

13.4 Raken en loodrecht snijden

Opgave 40:

a. $f(1) = 0,5 \cdot 1^2 + 1 + 1,5 = 3$

$$g(1) = -1^2 + 4 \cdot 1 = 3$$

b. $f'(x) = x + 1$

$$f'(1) = 2$$

k : $y = 2x + 1$ door $(1,3)$

$$3 = 2 + b$$

$$b = 1$$

$$y = 2x + 1$$

$$g'(x) = -2x + 4$$

$$g'(1) = 2$$

l : $y = 2x + b$ door $(1,3)$

$$3 = 2 + b$$

$$b = 1$$

$$y = 2x + 1$$

c. punt A is het raakpunt van f en g

Opgave 41:

De grafieken van f en g hebben in het punt met $x = -3$ dezelfde helling maar de grafieken snijden elkaar niet.

Opgave 42:

a. $f'(x) = 3x^2 + 8x + 2$

$$g'(x) = 2x + 11$$

$f'(x) = g'(x)$ geeft: $3x^2 + 8x + 2 = 2x + 11$

$$3x^2 + 6x - 9 = 0$$

$$x^2 + 2x - 3 = 0$$

$$(x + 3)(x - 1) = 0$$

$$x = -3 \quad \vee \quad x = 1$$

$$f(-3) = 4 \quad f(1) = 8$$

$$g(-3) = 4 \quad g(1) = 40$$

dus de grafieken van f en g raken elkaar in het punt $(-3,4)$

b. $f'(-3) = 5$

$$y = 5x + b \text{ door } (-3,4)$$

$$4 = -15 + b$$

$$b = 19$$

$$y = 5x + 19$$

Opgave 43:

Met de GR krijg je het vermoeden dat het raakpunt het punt $(-1,2)$ is.

$$f(-1) = 2 \text{ en } g(-1) = 2$$

$$f'(x) = \frac{1}{2\sqrt{2x+6}} \cdot 2 = \frac{1}{\sqrt{2x+6}} \quad \text{dus } f'(-1) = \frac{1}{2}$$

$$g'(x) = 2x + 2\frac{1}{2} \text{ dus } g'(-1) = \frac{1}{2}$$

dus de grafieken van f en g raken elkaar in het punt $(-1, 2)$

Opgave 44:

$$f'(x) = \frac{(2x+5) \cdot 4 - 4x \cdot 2}{(2x+5)^2} = \frac{8x+20-8x}{(2x+5)^2} = \frac{20}{(2x+5)^2}$$

$$g'(x) = 2x + 5$$

$$f'(x) = g'(x) \text{ geeft: } \frac{20}{(2x+5)^2} = 2x+5$$

$$(2x+5)^3 = 20$$

$$2x+5 = \sqrt[3]{20}$$

$$2x = -5 + \sqrt[3]{20}$$

$$x = -2\frac{1}{2} + \frac{1}{2}\sqrt[3]{20}$$

$$f(-2\frac{1}{2} + \frac{1}{2}\sqrt[3]{20}) = -1,684$$

$$g(-2\frac{1}{2} + \frac{1}{2}\sqrt[3]{20}) = -1,658$$

dus de grafieken van f en g raken elkaar niet

Opgave 45:

a. $f'(x) = e^x$

$$g'(x) = e^{-x+2}$$

$$f'(x) = g'(x) \text{ geeft: } e^x = e^{-x+2}$$

$$x = -x + 2$$

$$2x = 2$$

$$x = 1$$

$$f(1) = e + 2$$

$$g(1) = 2e + 2 - e = e + 2$$

dus de grafieken van f en g raken elkaar in het punt $(1, e + 2)$

b.
$$\begin{aligned} Opp(V) &= \int_0^1 (f(x) - g(x)) dx \\ &= \int_0^1 (e^x + 2 - (2e + 2 - e^{-x+2})) dx \\ &= \int_0^1 (e^x - 2e + e^{-x+2}) dx \\ &= [e^x - 2e \cdot x - e^{-x+2}]_0^1 \\ &= e - 2e - e - (1 - 0 - e^2) \\ &= e^2 - 2e - 1 \end{aligned}$$

Opgave 46:

a.
$$f'(x) = \frac{1}{2\sqrt{2x}} \cdot 2 = \frac{1}{\sqrt{2x}}$$

$$g_1'(x) = 2x$$

$$f'(x) = g'(x) \text{ geeft: } \frac{1}{\sqrt{2x}} = 2x$$

$$2x\sqrt{2x} = 1$$

$$8x^3 = 1$$

$$x^3 = \frac{1}{8}$$

$$x = \frac{1}{2}$$

$f(\frac{1}{2}) = 1$ $g_1(\frac{1}{2}) = 1\frac{1}{4}$ dus de grafieken van f en g raken elkaar niet

b. $g'_p(x) = 2x$

$$f'(x) = g'(x) \text{ geeft: } \frac{1}{\sqrt{2x}} = 2x$$

$$x = \frac{1}{2} \text{ (zie opgave a)}$$

$$f(\frac{1}{2}) = 1$$

$$g(\frac{1}{2}) = \frac{1}{4} + p = 1$$

$$p = \frac{3}{4}$$

dus de grafieken van f en g raken elkaar voor $p = \frac{3}{4}$

Opgave 47:

a. $1 - \frac{1}{x} = p$ links en rechts vermenigvuldigen met x geeft:

$$x - 1 = px \text{ invullen in } x - \ln x = px \text{ geeft:}$$

$$x - \ln x = x - 1$$

b. $x - \ln x + 2 = p\sqrt{x}$ $1 - \frac{1}{x} = \frac{p}{2\sqrt{x}}$

$$p = \frac{x - \ln x + 2}{\sqrt{x}} \quad 2\sqrt{x} \cdot (1 - \frac{1}{x}) = p$$

$$\frac{x - \ln x + 2}{\sqrt{x}} = 2\sqrt{x} \cdot (1 - \frac{1}{x})$$

$$x - \ln x + 2 = 2x(1 - \frac{1}{x})$$

$$x - \ln x + 2 = 2x - 2$$

$$4 - x - \ln x = 0$$

neem $y_1 = 4 - x - \ln x$ optie zero geeft: $x = 2,926$ dus $p = 2,25$

Opgave 48:

$$f'(x) = -2x + 8$$

$$g'_p(x) = 2x + p$$

$$2x + p = -2x + 8$$

$$p = -4x + 8$$

$$-x^2 + 8x - 12 = x^2 + x(-4x + 8)$$

$$-x^2 + 8x - 12 = x^2 - 4x^2 + 8x$$

$$2x^2 = 12$$

$$x^2 = 6$$

$$x = \sqrt{6} \quad \vee \quad x = -\sqrt{6}$$

$$p = 8 - 4\sqrt{6} \quad \vee \quad p = 8 + 4\sqrt{6}$$

Opgave 49:

$$f'(x) = 1 - e^x$$

$$g'(x) = 2x + p$$

$$2x + p = 1 - e^x$$

$$p = 1 - 2x - e^x$$

$$x - e^x = x^2 + x(1 - 2x - e^x)$$

$$x - e^x = x^2 + x - 2x^2 - xe^x$$

$$xe^x - e^x = -x^2$$

$$\text{neem } y_1 = xe^x - e^x \text{ en } y_2 = -x^2$$

$$\text{intersect geeft: } x = -0,883 \quad \vee \quad x = 0,739$$

$$p = 2,351 \quad \vee \quad p = -2,572$$

Opgave 50:

a. $f'_3(x) = \frac{2}{x} + 3$

$$g'_q(x) = 2x$$

$$2x = \frac{2}{x} + 3$$

$$2x^2 = 2 + 3x$$

$$2x^2 - 3x - 2 = 0$$

$$x^2 - 1\frac{1}{2}x - 1 = 0$$

$$(x - 2)(x + \frac{1}{2}) = 0$$

$$x = 2 \quad \vee \quad x = -\frac{1}{2} \text{ (vervalt)}$$

$$f_3(2) = 2\ln 2 + 6 \text{ en } g_q(2) = 4 + q$$

$$4 + q = 2\ln 2 + 6$$

$$q = 2\ln 2 + 2$$

b. $f'_p(x) = \frac{2}{x} + p$

$$g'_2(x) = 2x$$

$$\frac{2}{x} + p = 2x$$

$$p = 2x - \frac{2}{x}$$

$$2\ln x + x(2x - \frac{2}{x}) = x^2 + 2$$

$$2\ln x + 2x^2 - 2 = x^2 + 2$$

$$\text{neem } y_1 = 2\ln x + 2x^2 - 2 \text{ en } y_2 = x^2 + 2$$

$$\text{intersect geeft } x = 1,711 \text{ dus } p = 2,252$$

Opgave 51:

$$f'_p(x) = 2 + \frac{p}{x}$$

$$g'_p(x) = 2px$$

$$2px = 2 + \frac{p}{x}$$

$$2px - \frac{p}{x} = 2$$

$$p\left(2x - \frac{1}{x}\right) = 2$$

$$p \cdot \frac{2x^2 - 1}{x} = 2$$

$$p = \frac{2x}{2x^2 - 1}$$

$$2x + \frac{2x}{2x^2 - 1} \cdot \ln x = \frac{2x}{2x^2 - 1} \cdot x^2$$

neem $y_1 = 2x + \frac{2x}{2x^2 - 1} \cdot \ln x$ en $y_2 = \frac{2x^3}{2x^2 - 1}$

intersect geeft: $x = 1$
 $p = 2$

Opgave 52:

$$f'(x) = 2xe^{x^2-2} + 1$$

$$g'(x) = \frac{1}{x}$$

$$2xe^{x^2-2} + 1 = \frac{1}{x}$$

neem $y_1 = 2xe^{x^2-2} + 1$ en $y_2 = \frac{1}{x}$

intersect geeft $x = -1,280$ (vervalt) \vee $x = 0,742$
 $q = 1,275$

Opgave 53:

$$rc_l = -\frac{1}{3}$$

Opgave 54:

$$f'(x) = \frac{1}{2\sqrt{x}} \text{ en } g'(x) = -x$$

$$f'(x) \cdot g'(x) = -1 \text{ geeft:}$$

$$\frac{1}{2\sqrt{x}} \cdot -x = -1$$

$$\frac{x}{2\sqrt{x}} = 1$$

$$\frac{1}{2}\sqrt{x} = 1$$

$$\sqrt{x} = 2$$

$$x = 4$$

$f(4) = 2$ en $g(4) = 2$ dus de grafieken van f en g snijden elkaar loodrecht in $(4,2)$

Opgave 55:

$$f'(x) = \sqrt{2} \cdot \cos x$$

$$g'(x) = -\sqrt{2} \cdot \sin x$$

$$f'(x) \cdot g'(x) = -1 \text{ geeft:}$$

$$\sqrt{2} \cdot \cos x \cdot -\sqrt{2} \cdot \sin x = -1$$

$$2 \cos x \sin x = 1$$

$$\sin 2x = 1$$

$$2x = \frac{1}{2}\pi + k \cdot 2\pi$$

$$x = \frac{1}{4}\pi + k \cdot \pi$$

$f(\frac{1}{4}\pi) = 1$ en $g(\frac{1}{4}\pi) = 1$ dus de grafieken van f en g snijden elkaar loodrecht in $(\frac{1}{4}\pi, 1)$

$f(1\frac{1}{4}\pi) = -1$ en $g(1\frac{1}{4}\pi) = -1$ dus de grafieken van f en g snijden elkaar loodrecht in $(1\frac{1}{4}\pi, -1)$

Opgave 56:

$$f'_p(x) = p \cdot \frac{1}{2\sqrt{x}}$$

$$g'(x) = -\frac{8}{x^2}$$

$$f'_p(x) \cdot g'(x) = -1 \text{ geeft:}$$

$$\frac{p}{2\sqrt{x}} \cdot -\frac{8}{x^2} = -1$$

$$8p = 2x^2 \sqrt{x}$$

$$p = \frac{1}{4}x^2 \sqrt{x}$$

$$f_p(x) = g(x) \text{ geeft:}$$

$$\frac{1}{4}x^3 = \frac{8}{x}$$

$$x^4 = 32$$

$$x = \sqrt[4]{32} = 2^{\frac{5}{4}} = 2 \cdot \sqrt[4]{2}$$

$$y = \frac{8}{2^{\frac{5}{4}}} = \frac{2^3}{2^{\frac{5}{4}}} = 2^{\frac{7}{4}} = 2 \cdot \sqrt[4]{2^3} = 2 \cdot \sqrt[4]{8}$$

$$p = 2^{\frac{11}{8}} = 2 \cdot \sqrt[8]{2}$$

Opgave 57:

a. $f'(x) = 2x - 4$

$$f'(5) = 6 \text{ dus } rc_k = -\frac{1}{6}$$

$$k: y = -\frac{1}{6}x + b \text{ door } (5,5)$$

$$5 = -\frac{5}{6} + b$$

$$b = 5\frac{5}{6}$$

$$y = -\frac{1}{6}x + 5\frac{5}{6}$$

b. $rc_l = -5$ dus $g'(x) = \frac{1}{5}$

$$g'(x) = \frac{(x+2) \cdot 2 - (2x-1) \cdot 1}{(x+2)^2} = \frac{2x+4-2x+1}{(x+2)^2} = \frac{5}{(x+2)^2} = \frac{1}{5}$$

$$(x+2)^2 = 25$$

$$x+2 = 5 \quad \vee \quad x+2 = -5$$

$$x = 3 \quad \vee \quad x = -7$$

$$y = 1 \quad \vee \quad y = 3$$

$$p = 16 \quad \vee \quad p = -32$$

c. $rc_m = -8$ dus $h'(x) = \frac{1}{8}$

$$h'(x) = \frac{\sqrt{x^2+4} \cdot 2 - 2x \cdot \frac{1}{2\sqrt{x^2+4}} \cdot 2x}{x^2+4}$$

$$= \frac{2\sqrt{x^2+4} - \frac{2x^2}{\sqrt{x^2+4}}}{x^2+4}$$

$$= \frac{2(x^2+4) - 2x^2}{\sqrt{x^2+4} \cdot (x^2+4)}$$

$$= \frac{8}{(x^2+4)\sqrt{x^2+4}} = \frac{1}{8}$$

$$(x^2+4)\sqrt{x^2+4} = 64$$

$$(x^2+4)^3 = 4096$$

$$x^2+4 = 16$$

$$x^2 = 12$$

$$x = \sqrt{12} = 2\sqrt{3} \quad \vee \quad x = -2\sqrt{3}$$

$$y = \sqrt{3} \quad \vee \quad y = -\sqrt{3}$$

$$q = 17\sqrt{3} \quad \vee \quad q = -17\sqrt{3}$$

Opgave 58:

$$rc_k = -\frac{1}{2} \text{ dus } f'_p(x) = 2$$

$$f'_p(x) = 2xe^x + (x^2 - p)e^x = (x^2 + 2x - p)e^x = 2$$

$$x^2e^x + 2xe^x - pe^x = 2$$

$$-pe^x = -x^2e^x - 2xe^x + 2$$

$$p = x^2 + 2x - \frac{2}{e^x}$$

$$(x^2 - (x^2 + 2x - \frac{2}{e^x}))e^x = -\frac{1}{2}x - \frac{1}{2}$$

$$(\frac{2}{e^x} - 2x)e^x = -\frac{1}{2}x - \frac{1}{2}$$

$$2 - 2xe^x = -\frac{1}{2}x - \frac{1}{2}$$

$$\text{neem } y_1 = 2 - 2xe^x \text{ en } y_2 = -\frac{1}{2}x - \frac{1}{2}$$

$$\text{intersect geeft } x = -5,12 \quad \vee \quad x = 0,70$$

$$y = 2,06 \quad y = -0,85$$

dus $A(-5,12; 2,06)$ of $A(0,70; -0,85)$

Opgave 59:

a. $f(x) = e^{1-x^2}$

$$f'(x) = -2xe^{1-x^2}$$

$$f''(x) = -2e^{1-x^2} + -2x \cdot -2xe^{1-x^2} = (4x^2 - 2)e^{1-x^2} = 0$$

$$4x^2 - 2 = 0 \quad \vee \quad e^{1-x^2} = 0$$

$$4x^2 = 2 \quad \text{k.n.}$$

$$x^2 = \frac{1}{2}$$

$$x = \sqrt{\frac{1}{2}} = \frac{1}{2}\sqrt{2} \quad \vee \quad x = -\frac{1}{2}\sqrt{2}$$

$$y = e^{\frac{1}{2}} = \sqrt{e} \quad y = \sqrt{e}$$

$$\left(\frac{1}{2}\sqrt{2}, \sqrt{e}\right) \text{ en } \left(-\frac{1}{2}\sqrt{2}, \sqrt{e}\right)$$

b. $f'(x) = -2x \cdot e^{1-x^2} = -\frac{1}{a}$

$$a = \frac{1}{2xe^{1-x^2}}$$

$$e^{1-x^2} = \frac{x}{2xe^{1-x^2}}$$

$$e^{1-x^2} = \frac{1}{2e^{1-x^2}}$$

$$\text{neem } y_1 = e^{1-x^2} \text{ en } y_2 = \frac{1}{2y_1}$$

$$\text{intersect geeft } x = 1,16 \quad \vee \quad x = -1,16$$

$$a = 0,61 \quad \vee \quad a = -0,61$$

Opgave 60:

a. $f'(x) = 3x^2 - 1$

$$g'_p(x) = -\frac{p}{x^2}$$

$$f'(x) = g'_p(x) \text{ geeft:}$$

$$3x^2 - 1 = -\frac{p}{x^2}$$

$$3x^4 - x^2 = -p$$

$$p = -3x^4 + x^2$$

$$x^3 - x = \frac{-3x^4 + x^2}{x}$$

$$x^3 - x = -3x^3 + x$$

$$4x^3 - 2x = 0$$

$$2x(2x^2 - 1) = 0$$

$$x = 0 \quad \vee \quad 2x^2 = 1$$

$$x^2 = \frac{1}{2}$$

$$x = \frac{1}{2}\sqrt{2} \quad \vee \quad x = -\frac{1}{2}\sqrt{2}$$

$$p = -\frac{1}{4} \quad \vee \quad p = -\frac{1}{4}$$

b. $f'(x) \cdot g'_p(x) = -1$

$$(3x^2 - 1) \cdot -\frac{p}{x^2} = -1$$

$$p = \frac{x^2}{3x^2 - 1}$$

$$x^3 - x = \frac{x^2}{3x^2 - 1}$$

$$x^4 - x^2 = \frac{x^2}{3x^2 - 1}$$

$$x^2(x^2 - 1)(3x^2 - 1) = x^2$$

$$x^2 = 0 \quad \vee \quad (x^2 - 1)(3x^2 - 1) = 1$$

$$3x^4 - 4x^2 + 1 = 1$$

$$3x^4 - 4x^2 = 0$$

$$x^2(3x^2 - 4) = 0$$

$$x = 0 \quad \vee \quad 3x^2 = 4$$

$$x^2 = \frac{4}{3}$$

$$x = \sqrt{\frac{4}{3}} \quad \vee \quad x = -\sqrt{\frac{4}{3}}$$

$$y = \frac{1}{3}\sqrt{\frac{4}{3}} \quad y = \frac{1}{3}\sqrt{\frac{4}{3}}$$

$$p = \frac{4}{9} \quad B(\sqrt{\frac{4}{3}}, \frac{1}{3}\sqrt{\frac{4}{3}}) \text{ of } B(-\sqrt{\frac{4}{3}}, \frac{1}{3}\sqrt{\frac{4}{3}})$$