\right]_1^e=\pi\cdot(\tfrac{1}{5}-0)=\tfrac{1}{5}\pi$$