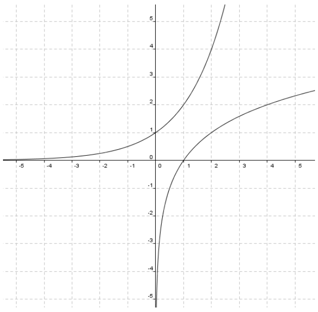
K3 Cyclometrische functies

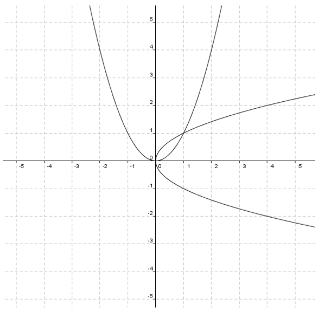
Opgave 27:

a.



b. ja, $y={}^2\log x$

c.



d. bij een functie hoort bij iedere x één of geen y

e. ja, $y=\sqrt{x}$

Opgave 28:

a. $\frac{1}{\tan^2 t+1}d(\tan t)=\frac{1}{\tan^2 t+1}\cdot(\tan^2 t+1)dt=1dt=dt$

b. $F(x)=\tan^{-1}(x)$ en $x=\tan t$ ofwel $t=\tan^{-1}(x)$ geeft:

$$F'(x)=\frac{dF(x)}{dx}=\frac{dt}{dx}=\frac{1}{\frac{dx}{dt}}=\frac{1}{\tan^2 t+1}=\frac{1}{x^2+1}$$

Opgave 29:

a. er moet gelden: $-\frac{1}{2}\pi<\arctan x<\frac{1}{2}\pi$

b. $\sqrt{3}\approx 1{,}73>\frac{1}{2}\pi$

Opgave 30:

x	$-\sqrt{3}$	-1	$-\frac{1}{3}\sqrt{3}$	0	$\frac{1}{3}\sqrt{3}$	1	$\sqrt{3}$
$\arctan x$	$-\frac{1}{3}\pi$	$-\frac{1}{4}\pi$	$-\frac{1}{6}\pi$	0	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$

Opgave 31:

a. $\arctan x=\frac{1}{3}\pi$

$x=\tan\frac{1}{3}\pi=\sqrt{3}$

b. $\arctan(x-2)=-\frac{1}{4}\pi$

$x-2=\tan(-\frac{1}{4}\pi)=-1$

$x=1$

c. $\arctan(x^2-1)=\frac{1}{4}\pi$

$x^2-1=\tan\frac{1}{4}\pi=1$

$x^2=2$

$x=\sqrt{2} \quad \vee \quad x=-\sqrt{2}$

d. $\arctan x=\frac{2}{3}\pi$

geen oplossingen

e. $\arctan x=\sqrt{2}$

$x=\tan\sqrt{2}=6{,}334$

f. $\arctan(x^2-1)=1$

$x^2-1=\tan 1=1{,}557$

$x^2=2{,}557$

$x=1{,}599 \quad \vee \quad x=-1{,}599$

<u>Opgave 32:</u>

a. $f(x)=2\arctan(\frac{1}{2}x)=2\arctan u$ met $u=\frac{1}{2}x$ dus $u'=\frac{1}{2}$

$$f'(x)=2\cdot\frac{1}{u^2+1}\cdot u'=2\cdot\frac{1}{(\frac{1}{2}x)^2+1}\cdot\frac{1}{2}=\frac{1}{\frac{1}{4}x^2+1}=\frac{4}{x^2+4}$$

b. $g(x)=\arctan(x-2)=\arctan u$ met $u=x-2$ dus $u'=1$

$$g'(x)=\frac{1}{u^2+1}\cdot u'=\frac{1}{(x-2)^2+1}\cdot 1=\frac{1}{(x-2)^2+1}$$

c. $h(x)=\arctan(x^2)=\arctan u$ met $u=x^2$ dus $u'=2x$

$$h'(x)=\frac{1}{u^2+1}\cdot u'=\frac{1}{(x^2)^2+1}\cdot 2x=\frac{2x}{x^4+!}$$

<u>Opgave 33:</u>

a. $$\int_{-\frac{1}{3}\sqrt{3}}^{\sqrt{3}}\frac{1}{x^2+1}dx=\left[\arctan x\right]_{-\frac{1}{3}\sqrt{3}}^{\sqrt{3}}=\arctan(\sqrt{3})-\arctan(-\tfrac{1}{3}\sqrt{3})=\tfrac{1}{3}\pi--\tfrac{1}{6}\pi=\tfrac{1}{2}\pi$$

b. $$\int_{-2}^{-1}\frac{1}{(x+1)^2+1}dx=\left[\arctan(x+1)\right]_{-2}^{-1}=\arctan(0)-\arctan(-1)=0--\tfrac{1}{4}\pi=\tfrac{1}{4}\pi$$

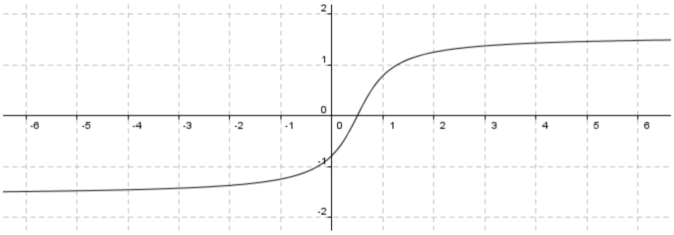
c. $$\int_{0}^{\frac{1}{3}\sqrt{3}}\frac{3}{(3x)^2+1}dx=\left[\arctan(3x)\right]_{0}^{\frac{1}{3}\sqrt{3}}=\arctan(\sqrt{3})-\arctan(0)=\tfrac{1}{3}\pi-0=\tfrac{1}{3}\pi$$

<u>Opgave 34:</u>

a. $f(x)=\arctan(2x-1)=\arctan(2(x-\frac{1}{2}))$

$$y=\arctan(x)\xrightarrow{V_{y-as,\frac{1}{2}}}y=\arctan(2x)\xrightarrow{T(\frac{1}{2},0)}y=\arctan(2(x-\tfrac{1}{2}))$$

dus de asymptoten blijven gelijk, dus $y=-\frac{1}{2}\pi$ en $y=\frac{1}{2}\pi$



b. $f(x) = \arctan(2x-1) = 0$

$2x-1 = \arctan(0) = 0$

$2x = 1$

$x = \frac{1}{2}$

$f'(x) = \dfrac{1}{(2x-1)^2+1} \cdot 2 = \dfrac{2}{(2x-1)^2+1}$

$f'(\frac{1}{2}) = 2$

$y = 2x + b$ door $(\frac{1}{2},0)$

$0 = 1 + b$

$b = -1$

$k: \quad y = 2x - 1$

c. $\arctan(2x-1) = -\frac{1}{4}\pi$

$2x-1 = \tan(-\frac{1}{4}\pi) = -1$

$2x = 0$

$x = 0$

$\arctan(2x-1) = \frac{1}{3}\pi$

$2x-1 = \tan(\frac{1}{3}\pi) = \sqrt{3}$

$2x = 1 + \sqrt{3}$

$x = \frac{1}{2} + \frac{1}{2}\sqrt{3}$

dus $0 < x < \frac{1}{2} + \frac{1}{2}\sqrt{3}$

d. $f(1) = \arctan(1) = \frac{1}{4}\pi$

dus $-\frac{1}{2}\pi < x \le \frac{1}{4}\pi$

Opgave 35:

a. $f(x) = \dfrac{1}{4x^2+1} = \dfrac{1}{(2x)^2+1}$

neem $u = 2x$ dan $u' = 2$

$1 = \frac{1}{2}u'$

$f(x) = \dfrac{\frac{1}{2}u'}{u^2+1} = \frac{1}{2} \cdot \dfrac{1}{u^2+1} \cdot u'$

$F(x) = \frac{1}{2}\arctan u + c = \frac{1}{2}\arctan(2x) + c$

b. $g(x) = \dfrac{4}{x^2+4} = \dfrac{1}{\frac{1}{4}x^2+1} = \dfrac{1}{(\frac{1}{2}x)^2+1}$

neem $u = \frac{1}{2}x$ dan $u' = \frac{1}{2}$

$1 = 2u'$

$g(x) = \dfrac{2u'}{u^2+1} = 2 \cdot \dfrac{1}{u^2+1} \cdot u'$

$G(x) = 2\arctan u + c = 2\arctan(\frac{1}{2}x) + c$

Opgave 36:

a. $f(x)=\frac{12}{16x^2+!}=\frac{12}{(4x)^2+1}$

neem $u=4x$ dan $u'=4$

$12=3u'$

$f(x)=\frac{3u'}{u^2+1}=3\cdot\frac{1}{u^2+1}\cdot u'$

$F(x)=3\arctan u+c=3\arctan(4x)+c$

b. $g(x)=\frac{4}{x^2+4}=\frac{1}{\frac{1}{4}x^2+1}=\frac{1}{(\frac{1}{2}x)^2+1}$

neem $u=\frac{1}{2}x$ dan $u'=\frac{1}{2}$

$1=2u'$

$g(x)=\frac{2u'}{u^2+1}=2\cdot\frac{1}{u^2+1}\cdot u'$

$G(x)=2\arctan u+c=2\arctan(\frac{1}{2}x)+c$

c. $h(x)=\arctan(2x)$

$$\int 1\cdot\arctan(2x)dx=x\cdot\arctan(2x)-\int x\cdot\frac{1}{(2x)^2+1}\cdot 2dx$$

$$=x\cdot\arctan(2x)-\int\frac{2x}{(2x)^2+1}dx$$

$$=x\cdot\arctan(2x)-\tfrac{1}{4}\ln((2x)^2+1)+c$$

d. $j(x)=\frac{3}{x^2+4x+5}=\frac{3}{(x+2)^2+1}$

neem $u=x+2$ dan $u'=1$

$j(x)=\frac{3u'}{u^2+1}=3\cdot\frac{1}{u^2+1}\cdot u'$

$J(x)=3\arctan u+c=3\arctan(x+2)+c$

Opgave 37:

a. $f(x)=\frac{3}{x^2+3}=\frac{1}{\frac{1}{3}x^2+1}=\frac{1}{(\frac{1}{3}\sqrt{3}\cdot x)^2+1}$

neem $u=\frac{1}{3}\sqrt{3}\cdot x$ dan $u'=\frac{1}{3}\sqrt{3}$

$1=\frac{1}{\frac{1}{3}\sqrt{3}}\cdot u'=\frac{3}{\sqrt{3}}\cdot u'=\sqrt{3}\cdot u'$

$f(x)=\frac{\sqrt{3}\cdot u'}{u^2+1}=\sqrt{3}\cdot\frac{1}{u^2+1}\cdot u'$

$F(x)=\sqrt{3}\cdot\arctan u=\sqrt{3}\cdot\arctan(\frac{1}{3}\sqrt{3}\cdot x)$

$$\int_0^1\frac{3}{x^2+3}dx=\left[\sqrt{3}\cdot\arctan(\tfrac{1}{3}\sqrt{3}\cdot x)\right]_0^1=\sqrt{3}\cdot\arctan(\tfrac{1}{3}\sqrt{3})-\sqrt{3}\cdot\arctan(0)=\sqrt{3}\cdot\tfrac{1}{6}\pi-\sqrt{3}\cdot 0$$

$$=\tfrac{1}{6}\pi\sqrt{3}$$

b. $f(x)=\frac{x}{x^4+1}=\frac{x}{(x^2)^2+1}$

neem $u = x^2$ dan $u' = 2x$

$x = \frac{1}{2}u'$

$f(x) = \dfrac{\frac{1}{2}u'}{u^2+1} = \frac{1}{2} \cdot \dfrac{1}{u^2+1} \cdot u'$

$F(x) = \frac{1}{2}\arctan u = \frac{1}{2}\arctan(x^2)$

$$\int_0^1 \frac{x}{x^4+1}\,dx = \left[\tfrac{1}{2}\arctan(x^2)\right]_0^1 = \tfrac{1}{2}\arctan(1) - \tfrac{1}{2}\arctan(0) = \tfrac{1}{2}\cdot\tfrac{1}{4}\pi - \tfrac{1}{2}\cdot 0 = \tfrac{1}{8}\pi$$

c. $f(x) = \dfrac{5}{x^2-6x+18} = \dfrac{5}{(x-3)^2+9} = \dfrac{\frac{5}{9}}{\frac{1}{9}(x-3)^2+1} = \dfrac{\frac{5}{9}}{(\frac{1}{3}(x-3))^2+1} = \dfrac{\frac{5}{9}}{(\frac{1}{3}x-1)^2+1}$

neem $u = \frac{1}{3}x - 1$ dan $u' = \frac{1}{3}$

$\frac{5}{9} = \frac{5}{3}u'$

$f(x) = \dfrac{\frac{5}{3}u'}{u^2+1} = \frac{5}{3} \cdot \dfrac{1}{u^2+1} \cdot u'$

$F(x) = \frac{5}{3}\arctan u = \frac{5}{3}\arctan(\frac{1}{3}x-1)$

$$\int_3^6 \frac{5}{x^2-6x+18}\,dx = \left[\tfrac{5}{3}\arctan(\tfrac{1}{3}x-1)\right]_3^6 = \tfrac{5}{3}\arctan(1) - \tfrac{5}{3}\arctan(0) = \tfrac{5}{3}\cdot\tfrac{1}{4}\pi - 0 = \tfrac{5}{12}\pi$$

d. $f(x) = \dfrac{\arctan x}{x^2+1} = \arctan x \cdot \dfrac{1}{x^2+1}$

neem $u = \arctan x$ dan $u' = \dfrac{1}{x^2+1}$

$f(x) = u \cdot u'$

$F(x) = \frac{1}{2}u^2 = \frac{1}{2}(\arctan x)^2$

$$\int_0^{\sqrt{3}} \frac{\arctan x}{x^2+1}dx = \left[\tfrac{1}{2}(\arctan x)^2\right]_0^{\sqrt{3}} = \tfrac{1}{2}(\arctan(\sqrt{3}))^2 - \tfrac{1}{2}(\arctan(0))^2 = \tfrac{1}{2}(\tfrac{1}{3}\pi)^2 - 0 = \tfrac{1}{18}\pi^2$$

<u>Opgave 38:</u>

a. $f(x) = \dfrac{2}{4x^2+9} = \dfrac{\frac{2}{9}}{\frac{4}{9}x^2+1} = \dfrac{\frac{2}{9}}{(\frac{2}{3}x)^2+1}$

neem $u = \frac{2}{3}x$ dan $u' = \frac{2}{3}$

$\frac{2}{9} = \frac{1}{3}u'$

$f(x) = \dfrac{\frac{1}{3}u'}{u^2+1} = \frac{1}{3} \cdot \dfrac{1}{u^2+1} \cdot u'$

$F(x) = \frac{1}{3}\arctan u = \frac{1}{3}\arctan(\frac{2}{3}x)$

$$\int_0^{1\frac{1}{2}} \frac{2}{4x^2+9}\,dx = \left[\tfrac{1}{3}\arctan(\tfrac{2}{3}x)\right]_0^{1\frac{1}{2}} = \tfrac{1}{3}\arctan 1 - \tfrac{1}{3}\arctan 0 = \tfrac{1}{3}\cdot\tfrac{1}{4}\pi - 0 = \tfrac{1}{12}\pi$$

b. $f(x) = \dfrac{e^x}{e^{2x}+1} = \dfrac{e^x}{(e^x)^2+1} = \dfrac{1}{(e^x)^2+1} \cdot e^x$

neem $u = e^x$ dan $u' = e^x$

$f(x) = \dfrac{1}{u^2+1} \cdot u'$

$F(x) = \arctan u = \arctan(e^x)$

$$\int_0^{\frac{1}{2}\ln 3} \frac{e^x}{e^{2x}+1}\,dx = \left[\arctan(e^x)\right]_0^{\frac{1}{2}\ln 3} = \arctan(e^{\frac{1}{2}\ln 3}) - \arctan e^0 = \arctan(e^{\ln\sqrt{3}}) - \arctan 1$$

$$= \arctan(\sqrt{3}) - \arctan 1 = \tfrac{1}{3}\pi - \tfrac{1}{4}\pi = \tfrac{1}{12}\pi$$

c. $f(x) = \dfrac{1}{2x^2-2x+1} = \dfrac{2}{4x^2-4x+2} = \dfrac{2}{(2x-1)^2-1+2} = \dfrac{2}{(2x-1)^2+1}$

neem $u = 2x-1$ dan $u' = 2$

$f(x) = \dfrac{u'}{u^2+1} = \dfrac{1}{u^2+1}\cdot u'$

$F(x) = \arctan u = \arctan(2x-1)$

$$\int_0^1 \frac{1}{2x^2-2x+1}\,dx = \left[\arctan(2x-1)\right]_0^1 = \arctan 1 - \arctan(-1) = \tfrac{1}{4}\pi - -\tfrac{1}{4}\pi = \tfrac{1}{2}\pi$$

d. $f(x) = \dfrac{\ln(x^2+1)}{x^2} = \dfrac{1}{x^2}\cdot\ln(x^2+1)$

partieel integreren geeft:

$$F(x) = \frac{-1}{x}\cdot\ln(x^2+1) - \int \frac{-1}{x}\cdot\frac{1}{x^2+1}\cdot 2x\,dx = \frac{-\ln(x^2+1)}{x} + \int\frac{2}{x^2+1}\,dx =$$

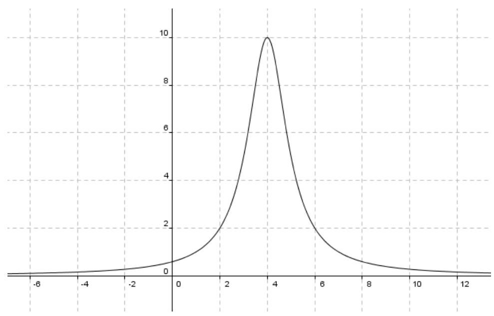
$$= \frac{-\ln(x^2+1)}{x} + 2\arctan(x)$$

$$\int_1^{\sqrt{3}} \frac{\ln(x^2+1)}{x^2}\,dx = \left[\frac{-\ln(x^2+1)}{x} + 2\arctan(x)\right]_1^{\sqrt{3}}$$

$$= \frac{-\ln 4}{\sqrt{3}} + 2\arctan(\sqrt{3}) - (-\ln 2 + 2\arctan 1) = \frac{-\ln 4}{\sqrt{3}} + \tfrac{2}{3}\pi + \ln 2 - \tfrac{1}{2}\pi$$

$$= \frac{-\ln 4}{\sqrt{3}} + \ln 2 + \tfrac{1}{6}\pi$$

Opgave 39:



a. $f(x) = \dfrac{10}{x^2-8x+17}$

$f'(x) = \dfrac{0-10\cdot(2x-8)}{(x^2-8x+17)^2} = \dfrac{-20x+80}{(x^2-8x+17)^2} = 0$

$-20x+80 = 0$

$-20x = -80$

$x = 4$

$f(x) = 10$

$B_f = \langle 0{,}10]$

b. $f(4+p) = \dfrac{10}{(4+p)^2-8(4+p)+17} = \dfrac{10}{16+8p+p^2-32-8p+17} = \dfrac{10}{p^2+1}$

$f(4-p) = \dfrac{10}{(4-p)^2-8(4-p)+17} = \dfrac{10}{16-8p+p^2-32+8p+17} = \dfrac{10}{p^2+1}$

dus voor iedere p geldt: $f(4+p) = f(4-p)$

dus de grafiek van *f* is symmetrisch in de lijn $x = 4$

c. $f(x) = \dfrac{10}{x^2 - 8x + 17} = \dfrac{10}{(x-4)^2 - 16 + 17} = \dfrac{10}{(x-4)^2 + 1}$

$F(x) = 10\arctan(x-4)$

$Opp(V) = \int_4^5 \dfrac{10}{x^2 - 8x + 17}\,dx = \left[10\arctan(x-4)\right]_4^5 = 10\arctan 1 - 10\arctan 0 = 2\tfrac{1}{2}\pi$

d. $\int_4^p \dfrac{10}{x^2 - 8x + 17}\,dx = \left[10\arctan(x-4)\right]_4^p = 10\arctan(p-4) - 10\arctan 0 = 10$

$10\arctan(p-4) = 10$

$\arctan(p-4) = 1$

$p - 4 = \tan(1)$

$p = 4 + \tan(1) = 5{,}557$

Opgave 40:

a. voor iedere *x* met $D_f = [-\tfrac{1}{2}\pi, \tfrac{1}{2}\pi]$ is er precies één functiewaarde $f(x)$

b. II $[0, \pi]$

Opgave 41:

x	-1	$-\frac{1}{2}\sqrt{3}$	$-\frac{1}{2}\sqrt{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1
$\arcsin(x)$	$-\frac{1}{2}\pi$	$-\frac{1}{3}\pi$	$-\frac{1}{4}\pi$	$-\frac{1}{6}\pi$	0	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$	$\frac{1}{2}\pi$
$\arccos(x)$	π	$\frac{5}{6}\pi$	$\frac{3}{4}\pi$	$\frac{2}{3}\pi$	$\frac{1}{2}\pi$	$\frac{1}{3}\pi$	$\frac{1}{4}\pi$	$\frac{1}{6}\pi$	0

Opgave 42:

a. $\arcsin x = \tfrac{1}{2}\pi$

$x = \sin(\tfrac{1}{2}\pi) = 1$

b. $\arccos(x) = \tfrac{1}{2}\pi$

$x = \cos(\tfrac{1}{2}\pi) = 0$

c. $\arcsin x = -\tfrac{1}{6}\pi$

$x = \sin(-\tfrac{1}{6}\pi) = -\tfrac{1}{2}$

d. $\arccos x = -\tfrac{1}{6}\pi$ bestaat niet want $-\tfrac{1}{6}\pi < 0$

e. $\arcsin x = 2$ bestaat niet want $2 > \tfrac{1}{2}\pi$

f. $\arccos x = 2$
$x = \cos(2) = -0{,}416$

g. $3\arcsin(x - \sqrt{3}) = \pi$

$\arcsin(x - \sqrt{3}) = \tfrac{1}{3}\pi$

$x - \sqrt{3} = \sin(\tfrac{1}{3}\pi) = \tfrac{1}{2}\sqrt{3}$

$x = 1\tfrac{1}{2}\sqrt{3}$

h. $3\arccos(x - \sqrt{3}) = \pi$

$\arccos(x - \sqrt{3}) = \tfrac{1}{3}\pi$

$x - \sqrt{3} = \cos(\frac{1}{3}\pi) = \frac{1}{2}$

$x = \frac{1}{2} + \sqrt{3}$

Opgave 43:

a. $-1 < 2 - x^2 < 1$

$-1 < 2 - x^2 \quad \wedge \quad 2 - x^2 < 1$

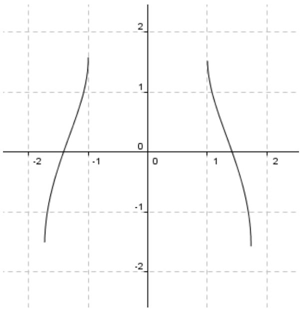
$x^2 < 3 \quad \wedge \quad -x^2 < -1$

$x^2 < 3 \quad \wedge \quad x^2 > 1$

$-\sqrt{3} \leq x \leq -1 \quad \vee \quad 1 \leq x \leq \sqrt{3}$

b. $B_f = [-\frac{1}{2}\pi, \frac{1}{2}\pi]$

c.



d. $f(x) = \arcsin(2 - x^2) = 0$

$2 - x^2 = \sin(0) = 0$

$-x^2 = -2$

$x^2 = 2$

$x_A = \sqrt{2}$

$f'(x) = \dfrac{1}{\sqrt{1-(2-x^2)^2}} \cdot -2x = \dfrac{-2x}{\sqrt{1-(2-x^2)^2}}$

$f'(\sqrt{2}) = \dfrac{-2\sqrt{2}}{\sqrt{1}} = -2\sqrt{2}$

$k: \quad y = -2\sqrt{2} \cdot x + b$ door $(\sqrt{2}, 0)$

$0 = -4 + b$

$b = 4$

$k: \quad y = -2\sqrt{2} \cdot x + 4$

e. $\arcsin(2 - x^2) < \frac{1}{6}\pi$

$\arcsin(2 - x^2) = \frac{1}{6}\pi$

$2 - x^2 = \sin(\frac{1}{6}\pi) = \frac{1}{2}$

$-x^2 = -1\frac{1}{2}$

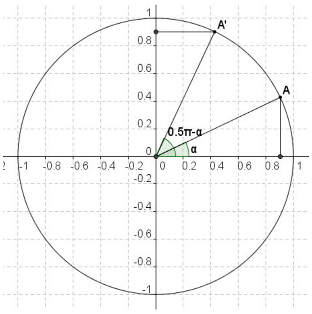
$x^2 = 1\frac{1}{2}$

$x = \sqrt{1\frac{1}{2}} = \sqrt{\frac{6}{4}} = \frac{1}{2}\sqrt{6} \quad \vee \quad x = -\frac{1}{2}\sqrt{6}$

$-\sqrt{3} \leq x < -\frac{1}{2}\sqrt{6} \quad \vee \quad \frac{1}{2}\sqrt{6} < x \leq \sqrt{3}$

Opgave 44:

a.



b. als $\cos\alpha = x$ dan $\sin(\frac{1}{2}\pi - \alpha) = x$

als $\cos\alpha = x$ dan $\arccos x = \alpha$

als $\sin(\frac{1}{2}\pi - \alpha) = x$ dan $\arcsin x = \frac{1}{2}\pi - \alpha$

$\arcsin x + \arccos x = \frac{1}{2}\pi - \alpha + \alpha = \frac{1}{2}\pi$

c. $[\arcsin x + \arccos x]' = [\frac{1}{2}\pi]'$

$\frac{1}{\sqrt{1-x^2}} + [\arccos x]' = 0$

$[\arccos x]' = \frac{-1}{\sqrt{1-x^2}}$

Opgave 45:

a. $f(x) = \frac{1}{\sqrt{1-9x^2}} = \frac{1}{\sqrt{1-(3x)^2}}$

neem $u = 3x$ dan $u' = 3$

$1 = \frac{1}{3}u'$

$f(x) = \frac{\frac{1}{3}u'}{\sqrt{1-u^2}} = \frac{1}{3} \cdot \frac{1}{\sqrt{1-u^2}} \cdot u'$

$F(x) = \frac{1}{3} \cdot \arcsin u = \frac{1}{3}\arcsin(3x)$

$\int_{\frac{1}{6}}^{\frac{1}{6}\sqrt{3}} \frac{1}{\sqrt{1-9x^2}}\,dx = \left[\frac{1}{3}\arcsin(3x)\right]_{\frac{1}{6}}^{\frac{1}{6}\sqrt{3}} = \frac{1}{3}\arcsin(\frac{1}{2}\sqrt{3}) - \frac{1}{3}\arcsin(\frac{1}{2}) = \frac{1}{9}\pi - \frac{1}{18}\pi = \frac{1}{18}\pi$

b. $f(x) = \frac{1}{\sqrt{9-x^2}} = \frac{1}{\sqrt{9(1-\frac{1}{9}x^2)}} = \frac{1}{3\sqrt{1-\frac{1}{9}x^2}} = \frac{1}{3} \cdot \frac{1}{\sqrt{1-(\frac{1}{3}x)^2}}$

neem $u = \frac{1}{3}x$ dan $u' = \frac{1}{3}$

$f(x) = u' \cdot \frac{1}{\sqrt{1-u^2}}$

$F(x) = \arcsin(u) = \arcsin(\frac{1}{3}x)$

$\int_{-1\frac{1}{2}\sqrt{2}}^{1\frac{1}{2}\sqrt{2}} \frac{1}{\sqrt{9-x^2}}\,dx = \left[\arcsin(\frac{1}{3}x)\right]_{-1\frac{1}{2}\sqrt{2}}^{1\frac{1}{2}\sqrt{2}} = \arcsin(\frac{1}{2}\sqrt{2}) - \arcsin(-\frac{1}{2}\sqrt{2}) = \frac{1}{4}\pi - -\frac{1}{4}\pi = \frac{1}{2}\pi$

c. $f(x) = \frac{x}{\sqrt{1-x^4}} = \frac{x}{\sqrt{1-(x^2)^2}}$

neem $u = x^2$ dan $u' = 2x$

$x = \frac{1}{2}u'$

$$f(x) = \frac{\frac{1}{2}u'}{\sqrt{1-u^2}} = \frac{1}{2} \cdot \frac{1}{\sqrt{1-u^2}} \cdot u'$$

$F(x) = \frac{1}{2}\arcsin u = \frac{1}{2}\arcsin(x^2)$

$$\int_0^{\frac{1}{2}\sqrt{2}} \frac{x}{\sqrt{1-x^4}}\,dx = \left[\tfrac{1}{2}\arcsin(x^2)\right]_0^{\frac{1}{2}\sqrt{2}} = \tfrac{1}{2}\arcsin(\tfrac{1}{2}) - \tfrac{1}{2}\arcsin(0) = \tfrac{1}{12}\pi$$

d. $$f(x) = \frac{x}{\sqrt{4-x^4}} = \frac{x}{\sqrt{4(1-\frac{1}{4}x^4)}} = \frac{x}{2\sqrt{1-(\frac{1}{2}x^2)^2}}$$

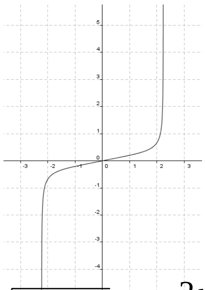
neem $u = \frac{1}{2}x^2$ dan $u' = x$

$$f(x) = \frac{u'}{2\sqrt{1-u^2}} = \frac{1}{2} \cdot \frac{1}{\sqrt{1-u^2}} \cdot u'$$

$F(x) = \frac{1}{2}\arcsin u = \frac{1}{2}\arcsin(\frac{1}{2}x^2)$

$$\int_0^1 \frac{x}{\sqrt{4-x^4}}\,dx = \left[\tfrac{1}{2}\arcsin(\tfrac{1}{2}x^2)\right]_0^1 = \tfrac{1}{2}\arcsin(\tfrac{1}{2}) - \tfrac{1}{2}\arcsin(0) = \tfrac{1}{12}\pi$$

Opgave 46:



a. $25 - x^4 > 0$

$-x^4 > -25$

$x^4 < 25$

$-\sqrt{5} < x < \sqrt{5}$

$D_f = \langle -\sqrt{5}, \sqrt{5} \rangle$

b. $$f'(x) = \frac{\sqrt{25-x^4}\cdot 1 - x \cdot \frac{1}{2\sqrt{25-x^4}} \cdot -4x^3}{25-x^4} = \frac{\sqrt{25-x^4} + \frac{2x^4}{\sqrt{25-x^4}}}{25-x^4}$$

$f'(\sqrt{3}) = \frac{4 + 4\frac{1}{2}}{16} = \frac{17}{32}$

$y_A = f(\sqrt{3}) = \frac{\sqrt{3}}{4} = \frac{1}{4}\sqrt{3}$

$k:\quad y = \frac{17}{32}x + b$ door $(\sqrt{3}, \frac{1}{4}\sqrt{3})$

$\frac{1}{4}\sqrt{3} = \frac{17}{32}\sqrt{3} + b$

$b = -\frac{9}{32}\sqrt{3}$

$k:\quad y = \frac{17}{32}x - \frac{9}{32}\sqrt{3}$

c. $$f(p) = \frac{p}{\sqrt{25-p^4}}$$

$$f(-p) = \frac{-p}{\sqrt{25-p^4}} = -f(p)$$

voor iedere p geldt: $f(p) = -f(-p)$

dus de grafiek van f is puntsymmetrisch ten opzichte van $(0,0)$

d. $f(x)=\dfrac{x}{\sqrt{25-x^4}}=\dfrac{x}{\sqrt{25(1-\frac{1}{25}x^4)}}=\dfrac{x}{5\sqrt{1-(\frac{1}{5}x^2)^2}}$

neem $u=\frac{1}{5}x^2$ dan $u'=\frac{2}{5}x$

$\frac{1}{5}x=\frac{1}{2}u'$

$f(x)=\dfrac{\frac{1}{2}u'}{\sqrt{1-u^2}}=\frac{1}{2}\cdot\dfrac{1}{\sqrt{1-u^2}}\cdot u'$

$F(x)=\frac{1}{2}\arcsin u=\frac{1}{2}\arcsin(\frac{1}{5}x^2)$

$OppV=\displaystyle\int_0^{\frac{1}{2}\sqrt{10}}\frac{x}{\sqrt{25-x^4}}dx=\left[\tfrac{1}{2}\arcsin(\tfrac{1}{5}x^2)\right]_0^{\frac{1}{2}\sqrt{10}}=\tfrac{1}{2}\arcsin(\tfrac{1}{2})-\tfrac{1}{2}\arcsin(0)=\tfrac{1}{12}\pi$

Opgave 47:

a. $\displaystyle\int\arcsin x\,dx=x\cdot\arcsin x-\int x\cdot\frac{1}{\sqrt{1-x^2}}dx=x\cdot\arcsin x+\sqrt{1-x^2}+c$

b. $g(x)=\dfrac{\arcsin x}{\sqrt{1-x^2}}=\arcsin x\cdot\dfrac{1}{\sqrt{1-x^2}}$

neem $u=\arcsin x$ dan $u'=\dfrac{1}{\sqrt{1-x^2}}$

$g(x)=u\cdot u'$

$G(x)=\frac{1}{2}u^2=\frac{1}{2}(\arcsin x)^2+c$