15.4 Lissajous-figuren.

Opgave 46:

a. $x(\frac{1}{4}\pi) = \sin\frac{1}{4}\pi = \frac{1}{2}\sqrt{2}$

$y(\frac{1}{4}\pi) = \sin\frac{1}{2}\pi = 1$

dus voor $t = \frac{1}{4}\pi$ krijg je het punt $(\frac{1}{2}\sqrt{2},1)$

b. $t = 0 \quad x = \sin 0 = 0$

$y = \sin 0 = 0$

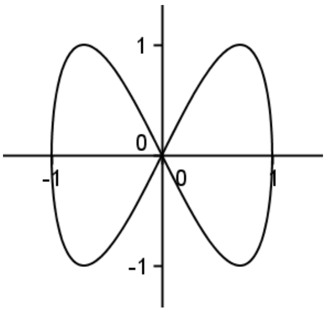
$t = \frac{1}{2}\pi \quad x = \sin\frac{1}{2}\pi = 1$

$y = \sin\pi = 0$

c.

t	0	$\frac{1}{4}\pi$	$\frac{1}{2}\pi$	$\frac{3}{4}\pi$	π	$1\frac{1}{4}\pi$	$1\frac{1}{2}\pi$	$1\frac{3}{4}\pi$	2π
x	0	$\frac{1}{2}\sqrt{2}$	1	$\frac{1}{2}\sqrt{2}$	0	$-\frac{1}{2}\sqrt{2}$	-1	$-\frac{1}{2}\sqrt{2}$	0
y	0	1	0	-1	0	1	0	-1	0

d.



Opgave 47:

a. er is 1 punt met $x = 1$ en 1 punt met $x = -1$ en de grafiek snijdt de evenwichts-as 2 keer
er zijn 4 punten met $y = 1$ en 4 punten met $y = -1$ en de grafiek snijdt de evenwichts-as 8 keer

b. $a = 2 \quad b = 8$

c. $a = 3 \quad b = 12$

Opgave 48:

$c = 2$

Opgave 49:

$a = 2 \quad b = 5$

Opgave 50:

a. $a = 2 \quad b = 3$

b. snijpunt x-as: $t = 0 \quad \vee \quad t = \frac{2}{5}\pi \quad \vee \quad t = \frac{4}{5}\pi \quad \vee \quad t = 1\frac{1}{5}\pi \quad \vee \quad t = 1\frac{3}{5}\pi \quad \vee \quad t = 2\pi$

$x = 1: \quad t = \frac{1}{4}\pi \quad \vee \quad t = 1\frac{1}{4}\pi$

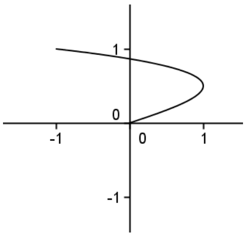
$x = -1: \quad t = \frac{3}{4}\pi \quad \vee \quad t = 1\frac{3}{4}\pi$

$y = 1: \quad t = \frac{1}{6}\pi \quad \vee \quad t = \frac{5}{6}\pi \quad \vee \quad t = 1\frac{1}{2}\pi$

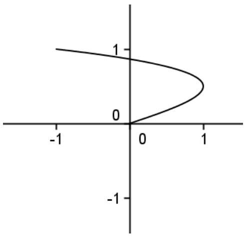
$y = -1: \quad t = \frac{1}{2}\pi \quad \vee \quad t = 1\frac{1}{6}\pi \quad \vee \quad t = 1\frac{5}{6}\pi$

Opgave 51:

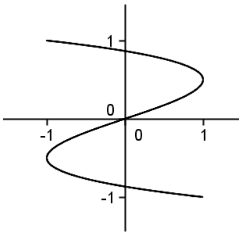
a.



b.



c.



2 keer doorlopen

d. $[-\frac{1}{2}\pi, \frac{1}{2}\pi]$

Opgave 52:

a. $c = 5$

b. op $[\frac{1}{2}\pi, 1\frac{1}{2}\pi]$ loopt de grafiek van rechts naar links

op $[-\frac{1}{2}\pi, \frac{1}{2}\pi]$ loopt de grafiek van links naar rechts

Opgave 53:

a. $y = 0$ dus $\sin 2t = 0$

$2t = 0 + k \cdot \pi$

$t = 0 + k \cdot \frac{1}{2}\pi$

$A(-\frac{1}{2},0)$ $D(\frac{1}{2}\sqrt{3},0)$ $G(\frac{1}{2},0)$ $I(-\frac{1}{2}\sqrt{3},0)$

$x = 0$ dus $\sin(t - \frac{1}{6}\pi) = 0$

$t - \frac{1}{6}\pi = 0 + k \cdot \pi$

$t = \frac{1}{6}\pi + k \cdot \pi$

$B(0, \frac{1}{2}\sqrt{3})$

$y = 1$ dus $\sin 2t = 1$

$2t = \frac{1}{2}\pi + k \cdot 2\pi$

$t = \frac{1}{4}\pi + k \cdot \pi$

$C(\sin \frac{1}{12}\pi, 1)$ $H(\sin 1\frac{1}{12}\pi, 1)$

$y = -1$ dus $\sin 2t = -1$

$2t = 1\frac{1}{2}\pi + k \cdot 2\pi$

$t = \frac{3}{4}\pi + k \cdot \pi$

$F(\sin \frac{7}{12}\pi, -1)$ $K(\sin 1\frac{7}{12}\pi, -1)$

$x = 1$ dus $\sin(t - \frac{1}{6}\pi) = 1$

$t - \frac{1}{6}\pi = \frac{1}{2}\pi + k \cdot 2\pi$

$t = \frac{2}{3}\pi + k \cdot 2\pi$

$E(1, -\frac{1}{2}\sqrt{3})$

$x = -1$ dus $\sin(t - \frac{1}{6}\pi) = -1$

$t - \frac{1}{6}\pi = 1\frac{1}{2}\pi + k \cdot 2\pi$

$t = 1\frac{2}{3}\pi + k \cdot 2\pi$

$J(-1, -\frac{1}{2}\sqrt{3})$

b. $\sin(t - \frac{1}{6}\pi) = \frac{1}{2}$

$t - \frac{1}{6}\pi = \frac{1}{6}\pi \quad \vee \quad t - \frac{1}{6}\pi = \frac{5}{6}\pi$

$t = \frac{1}{3}\pi \quad \vee \quad t = \pi$

$y = \frac{1}{2}\sqrt{3} \qquad y = 0$

$L = \frac{1}{2}\sqrt{3}$

c. $\sin 2t = \sin(t - \frac{1}{6}\pi)$

$2t = t - \frac{1}{6}\pi + k \cdot 2\pi \quad \vee \quad 2t = \pi - (t - \frac{1}{6}\pi) + k \cdot 2\pi$

$t = -\frac{1}{6}\pi + k \cdot 2\pi \quad \vee \quad 2t = \pi - t + \frac{1}{6}\pi + k \cdot 2\pi$

$t = -\frac{1}{6}\pi + k \cdot 2\pi \quad \vee \quad 3t = \frac{7}{6}\pi + k \cdot 2\pi$

$t = -\frac{1}{6}\pi + k \cdot 2\pi \quad \vee \quad t = \frac{7}{18}\pi + k \cdot \frac{2}{3}\pi$

$t = 1\frac{5}{6}\pi \quad \vee \quad t = \frac{7}{18}\pi \quad \vee \quad t = \frac{19}{18}\pi \quad \vee \quad t = \frac{31}{18}\pi$

Opgave 54:

$a = 3$

$y_B = \sin(\frac{3}{4}\pi + b) = 0$

$\frac{3}{4}\pi + b = 0 \quad \vee \quad \frac{3}{4}\pi + b = \pi$

$b = -\frac{3}{4}\pi \quad \vee \quad b = \frac{1}{4}\pi$

$b = -\frac{3}{4}\pi$ levert de gespiegelde grafiek op dus alleen $b = \frac{1}{4}\pi + k \cdot 2\pi$

Opgave 55:

a. $x = \sin 2t = 0$

$2t = 0 + k \cdot \pi$

$t = 0 + k \cdot \frac{1}{2}\pi$

$A(0, \frac{1}{2}\sqrt{3}) \quad B(0, \frac{1}{2}) \quad C(0, -\frac{1}{2}) \quad D(0, -\frac{1}{2}\sqrt{3})$

b. $x = \sin 2t = -\frac{1}{2}$

$2t = 1\frac{1}{6}\pi + k \cdot 2\pi \quad \vee \quad 2t = 1\frac{5}{6}\pi + k \cdot 2\pi$

$t = \frac{7}{12}\pi + k \cdot \pi \quad \vee \quad t = \frac{11}{12}\pi + k \cdot \pi$

$t = \frac{11}{12}\pi \qquad t = 1\frac{11}{12}\pi$

$y_A = \sin 1\frac{1}{4}\pi = -\frac{1}{2}\sqrt{2} \qquad y_E = \sin 2\frac{1}{4}\pi = \frac{1}{2}\sqrt{2}$

$EH = \sqrt{2}$

Opgave 56:

a. $\begin{cases} x = r \cdot \cos t = r \cdot \sin(t + \frac{1}{2}\pi) \\ y = r \cdot \sin t \end{cases}$

b. $\begin{cases} x = \sin(t + \frac{1}{2}\pi) \\ y = r \cdot \sin t \end{cases}$

Opgave 57:

Door $(0,-1)$ dus $-1 = 0 + q$
dus $q = -1$

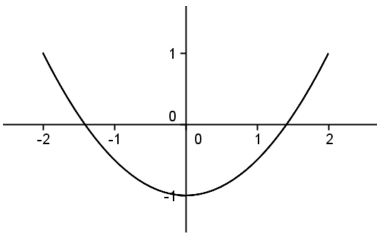
Door $(1,1)$ dus $1 = p - 1$
dus $p = 2$

Opgave 58:

$$x = \sin(t - \tfrac{1}{4}\pi) = \sin t \cdot \cos \tfrac{1}{4}\pi - \cos t \cdot \sin \tfrac{1}{4}\pi$$
$$= \tfrac{1}{2}\sqrt{2} \cdot \sin t - \tfrac{1}{2}\sqrt{2} \cdot \cos t$$
$$= \tfrac{1}{2}\sqrt{2}(\sin t - \cos t)$$
$$x^2 = \tfrac{1}{2}(\sin t - \cos t)^2$$
$$-2x^2 = -(\sin t - \cos t)^2$$
$$= -\sin^2 t + 2\sin t \cos t - \cos^2 t$$
$$= -1 + 2\sin t \cos t$$
$$-2x^2 + 1 = -1 + 2\sin t \cos t + 1$$
$$= 2\sin t \cos t$$
$$= \sin 2t = y$$

Opgave 59:

a.



de keerpunten zijn $(-2,1)$ en $(2,1)$

b. $y = ax^2 + b$ door $(0,-1)$

dus $b = -1$

$y = ax^2 - 1$ door $(2,1)$

$1 = 4a - 1$

$-4a = -2$

$a = \frac{1}{2}$

$K: \quad y = \frac{1}{2}x^2 - 1$

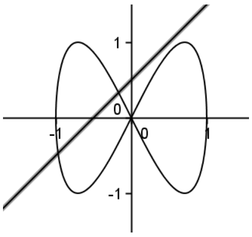
$\frac{1}{2}x^2 - 1 = \frac{1}{2}(2\sin t)^2 - 1 = 2\sin^2 t - 1$

$y = \sin(2t - \frac{1}{2}\pi) = \sin 2t \cdot \cos\frac{1}{2}\pi - \cos 2t \cdot \sin\frac{1}{2}\pi = 0 - \cos 2t = -(1 - 2\sin^2 t) = 2\sin^2 t - 1$

dus $y = 2\sin^2 t - 1 = \frac{1}{2}x^2 - 1$

Opgave 60:

a.



b. $\sin 2t = \sin t + \frac{1}{2}$

$y_1 = \sin 2x \quad y_2 = \sin x + \frac{1}{2}$ intersect geeft $x = 3{,}31 \quad \vee \quad x = 4{,}96$

dus $t = 3{,}31 \quad \vee \quad t = 4{,}96$

$t = 3{,}31$ geeft $(-0{,}17\,;0{,}33)$

$t = 4{,}96$ geeft $(-0{,}97\,;-0{,}47)$

c. $4x^2 - 4x^4 = 4\sin^2 t - 4\sin^4 t$

$= 4\sin^2 t \cdot (1 - \sin^2 t)$

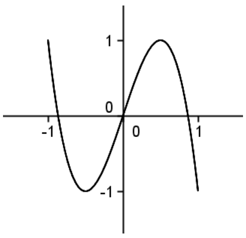
$= 4\sin^2 t \cdot \cos^2 t$

$y^2 = (\sin 2t)^2 = (2\sin t \cos t)^2 = 4\sin^2 t \cdot \cos^2 t$

$y^2 = 4\sin^2 t \cdot \cos^2 t = 4x^2 - 4x^4$

Opgave 61:

a.



de keerpunten zijn $(-1{,}1)$ en $(1{,}-1)$

b. $3x - 4x^3 = 3\sin t - 4\sin^3 t$

$y = \sin 3t = \sin(2t + t)$

$= \sin 2t \cdot \cos t + \cos 2t \cdot \sin t$

$= 2\sin t \cos t \cdot \cos t + (1 - 2\sin^2 t) \cdot \sin t$

$= 2\sin t \cdot \cos^2 t + \sin t - 2\sin^3 t$

$= 2\sin t \cdot (1 - \sin^2 t) + \sin t - 2\sin^3 t$

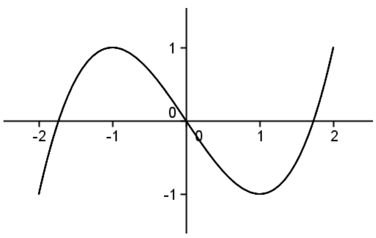
$= 2\sin t - 2\sin^3 t + \sin t - 2\sin^3 t$

$= 3\sin t - 4\sin^3 t$

$= 3x - 4x^3$

Opgave 62:

a.



b. $y = ax^3 + bx$ door $(2,1)$ en $(1,-1)$ dus

$$\begin{cases} 1 = 8a + 2b & \times 1 \\ -1 = a + b & \times 2 \end{cases}$$

$$\begin{cases} 1 = 8a + 2b \\ -2 = 2a + 2b & - \end{cases}$$

$$3 = 6a$$

$$a = \tfrac{1}{2}$$

$$b = -1\tfrac{1}{2}$$

$c = -2$ en $d = 2$

$K: \quad y = \tfrac{1}{2}x^3 - 1\tfrac{1}{2}x$

$$\tfrac{1}{2}x^3 - 1\tfrac{1}{2}x = \tfrac{1}{2}(2\cos t)^3 - 1\tfrac{1}{2} \cdot 2\cos t = 4\cos^3 t - 3\cos t$$

$$\begin{aligned} y = \cos 3t &= \cos(2t + t) \\ &= \cos 2t \cdot \cos t - \sin 2t \cdot \sin t \\ &= (2\cos^2 t - 1) \cdot \cos t - 2\sin t \cos t \cdot \sin t \\ &= 2\cos^3 t - \cos t - 2\sin^2 t \cdot \cos t \\ &= 2\cos^3 t - \cos t - 2 \cdot (1 - \cos^2 t) \cdot \cos t \\ &= 2\cos^3 t - \cos t - 2\cos t + 2\cos^3 t \\ &= 4\cos^3 t - 3\cos t \\ &= \tfrac{1}{2}x^3 - 1\tfrac{1}{2}x \end{aligned}$$