7.2 Directe formules

Opgave 15:

a. $u_0 = 20$, $u_1 = 26$, $u_2 = 32$, $u_3 = 38$, $u_4 = 44$, $u_5 = 50$

b. $a = 20$ $b = 6$

c. $u_{25} = 20 + 6 \cdot 25 = 170$

Opgave 16:

a. het verschil tussen twee opeenvolgende termen is 5

b. recursieve formule: $u_n = u_{n-1} + 5$ met $u_0 = 13$

directe formule: $u_n = 13 + 5n$

c. $u_{49} = 13 + 5 \cdot 49 = 258$

d. $13 + 5n = 633$

$5n = 620$

$n = 124$ dus de 125^{e} term

Opgave 17:

a. $u_n = 1023 - 7n$

$1023 - 7n = 246$

$-7n = -777$

$n = 111$ dus de 112^{e} term

b. $1023 - 7n = 0$

$-7n = -1023$

$n = 146{,}1$ dus u_{146} is nog positief

dus 147 termen zijn positief

Opgave 18:

a. $2\times$ de gevraagde som is $100 \cdot 101$

dus de gevraagde som is $\frac{1}{2} \cdot 100 \cdot 101$

b. $1 + 2 + 3 + ... + 50 = \frac{1}{2} \cdot 50 \cdot 51 = 1275$

Opgave 19:

a. $\sum_{k=0}^{50} (3k + 4) = \frac{1}{2} \cdot 51 \cdot (4 + 154) = 4029$

b. $\sum_{k=0}^{40} (100 - 2k) = \frac{1}{2} \cdot 41 \cdot (100 + 20) = 2460$

c. $\sum_{k=5}^{30} (6k - 12) = \frac{1}{2} \cdot 26 \cdot (18 + 168) = 2418$

d. $\sum_{k=12}^{36} (150 - 3k) = \frac{1}{2} \cdot 25 \cdot (114 + 42) = 1950$

Opgave 20:

a. $u_n = 12 + 4n$

$12 + 4n = 152$

$4n = 140$

$n = 35$

$$12 + 16 + 20 + 24 + ... + 152 = \sum_{n=0}^{35} (12 + 4n) = \tfrac{1}{2} \cdot 36 \cdot (12 + 152) = 2952$$

b. $u_n = 100 - 3n$

$u_{33} = 1$

$$100 + 97 + 94 + 91 + ... + 1 = \sum_{n=0}^{33} (100 - 3n) = \tfrac{1}{2} \cdot 34 \cdot (100 + 1) = 1717$$

c. $u_n = 18 + 7n$

$u_{24} = 186$

$$\sum_{n=0}^{24} (18 + 7n) = \tfrac{1}{2} \cdot 25 \cdot (18 + 186) = 2550$$

Opgave 21:

a. $u_n = 30{,}62 + 0{,}15n$

$$\sum_{n=0}^{24} (30{,}62 + 0{,}15n) = \tfrac{1}{2} \cdot 25 \cdot (30{,}62 + 34{,}22) = 810{,}5 \text{ sec} = 13 \text{ min } 30{,}5 \text{ sec}$$

b. $u_n = 35{,}76 - 0{,}22n$

$$\sum_{n=0}^{24} (35{,}76 - 0{,}22n) = \tfrac{1}{2} \cdot 25 \cdot (35{,}76 + 30{,}48) = 828 \text{ sec} = 13 \text{ min } 48 \text{ sec}$$

Opgave 22:

a. $u_n = 20 + n \cdot v$

$u_{29} = 20 + 29v$

$$\sum_{n=0}^{29} (20 + n \cdot v) = \tfrac{1}{2} \cdot 30 \cdot (20 + 20 + 29v) = 15(40 + 29v) = 2340$$

b. $600 + 435v = 2340$

$435v = 1740$

$v = 4$

c. $$\sum_{n=0}^{49} (20 + 4n) = \tfrac{1}{2} \cdot 50 \cdot (20 + 216) = 5900$$

Opgave 23:

$u_n = 4 + n \cdot v$

$u_{11} = 4 + 11v$

$$\sum_{n=0}^{11} (4 + n \cdot v) = \tfrac{1}{2} \cdot 12 \cdot (4 + 4 + 11v) = 6 \cdot (8 + 11v) = 147$$

$48 + 66v = 147$

$66v = 99$

$v = 1\tfrac{1}{2}$

$$\sum_{n=0}^{7} (4 + 1\tfrac{1}{2} n) = \tfrac{1}{2} \cdot 8 \cdot (4 + 14{,}5) = 74 \text{ m}$$

Opgave 24:

a. $u_n = 4{,}9 + 9{,}8n$

$$\sum_{n=0}^{5}(4{,}9+9{,}8n) = \tfrac{1}{2}\cdot 6\cdot(4{,}9+53{,}9) = 176{,}4 \text{ m}$$

b. $S_n = \sum_{n=0}^{n}(4{,}9+9{,}8n) = \tfrac{1}{2}\cdot(n+1)\cdot(4{,}9+4{,}9+9{,}8n) = \tfrac{1}{2}(n+1)(9{,}8+9{,}8n) =$

$\tfrac{1}{2}(9{,}8n+9{,}8n^2+9{,}8+9{,}8n) = \tfrac{1}{2}(9{,}8n^2+19{,}6n+9{,}8) = 4{,}9n^2+9{,}8n+4{,}9$

c. $4{,}9n^2+9{,}8n+4{,}9 = 1960$

neem $y_1 = 4{,}9x^2+9{,}8x+4{,}9$ en $y_2 = 1960$

intersect geeft $x = 19$ dus $S_{19} = 1960$

dus na 20 seconden

Opgave 25:

a. $u_0 = 400$, $u_1 = 600$, $u_2 = 900$, $u_3 = 1350$, $u_4 = 2025$

b. $a = 400$ $b = 1{,}5$

Opgave 26:

a. iedere volgende term vind je door de voorgaande term te vermenigvuldigen met 1,2

b. recursieve formule: $u_n = 1{,}2\cdot u_{n-1}$ met $u_0 = 1250$

directe formule: $u_n = 1250\cdot 1{,}2^n$

c. voer in:

$nMin = 0$

$u(n) = 1250\cdot 1{,}2^n$

$u(nMin) = 1250$

kijk in de tabel, dan geldt: $u_{13} = 13374$ en $u_{14} = 16049$, dus vanaf de 15e term

Opgave 27:

$u_3\cdot r^7 = u_{10}$

$54\cdot r^7 = 118098$

$r^7 = 2187$

$r = \sqrt[7]{2187} = 3$

$u_0 = \dfrac{u_3}{r^3} = \dfrac{54}{3^3} = 2$

$u_n = 2\cdot 3^n$

Opgave 28:

a. $u_3 + 5v = u_8$

$16 + 5v = 16384$

$5v = 16368$

$v = 3273{,}6$

$u_0 = u_3 - 3v = 16 - 3\cdot 3273{,}6 = -9804{,}8$

$u_n = -9804{,}8 + 3273{,}6n$

b. $u_3 \cdot r^5 = u_8$

$16 \cdot r^5 = 16384$

$r^5 = 1024$

$r = \sqrt[5]{1024} = 4$

$u_0 = \dfrac{u_3}{r^3} = \dfrac{16}{4^3} = 0{,}25$

$u_n = 0{,}25 \cdot 4^n$

Opgave 29:

Bij een meetkundige rij hoort exponentiële groei.
Bij een rekenkundige rij hoort lineaire groei.

Opgave 30:

a. $PQ = \sqrt{8^2 + 4^2} = \sqrt{80}$

$AB \cdot r = PQ$

$r = \dfrac{PQ}{AB} = \dfrac{\sqrt{80}}{12}$

b. $u_n = 12 \cdot \left(\frac{\sqrt{80}}{12}\right)^n$

c. $u_8 = 1{,}14$

$u_9 = 0{,}85$ dus vanaf het 10e vierkant

d. iedere zijde wordt $k = \frac{\sqrt{80}}{12} \times$zo groot

dus de oppervlakte wordt $k^2 = \left(\frac{\sqrt{80}}{12}\right)^2 = \frac{5}{9} \times$zo groot

$v_0 = 12^2 = 144$

$v_n = 144 \cdot \left(\frac{5}{9}\right)^n$

e. $v_{16} = 0{,}0119$ cm² = 1,19mm²

$v_{17} = 0{,}0066$ cm² = 0,66mm² , dus vanaf het 18e vierkant

Opgave 31:

$-2 \cdot S_{10} = 15 - 2657205$

$-2 \cdot S_{10} = -2657190$

$S_{10} = 1328595$

Opgave 32:

a. $\displaystyle\sum_{n=0}^{11} (0{,}001 \cdot 2^n = \frac{u_0 - u_{12}}{1-r} = \frac{0{,}001 - 0{,}001 \cdot 2^{12}}{1-2} = 4{,}095$

b. $\displaystyle\sum_{k=0}^{20} 100 \cdot 0{,}8^k = \frac{u_0 - u_{21}}{1-r} = \frac{100 - 100 \cdot 0{,}8^{21}}{1-0{,}8} = 495{,}39$

c. $\displaystyle\sum_{k=5}^{18} 200 \cdot 1{,}1^k = \frac{u_5 - u_{19}}{1-r} = \frac{200 \cdot 1{,}1^5 - 200 \cdot 1{,}1^{19}}{1-1{,}1} = 9011$

d. $u_n = 600 \cdot 0{,}75^n$

$$S_{14} = \frac{u_0 - u_{15}}{1-r} = \frac{600 - 600 \cdot 0{,}75^{15}}{1 - 0{,}75} = 2367{,}9$$

Opgave 33:

a. mr met $u_0 = 2000$ en $r = 1{,}5$

$$2000 + 3000 + 4500 + ... + 34171{,}875 = \frac{2000 - 1{,}5 \cdot 34171{,}875}{1 - 1{,}5} = 98515{,}625$$

b. $1{,}06 + 1{,}06^2 + 1{,}06^3 + ... + 1{,}06^{12} = \dfrac{1{,}06 - 1{,}06^{13}}{1 - 1{,}06} = 17{,}882$

c. $1 + 1{,}5 + 1{,}5^2 + 1{,}5^3 + ... + 1{,}5^{20} = \dfrac{1 - 1{,}5^{21}}{1 - 1{,}5} = 9973{,}770$

d. $1{,}2 - 1{,}2^2 + 1{,}2^3 - 1{,}2^4 + ... - 1{,}2^{24} = \dfrac{1{,}2 - (-1{,}2 \cdot -1{,}2^{24})}{1 - -1{,}2} = -42{,}816$

Opgave 34:

a. $u_n = 20 \cdot 1{,}1^n$

b. $u_7 = 38{,}974$

$u_8 = 42{,}872$ dus bij de 9^e duurloop

$$S_8 = \frac{u_0 - u_9}{1-r} = \frac{20 - 20 \cdot 1{,}1^9}{1 - 1{,}1} = 271{,}6 \text{ km}$$

Opgave 35:

$u_n = 11{,}3 \cdot 1{,}074^n$

$$S_{12} = \frac{u_0 - u_{13}}{1-r} = \frac{11{,}3 - 11{,}3 \cdot 1{,}074^{13}}{1 - 1{,}074} = 233{,}6 \text{ miljard dollar}$$

Opgave 36:

a. toename: $T_n = 5{,}2 \cdot 0{,}8^n$

$T_7 = 5{,}2 \cdot 0{,}8^7 = 1{,}1 \text{ cm} = 11 \text{mm}$

b. $S_7 = \dfrac{T_0 - T_8}{1-r} = \dfrac{5{,}2 - 5{,}2 \cdot 0{,}8^8}{1 - 0{,}8} = 21{,}6 \text{ cm} = 216 \text{mm}$

c. $S_9 = \dfrac{T_0 - T_{10}}{1-r} = \dfrac{5{,}2 - 5{,}2 \cdot 0{,}8^{10}}{1 - 0{,}8} = 23{,}2 \text{ cm}$

dus $18 + 23{,}2 = 41{,}2 \text{ cm} = 412 \text{mm}$

Opgave 37:

a. u_n is een rekenkundige rij met $u_0 = 100$ en $v = 15$

dus $u_n = 100 + 15n$

$S_n = \frac{1}{2} \cdot (n+1) \cdot (u_0 + u_n) = \frac{1}{2}(n+1)(100 + 100 + 15n) = \frac{1}{2}(n+1)(200 + 15n)$

v_n is een meetkundige rij met $u_0 = 10$ en $r = 1{,}5$

dus $v_n = 10 \cdot 1{,}5^n$

$$T_n = \frac{v_0 - v_{n+1}}{1-r} = \frac{10 - 10 \cdot 1{,}5^{n+1}}{1-1{,}5} = \frac{10 - 10 \cdot 1{,}5^{n+1}}{-0{,}5} = -20 + 20 \cdot 1{,}5^{n+1}$$

b. $S_{10} = 1925 \quad T_{10} = 1710$

$S_{11} = 2190 \quad T_{11} = 2574{,}9$ dus vanaf $n = 11$