K4 Breuksplitsen

Opgave 48:

a. $f(x) = \dfrac{1}{x^2+1}$

$F(x) = \arctan x$

$g(x) = \dfrac{x}{x^2+1}$

neem $u = x^2+1$ dan $u' = 2x$

$x = \tfrac{1}{2}u'$

$g(x) = \dfrac{\tfrac{1}{2}u'}{u} = \tfrac{1}{2}\cdot\dfrac{1}{u}\cdot u'$

$G(x) = \tfrac{1}{2}\ln u = \tfrac{1}{2}\ln(x^2+1)$

b. $h(x) = \dfrac{x+1}{x^2+1} = \dfrac{x}{x^2+1} + \dfrac{1}{x^2+1} = g(x) + f(x)$

c. $H(x) = F(x) + G(x) = \arctan x + \tfrac{1}{2}\ln(x^2+1)$

Opgave 49:

a. $f(x) = \dfrac{2x+5}{x+1} = \dfrac{2x}{x+1} + \dfrac{5}{x+1}$

$\dfrac{5}{x+1}$ is wel te primitiveren, maar $\dfrac{2x}{x+1}$ niet

b. $f(x) = \dfrac{2x+5}{x+1} = \dfrac{2x+2+3}{x+1} = \dfrac{2x+2}{x+1} + \dfrac{3}{x+1} = 2 + \dfrac{3}{x+1}$

$F(x) = 2x + 3\ln|x+1|$

Opgave 50:

a. $f(x) = \dfrac{2x+1}{x+1} = \dfrac{2x+2-1}{x+1} = \dfrac{2x+2}{x+1} - \dfrac{1}{x+1} = 2 - \dfrac{1}{x+1}$

$F(x) = 2x - \ln|x+1| + c$

b. $f(x) = \dfrac{x}{x+1} = \dfrac{x+1-1}{x+1} = \dfrac{x+1}{x+1} - \dfrac{1}{x+1} = 1 - \dfrac{1}{x+1}$

$F(x) = x - \ln|x+1| + c$

c. $f(x) = \dfrac{x+1}{2x+1} = \dfrac{x+\tfrac{1}{2}+\tfrac{1}{2}}{2x+1} = \dfrac{x+\tfrac{1}{2}}{2x+1} + \dfrac{\tfrac{1}{2}}{2x+1} = \tfrac{1}{2} + \dfrac{\tfrac{1}{2}}{2x+1}$

$F(x) = \tfrac{1}{2}x + \tfrac{1}{4}\ln|2x+1| + c$

d. $f(x) = \dfrac{2-x}{x+1} = \dfrac{-x-1+3}{x+1} = \dfrac{-x-1}{x+1} + \dfrac{3}{x+1} = -1 + \dfrac{3}{x+1}$

$F(x) = -x + 3\ln|x+1| + c$

e. $f(x) = \dfrac{3-4x}{2x+1} = \dfrac{-4x-2+5}{2x+1} = \dfrac{-4x-2}{2x+1} + \dfrac{5}{2x+1} = -2 + \dfrac{5}{2x+1}$

$F(x) = -2x + 2\tfrac{1}{2}\ln|2x+1| + c$

f. $f(x)=\dfrac{6x-1}{1-2x}=\dfrac{-3+6x+2}{1-2x}=\dfrac{-3+6x}{1-2x}+\dfrac{2}{1-2x}=-3+\dfrac{2}{1-2x}$

$F(x)=-3x-\ln|1-2x|+c$

Opgave 51:

a. $x-1\,/\,x^2-2x+3\,\backslash\,x-1$

$\underline{x^2-x}$

$-x+3$

$\underline{-x+1}$

2

$f(x)=\dfrac{x^2-2x+3}{x-1}=x-1+\dfrac{2}{x-1}$

$F(x)=\frac{1}{2}x^2-x+2\ln|x-1|$

$$\int_2^3 \frac{x^2-2x+3}{x-1}dx=\left[\tfrac{1}{2}x^2-x+2\ln|x-1|\right]_2^3=4\tfrac{1}{2}-3+2\ln 2-(2-2+2\ln 1)=1\tfrac{1}{2}+2\ln 2$$

b. $2x+1\,/\,x^2+7x\,\backslash\,\frac{1}{2}x+3\frac{1}{4}$

$\underline{x^2+\frac{1}{2}x}$

$6\frac{1}{2}x$

$\underline{6\frac{1}{2}x+3\frac{1}{4}}$

$-3\frac{1}{4}$

$f(x)=\dfrac{x^2+7x}{2x+1}=\frac{1}{2}x+3\frac{1}{4}-\dfrac{3\frac{1}{4}}{2x+1}$

$F(x)=\frac{1}{4}x^2+3\frac{1}{4}x-\frac{13}{8}\ln|2x+1|$

$$\int_1^2 \frac{x^2+7x}{2x+1}dx=\left[\tfrac{1}{4}x^2+3\tfrac{1}{4}x-\tfrac{13}{8}\ln|2x+1|\right]_1^2=1+6\tfrac{1}{2}-\tfrac{13}{8}\ln 5-(\tfrac{1}{4}+3\tfrac{1}{4}-\tfrac{13}{8}\ln 3)$$

$$=7\tfrac{1}{2}-\tfrac{13}{8}\ln 5-3\tfrac{1}{2}+\tfrac{13}{8}\ln 3=4-\tfrac{13}{8}\ln\tfrac{3}{5}$$

c. $x+1\,/-2x^2-x\,\backslash-2x+1$

$\underline{-2x^2-2x}$

x

$\underline{x+1}$

-1

$f(x)=\dfrac{-2x^2-x}{x+1}=-2x+1-\dfrac{1}{x+1}$

$F(x)=-x^2+x-\ln|x+1|$

$$\int_{-4}^{-2} \frac{-2x^2-x}{x+1}dx=\left[-x^2+x-\ln|x+1|\right]_{-4}^{-2}=-4-2-\ln 1-(-16-4-\ln 3)$$

$$=-6+16+4+\ln 3=14+\ln 3$$

Opgave 52:

a. $f(x)=\dfrac{x^3+x}{x+1}$

$$f'(x)=\frac{(x+1)(3x^2+1)-(x^3+x)\cdot 1}{(x+1)^2}=\frac{3x^3+3x^2+x+1-x^3-x}{(x+1)^2}=\frac{2x^3+3x^2+1}{(x+1)^2}$$

$f'(1)=1\tfrac{1}{2}$

$y_A=f(1)=1$

$k:\quad y=1\tfrac{1}{2}x+b$ door $(1{,}1)$

$1=1\tfrac{1}{2}+b$

$b=-\tfrac{1}{2}$

$k:\quad y=1\tfrac{1}{2}x-\tfrac{1}{2}$

b. $f(x)=\dfrac{x^3+x}{x+1}=0$

$x^3+x=0$

$x(x^2+1)=0$

$x=0 \quad \vee \quad x^2=-1$ (k.n.)

$x+1\ /\ x^3+x\ \backslash\ x^2-x+2$

$\underline{x^3+x^2}$

$-x^2+x$

$\underline{-x^2-x}$

$2x$

$\underline{2x+2}$

-2

$f(x)=\dfrac{x^3+x}{x+1}=x^2-x+2-\dfrac{2}{x+1}$

$F(x)=\tfrac{1}{3}x^3-\tfrac{1}{2}x^2+2x-2\ln|x+1|$

$$Opp(V)=\int_0^2\frac{x^3+x}{x+1}\,dx=\left[\tfrac{1}{3}x^3-\tfrac{1}{2}x^2+2x-2\ln|x+1|\right]_0^2=\tfrac{8}{3}-2+4-2\ln 3-(-2\ln 1)$$

$$=4\tfrac{2}{3}-2\ln 3$$

c. $\displaystyle\int_0^p\frac{x^3+x}{x+1}\,dx=\left[\tfrac{1}{3}x^3-\tfrac{1}{2}x^2+2x-2\ln|x+1|\right]_0^p=\tfrac{1}{3}p^3-\tfrac{1}{2}p^2+2p-2\ln(p+1)=10$

neem $y_1=\tfrac{1}{3}x^3-\tfrac{1}{2}x^2+2x-2\ln(x+1)$ en $y_2=10$

de optie intersect geeft $x=3{,}276$ dus $p=3{,}276$

Opgave 53:

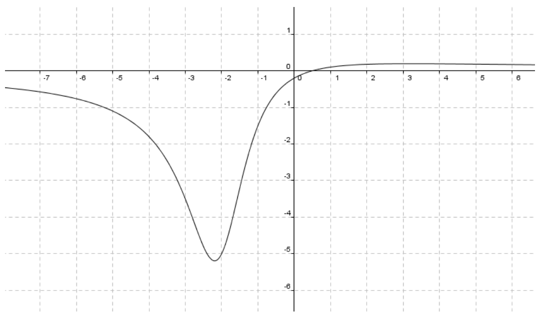
De noemer bepaalt de verticale asymptoten.

$f(x)=\dfrac{2x-1}{x^2+4x+5}$

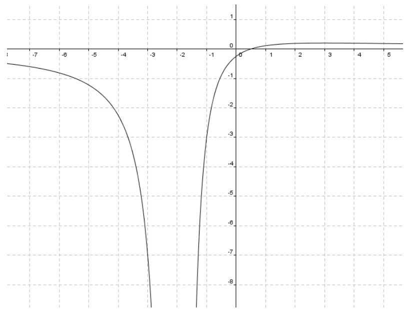
$x^2+4x+5=0$

$D=16-20=-4<0$ dus geen oplossingen,

dus de grafiek van f heeft geen verticale asymptoten.



$g(x) = \dfrac{2x-1}{x^2+4x+4}$

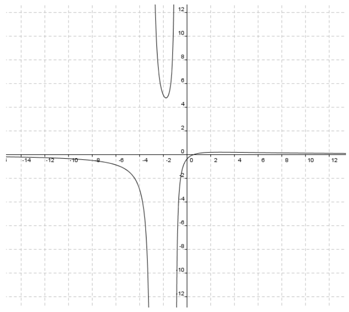


$x^2+4x+4=0$

$(x+2)^2=0$

$x=-2$ dus de grafiek van g heeft als verticale asymptoot de lijn $x=-2$

$h(x) = \dfrac{2x-1}{x^2+4x+3}$



$x^2+4x+3=0$

$(x+3)(x+1)=0$

$x=-3 \quad \vee \quad x=-1$

dus de grafiek van h heeft als verticale asymptoten de lijnen $x=-3$ en $x=-1$

Opgave 54:

a. $f(x) = \dfrac{x^2+x}{x^2+1} = \dfrac{x^2+1+x-1}{x^2+1} = 1 + \dfrac{x}{x^2+1} - \dfrac{1}{x^2+1}$

$F(x) = x + \frac{1}{2}\ln(x^2+1) - \arctan x + c$

b. $g(x) = \dfrac{x^2-6x+8}{x^2-6x+9} = \dfrac{x^2-6x+9-1}{x^2-6x+9} = 1 - \dfrac{1}{x^2-6x+9} = 1 - \dfrac{1}{(x-3)^2}$

$G(x) = x + \dfrac{1}{x-3} + c$

c. $h(x) = \dfrac{x^3+x}{x^2+1} = \dfrac{x(x^2+1)}{x^2+1} = x$

$H(x) = \frac{1}{2}x^2 + c$

d. $x^2-6x+9 \,/\, x^3+x \,\backslash\, x+6$

$\underline{x^3-6x^2+9x}$

$6x^2-8x$

$\underline{6x^2-36x+54}$

$28x-54$

$j(x) = \dfrac{x^3+x}{x^2-6x+9} = x+6+\dfrac{28x-54}{x^2-6x+9} = x+6+\dfrac{28x-84+30}{x^2-6x+9}$

$= x+6+\dfrac{28x-84}{x^2-6x+9} + \dfrac{30}{(x-3)^2}$

$J(x) = \frac{1}{2}x^2 + 6x + 14\ln(x^2-6x+9) - \dfrac{30}{x-3} + c$

Opgave 55:

a. $x^2+4 \,/\, x^3 \,\backslash\, x$

$\underline{x^3+4x}$

$-4x$

$$f(x)=\frac{x^3}{x^2+4}=x-\frac{4x}{x^2+4}$$
$$F(x)=\tfrac{1}{2}x^2-2\ln(x^2+4)$$
$$\int_0^2\frac{x^3}{x^2+4}dx=\left[\tfrac{1}{2}x^2-2\ln(x^2+4)\right]_0^2=2-2\ln 8-(0-2\ln 4)=2-2\ln 8+2\ln 4=2+2\ln\tfrac{1}{2}$$

b. $x^2+4x+4\ /\ x^3\ \backslash\ x-4$

$$\begin{array}{l}\underline{x^3+4x^2+4x}\\ -4x^2-4x\\ \underline{-4x^2-16x-16}\\ 12x+16\end{array}$$

$$f(x)=\frac{x^3}{x^2+4x+4}=x-4+\frac{12x+16}{x^2+4x+4}=x-4+\frac{12(x+2)-8}{(x+2)^2}=x-4+\frac{12}{x+2}-\frac{8}{(x+2)^2}$$
$$F(x)=\tfrac{1}{2}x^2-4x+12\ln(x+2)+\frac{8}{x+2}$$
$$\int_0^8\frac{x^3}{x^2+4x+4}dx=\left[\tfrac{1}{2}x^2-4x+12\ln(x+2)+\frac{8}{x+2}\right]_0^8=32-32+12\ln 10+\tfrac{8}{10}-(12\ln 2+4)$$
$$=12\ln 10+\tfrac{4}{5}-12\ln 2-4=12\ln 5-3\tfrac{1}{5}$$

c. $x^2+1\ /\ x^4+1\ \backslash\ x^2-1$

$$\begin{array}{l}\underline{x^4+x^2}\\ -x^2+1\\ \underline{-x^2-1}\\ 2\end{array}$$

$$f(x)=\frac{x^4+1}{x^2+1}=x^2-1+\frac{2}{x^2+1}$$
$$F(x)=\tfrac{1}{3}x^3-x+2\arctan x$$
$$\int_0^1\frac{x^4+1}{x^2+1}dx=\left[\tfrac{1}{3}x^3-x+2\arctan x\right]_0^1=\tfrac{1}{3}-1+2\arctan(1)-2\arctan(0)=\tfrac{1}{2}\pi-\tfrac{2}{3}$$

d. $x^2+2x+1\ /\ x^4+1\ \backslash\ x^2-2x+3$

$$\begin{array}{l}\underline{x^4+2x^3+x^2}\\ -2x^3-x^2+1\\ \underline{-2x^3-4x^2-2x}\\ 3x^2+2x+1\\ \underline{3x^2+6x+3}\\ -4x-2\end{array}$$

$$f(x)=\frac{x^4+1}{x^2+2x+1}=x^2-2x+3+\frac{-4x-2}{x^2+2x+1}=x^2-2x+3+\frac{-4(x+1)+2}{(x+1)^2}$$
$$=x^2-2x+3-\frac{4}{x+1}+\frac{2}{(x+1)^2}$$
$$F(x)=\tfrac{1}{3}x^3-x^2+3x-4\ln(x+1)-\frac{2}{x+1}$$

$$\int_0^1 \frac{x^4+1}{x^2+2x+1}\,dx = \left[\tfrac{1}{3}x^3 - x^2 + 3x - 4\ln(x+1) - \frac{2}{x+1}\right]_0^1$$
$$= \tfrac{1}{3} - 1 + 3 - 4\ln 2 - 1 - (-4\ln 1 - 2) = 3\tfrac{1}{3} - 4\ln 2$$

Opgave 56:

a. $f(x) = \dfrac{10x+5}{4x^2-4x+1} = 0$

$10x+5=0$

$10x=-5$

$x=-\tfrac{1}{2}$

$f'(x) = \dfrac{(4x^2-4x+1)\cdot 10 - (10x+5)\cdot(8x-4)}{(4x^2-4x+1)^2} = \dfrac{40x^2-40x+10-80x^2+20}{(4x^2-4x+1)^2}$

$= \dfrac{-40x^2-40x+30}{(4x^2-4x+1)^2}$

$f'(-\tfrac{1}{2}) = 2\tfrac{1}{2}$

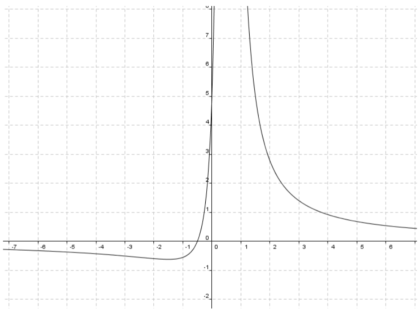
$k:\quad y = 2\tfrac{1}{2}x + b$ door $(-\tfrac{1}{2},0)$

$0 = -1\tfrac{1}{4} + b$

$b = 1\tfrac{1}{4}$

$k:\quad y = 2\tfrac{1}{2}x + 1\tfrac{1}{4}$

b. $f'(x) = \dfrac{-40x^2-40x+30}{(4x^2-4x+1)^2} = 0$



$-40x^2-40x+30=0$

$x^2+x-\tfrac{3}{4}=0$

$(x+\tfrac{3}{2})(x-\tfrac{1}{2})=0$

$x=-\tfrac{3}{2} \quad \vee \quad x=\tfrac{1}{2}$ (k.n.)

$y=-\tfrac{5}{8}$

$B_f = [-\tfrac{5}{8},\rightarrow\rangle$

c. $f(x) = \dfrac{10x+5}{4x^2-4x+1} = \dfrac{10x+5}{(2x-1)^2} = \dfrac{10x-5+10}{(2x-1)^2} = \dfrac{5(2x-1)+10}{(2x-1)^2} = \dfrac{5}{2x-1} + \dfrac{10}{(2x-1)^2}$

$F(x) = 2\tfrac{1}{2}\ln(2x-1) - \dfrac{5}{2x-1}$

$Opp(V) = \displaystyle\int_1^3 \frac{10x+5}{(4x^2-4x+1)}\,dx = \left[2\tfrac{1}{2}\ln(2x-1) - \frac{5}{2x-1}\right]_1^3 = 2\tfrac{1}{2}\ln 5 - 1 - (2\tfrac{1}{2}\ln 1 - 5)$

$= 2\tfrac{1}{2}\ln 5 + 4$

Opgave 57:

a. $\dfrac{-1\frac{1}{2}}{x+1} + \dfrac{3\frac{1}{2}}{x+3} = \dfrac{-1\frac{1}{2}(x+3)}{(x+1)(x+3)} + \dfrac{3\frac{1}{2}(x+1)}{(x+1)(x+3)} = \dfrac{-1\frac{1}{2}x - 4\frac{1}{2} + 3\frac{1}{2}x + 3\frac{1}{2}}{(x+1)(x+3)} = \dfrac{2x-1}{(x+1)(x+3)}$

b. $h(x) = \dfrac{2x-1}{(x+1)(x+3)} = \dfrac{-1\frac{1}{2}}{x+1} + \dfrac{3\frac{1}{2}}{x+3}$

$H(x) = -1\tfrac{1}{2}\ln|x+1| + 3\tfrac{1}{2}\ln|x+3| + c$

Opgave 58:

$$\frac{x+1}{(x-2)(x-2)} = \frac{A}{x-2} + \frac{B}{x-2} = \frac{A(x-2)}{(x-2)^2} + \frac{B(x-2)}{(x-2)^2} = \frac{(A+B)x - 2A - 2B}{(x-2)^2}$$

$$\begin{cases} A + B = 1 & \times 2 \\ -2A - 2B = 1 & \times 1 \end{cases}$$

$$\begin{cases} 2A + 2B = 2 \\ -2A - 2B = 1 \quad + \end{cases}$$

$0 = 3$ dus er zijn geen oplossingen

Opgave 59:

a. $f(x) = \dfrac{x^2 + 2x - 1}{x^2 - 5x + 6} = \dfrac{x^2 - 5x + 6 + 7x - 7}{x^2 - 5x + 6} = 1 + \dfrac{7x - 7}{x^2 - 5x + 6}$

$$\frac{7x-7}{x^2-5x+6} = \frac{7x-7}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3} = \frac{A(x-3) + B(x-2)}{(x-2)(x-3)} = \frac{(A+B)x - 3A - 2B}{(x-2)(x-3)}$$

$$\begin{cases} A + B = 7 & \times 2 \\ -3A - 2B = -7 & \times 1 \end{cases}$$

$$\begin{cases} 2A + 2B = 14 \\ -3A - 2B = -7 \quad + \end{cases}$$

$-A = 7$
$A = -7$
$B = 14$

$f(x) = 1 - \dfrac{7}{x-2} + \dfrac{14}{x-3}$

$F(x) = x - 7\ln|x-2| + 14\ln|x-3| + c$

b. $g(x) = \dfrac{x^3}{x^2 + 2x - 3}$

$$\begin{array}{l} x^2 + 2x - 3 \,/\, x^3 \quad \backslash\, x - 2 \\ \quad \underline{x^3 + 2x^2 - 3x} \\ \quad\quad -2x^2 + 3x \\ \quad\quad \underline{-2x^2 - 4x + 6} \\ \quad\quad\quad 7x - 6 \end{array}$$

$g(x) = \dfrac{x^3}{x^2 + 2x - 3} = x - 2 + \dfrac{7x - 6}{x^2 + 2x - 3}$

$$\frac{7x-6}{x^2+2x-3} = \frac{7x-6}{(x+3)(x-1)} = \frac{A}{x+3} + \frac{B}{x-1} = \frac{A(x-1) + B(x+3)}{(x+3)(x-1)} = \frac{(A+B)x - A + 3B}{(x+3)(x-1)}$$

$$\begin{cases} A + B = 7 \\ -A + 3B = -6 \quad + \end{cases}$$

$4B = 1$
$B = \tfrac{1}{4}$

$A = 6\frac{3}{4}$

$g(x) = \dfrac{x^3}{x^2+2x-3} = x - 2 + \dfrac{6\frac{3}{4}}{x+3} + \dfrac{\frac{1}{4}}{x-1}$

$G(x) = \frac{1}{2}x^2 - 2x + 6\frac{3}{4}\ln|x+3| + \frac{1}{4}\ln|x-1| + c$

c. $h(x) = \dfrac{2x^2+1}{2x^2-x} = \dfrac{2x^2-x+x+1}{2x^2-x} = 1 + \dfrac{x+1}{2x^2-x}$

$\dfrac{x+1}{2x^2-x} = \dfrac{x+1}{x(2x-1)} = \dfrac{A}{x} + \dfrac{B}{2x-1} = \dfrac{A(2x-1)+Bx}{x(2x-1)} = \dfrac{(2A+B)x-A}{x(2x-1)}$

$\begin{cases} 2A+B=1 \\ -A=1 \end{cases}$

$A = -1$
$B = 3$

$h(x) = 1 - \dfrac{1}{x} + \dfrac{3}{2x-1}$

$H(x) = x - \ln|x| + 1\frac{1}{2}\ln|2x-1| + c$

d. $j(x) = \dfrac{2x^3+1}{2x^2+x}$

$2x^2 + x \,/\, 2x^3 + 1 \qquad \backslash\, x - \frac{1}{2}$

$\underline{2x^3 + x^2}$

$-x^2 + 1$

$\underline{-x^2 - \frac{1}{2}x}$

$\frac{1}{2}x + 1$

$j(x) = \dfrac{2x^3+1}{2x^2+x} = x - \frac{1}{2} + \dfrac{\frac{1}{2}x+1}{2x^2+x}$

$\dfrac{\frac{1}{2}x+1}{2x^2+x} = \dfrac{\frac{1}{2}x+1}{x(2x+1)} = \dfrac{A}{x} + \dfrac{B}{2x+1} = \dfrac{A(2x+1)+Bx}{x(2x+1)} = \dfrac{(2A+B)x+A}{x(2x+1)}$

$\begin{cases} 2A+B=\frac{1}{2} \\ A=1 \end{cases}$

$B = -1\frac{1}{2}$

$j(x) = x - \frac{1}{2} + \dfrac{1}{x} - \dfrac{1\frac{1}{2}}{2x+1}$

$J(x) = \frac{1}{2}x^2 - \frac{1}{2}x + \ln|x| - \frac{3}{4}\ln|2x+1| + c$

Opgave 60:

a. $\dfrac{x}{x^2-x-2} = \dfrac{x}{(x-2)(x+1)} = \dfrac{A}{x-2} + \dfrac{B}{x+1} = \dfrac{A(x+1)+B(x-2)}{(x-2)(x+1)} = \dfrac{(A+B)x+A-2B}{(x-2)(x+1)}$

$\begin{cases} A+B=1 \\ \underline{A-2B=0} \quad - \end{cases}$

$3B = 1$

$B = \frac{1}{3}$

$A = \frac{2}{3}$

$f(x) = \dfrac{\frac{2}{3}}{x-2} + \dfrac{\frac{1}{3}}{x+1}$

$F(x) = \frac{2}{3}\ln|x-2| + \frac{1}{3}\ln|x+1|$

$$\int_3^4 \frac{x}{x^2-x-2}dx = \left[\tfrac{2}{3}\ln|x-2| + \tfrac{1}{3}\ln|x+1|\right]_3^4 = \tfrac{2}{3}\ln 2 + \tfrac{1}{3}\ln 5 - (\tfrac{2}{3}\ln 1 + \tfrac{1}{3}\ln 4)$$

$$= \tfrac{2}{3}\ln 2 + \tfrac{1}{3}\ln 5 - \tfrac{1}{3}\ln 4 = \tfrac{1}{3}\ln 5$$

b. $\dfrac{4x-8}{x^2-4x-5} = \dfrac{A}{x-5} + \dfrac{B}{x+1} = \dfrac{A(x+1)+B(x-5)}{(x-5)(x+1)} = \dfrac{(A+B)x + A - 5B}{(x-5)(x+1)}$

$$\begin{cases} A + B = 4 \\ A - 5B = -8 \quad - \end{cases}$$

$6B = 12$

$B = 2$

$A = 2$

$f(x) = \dfrac{2}{x-5} + \dfrac{2}{x+1}$

$F(x) = 2\ln|x-5| + 2\ln|x+1|$

$$\int_0^2 \frac{4x-8}{x^2-4x-5}dx = \left[2\ln|x-5|\right] + 2\ln|x+1|\Big|_0^2 = 2\ln 3 + 2\ln 3 - (2\ln 5 + 2\ln 1)$$

$$= 4\ln 3 - 2\ln 5 = \ln 3^4 - \ln 5^2 = \ln \tfrac{81}{25}$$

c. $\dfrac{-2}{x^2-2x-3} = \dfrac{A}{x-3} + \dfrac{B}{x+1} = \dfrac{A(x+1)+B(x-3)}{(x-3)(x+1)} = \dfrac{(A+B)x + A - 3B}{(x-3)(x+1)}$

$$\begin{cases} A + B = 0 \\ A - 3B = -2 \quad - \end{cases}$$

$4B = 2$

$B = \frac{1}{2}$

$A = -\frac{1}{2}$

$f(x) = \dfrac{-\frac{1}{2}}{x-3} + \dfrac{\frac{1}{2}}{x+1}$

$F(x) = -\frac{1}{2}\ln|x-3| + \frac{1}{2}\ln|x+1|$

$$\int_0^2 \frac{-2}{x^2-2x-3}dx = \left[-\tfrac{1}{2}\ln|x-3| + \tfrac{1}{2}\ln|x+1|\right]_0^2 = -\tfrac{1}{2}\ln 1 + \tfrac{1}{2}\ln 3 - (-\tfrac{1}{2}\ln 3 + \tfrac{1}{2}\ln 1)$$

$$= \tfrac{1}{2}\ln 3 + \tfrac{1}{2}\ln 3 = \ln 3$$

d. $x^2+4x+3\,/\,x^4 \qquad \backslash\, x^2-4x+13$

$$\begin{array}{l} \underline{x^4+4x^3+3x^2} \\ -4x^3-3x^2 \\ \underline{-4x^3-16x^2-12x} \\ 13x^2+12x \\ \underline{13x^2+52x+39} \\ -40x-39 \end{array}$$

$$f(x)=\frac{x^4}{x^2+4x+3}=x^2-4x+13+\frac{-40x-39}{x^2+4x+3}$$

$$\frac{-40x-39}{x^2+4x+3}=\frac{A}{x+3}+\frac{B}{x+1}=\frac{A(x+1)+B(x+3)}{(x+3)(x+1)}=\frac{(A+B)x+A+3B}{(x+3)(x+1)}$$

$$\begin{cases} A+B=-40 \\ A+3B=39 \quad - \end{cases}$$

$$-2B=-1$$

$$B=\tfrac{1}{2}$$

$$A=-40\tfrac{1}{2}$$

$$f(x)=x^2-4x+13+\frac{-40\frac{1}{2}}{x+3}+\frac{\frac{1}{2}}{x+1}$$

$$F(x)=\tfrac{1}{3}x^3-2x^2+13x-40\tfrac{1}{2}\ln|x+3|+\tfrac{1}{2}\ln|x+1|$$

$$\int_0^2\frac{x^4}{x^2+4x+3}\,dx=\left[\tfrac{1}{3}x^3-2x^2+13x-40\tfrac{1}{2}\ln|x+3|+\tfrac{1}{2}\ln|x+1|\right]_0^2$$

$$=\tfrac{8}{3}-8+26-40\tfrac{1}{2}\ln 5+\tfrac{1}{2}\ln 3-(-40\tfrac{1}{2}\ln 3+\tfrac{1}{2}\ln 1)$$

$$=20\tfrac{2}{3}-40\tfrac{1}{2}\ln 5+41\ln 3$$

Opgave 61:

Manier I:

$$f(x)=\frac{2x-5}{x^2-5x+6}=\frac{A}{x-3}+\frac{B}{x-2}=\frac{A(x-2)+B(x-3)}{(x-3)(x-2)}=\frac{(A+B)x-2A-3B}{(x-3)(x-2)}$$

$$\begin{cases} A+B=2 & \times 2 \\ -2A-3B=-5 & \times 1 \end{cases}$$

$$\begin{cases} 2A+2B=4 \\ -2A-3B=-5 \quad - \end{cases}$$

$$-B=-1$$

$$B=1$$

$$A=1$$

$$f(x)=\frac{1}{x-3}+\frac{1}{x-2}$$

$$F(x)=\ln|x-3|+\ln|x-2|=\ln|(x-3)(x-2)|=\ln|x^2-5x+6|$$

Manier II:

$$f(x)=\frac{2x-5}{x^2-5x+6} \quad \text{neem } u=x^2-5x+6 \text{ dan } u'=2x-5$$

$f(x) = \frac{u'}{u} = \frac{1}{u} \cdot u'$

$F(x) = \ln|u| = \ln|x^2 - 5x + 6|$

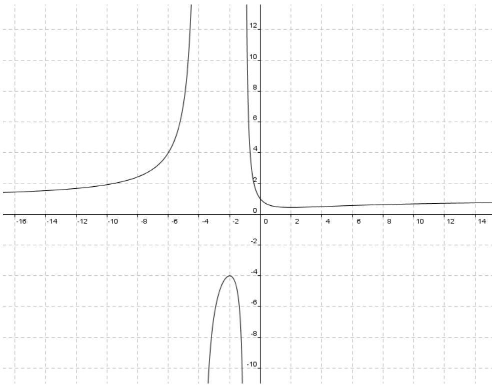
Opgave 62:

a. $f(x) = \frac{x^2 + 4}{x^2 + 5x + 4}$

$f'(x) = \frac{(x^2 + 5x + 4) \cdot 2x - (x^2 + 4) \cdot (2x + 5)}{(x^2 + 5x + 4)^2}$

$= \frac{2x^3 + 10x^2 + 8x - 2x^3 - 5x^2 - 8x - 20}{(x^2 + 5x + 4)^2}$

$= \frac{5x^2 - 20}{(x^2 + 5x + 4)^2} = 0$



$5x^2 - 20 = 0$

$5x^2 = 20$

$x^2 = 4$

$x = 2 \quad \vee \quad x = -2$

max $f(-2) = -4$

min $f(2) = \frac{4}{9}$

b. $x_A = -6$

$y_A = f(-6) = 4$

$f'(-6) = 1\frac{3}{5}$

$k: \quad y = 1\frac{3}{5}x + b \quad$ door $(-6,4)$

$4 = -9\frac{3}{5} + b$

$b = 13\frac{3}{5}$

$y = 1\frac{3}{5}x + 13\frac{3}{5}$

$C(0,13\frac{3}{5})$

snijpunt x-as: $1\frac{3}{5}x + 13\frac{3}{5} = 0$

$1\frac{3}{5}x = -13\frac{3}{5}$

$x = -8\frac{1}{2}$

$B(-8\frac{1}{2},0)$

$Opp(\Delta OBC) = \frac{1}{2} \cdot 8\frac{1}{2} \cdot 13\frac{3}{5} = 57\frac{4}{5}$

c. $Opp(V) = \int_0^6 \frac{x^2 + 4}{x^2 + 5x + 4}\,dx$

$x^2 + 5x + 4 \,/\, x^2 + 4 \qquad \backslash\, 1$

$\underline{x^2 + 5x + 4}$

$-5x$

$f(x) = \frac{x^2 + 4}{x^2 + 5x + 4} = 1 + \frac{-5x}{x^2 + 5x + 4}$

$$\frac{-5x}{x^2+5x+4} = \frac{A}{x+1} + \frac{B}{x+4} = \frac{A(x+4)+B(x+1)}{(x+1)(x+4)} = \frac{(A+B)x+4A+B}{(x+1)(x+4)}$$

$$\begin{cases} A+B=-5 \\ 4A+B=0 \quad - \end{cases}$$

$$-3A=-5$$
$$A=\tfrac{5}{3}$$
$$B=-6\tfrac{2}{3}$$

$$f(x)=1+\frac{-5x}{x^2+5x+4}=1+\frac{\frac{5}{3}}{x+1}-\frac{6\frac{2}{3}}{x+4}$$

$$F(x)=x+\tfrac{5}{3}\ln|x+1|-6\tfrac{2}{3}\ln|x+4|$$

$$Opp(V)=\int_0^6 \frac{x^2+4}{x^2+5x+4}\,dx=\left[x+\tfrac{5}{3}\ln|x+1|-6\tfrac{2}{3}\ln|x+4|\right]_0^6$$
$$=6+\tfrac{5}{3}\ln 7-6\tfrac{2}{3}\ln 10-(\tfrac{5}{3}\ln 1-6\tfrac{2}{3}\ln 4)$$
$$=6+\tfrac{5}{3}\ln 7-6\tfrac{2}{3}\ln 10+6\tfrac{2}{3}\ln 4$$
$$=6+6\tfrac{2}{3}\ln\tfrac{2}{5}+\tfrac{5}{3}\ln 7$$

Opgave 63:

a. $f(x)=\frac{x}{x+1}=\frac{x+1-1}{x+1}=1-\frac{1}{x+1}$

$F(x)=x-\ln|x+1|$

$Opp(V)=\int_0^3 \frac{x}{x+1}\,dx=\left[x-\ln|x+1|\right]_0^3=3-\ln 4-(0-\ln 1)=3-\ln 4$

b. $g(x)=\frac{x^2}{(x+1)^2}=\frac{x^2}{x^2+2x+1}=\frac{x^2+2x+1-2x-1}{x^2+2x+1}=1-\frac{2x+1}{x^2+2x+1}$

$$=1-\frac{2x+2-1}{x^2+2x+1}=1-\frac{2(x+1)}{(x+1)^2}+\frac{1}{(x+1)^2}=1-\frac{2}{x+1}+\frac{1}{(x+1)^2}$$

$G(x)=x-2\ln|x+1|-\frac{1}{x+1}$

$$Ih=\pi\int_0^3 \frac{x^2}{(x+1)^2}\,dx=\pi\left[x-2\ln|x+1|-\frac{1}{x+1}\right]_0^3$$
$$=\pi(3-2\ln 4-\tfrac{1}{4}-(0-2\ln 1-1))=\pi(3\tfrac{3}{4}-2\ln 4)$$

Opgave 64:

a. $f(x)=\ln(x^2+1)$

$f'(x)=\frac{1}{x^2+1}\cdot 2x=\frac{2x}{x^2+1}=\frac{3}{5}$

$3(x^2+1)=10x$

$3x^2-10x+3=0$

$x=\frac{10\pm\sqrt{100-36}}{6}=\frac{10\pm 8}{6}$

$x = 3 \quad \vee \quad x = \frac{1}{3}$

$y = \ln 10 \quad y = \ln \frac{10}{9}$

$(3, \ln 10)$ en $(\frac{1}{3}, \ln \frac{10}{9})$

b. $\ln(x^2 + 1) = \ln 2$

$x^2 + 1 = 2$

$x^2 = 1$

$x = 1 \quad \vee \quad x = -1$

$$\int \ln(x^2+1)dx = \int 1 \cdot \ln(x^2+1)dx = x \cdot \ln(x^2+1) - \int x \cdot \frac{1}{x^2+1} \cdot 2x dx$$

$$= x\ln(x^2+1) - \int \frac{2x^2}{x^2+1} dx$$

$$= x\ln(x^2+1) - \int \frac{2x^2+2-2}{x^2+1} dx$$

$$= x\ln(x^2+1) - \int 2 - \frac{2}{x^2+1} dx$$

$$= x\ln(x^2+1) - 2x + 2\arctan(x)$$

$$Opp(V) = \int_{-1}^{1} (\ln 2 - \ln(x^2+1))dx = \left[x\ln 2 - x\ln(x^2+1) + 2x - 2\arctan(x)\right]_{-1}^{1}$$

$$= \ln 2 - \ln 2 + 2 - 2\arctan(1) - (-\ln 2 + \ln 2 - 2 - 2\arctan(-1)$$

$$= 4 - 2 \cdot \tfrac{1}{4}\pi + 2 \cdot -\tfrac{1}{4}\pi$$

$$= 4 - \pi$$