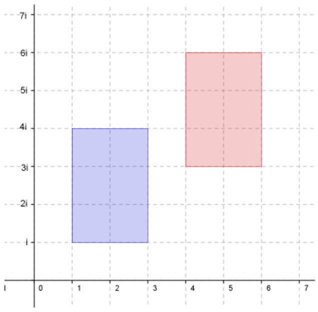
## 8.4 Complexe functies

**Opgave 47:**

a.



b. $z = 1 + i$ geeft $4 + 3i$

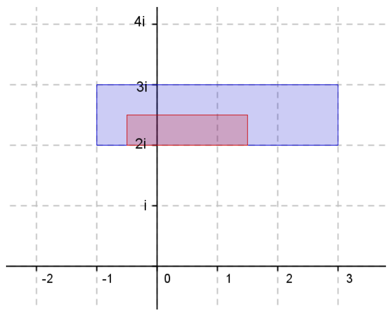
$z = 3 + i$ geeft $6 + 3i$

$z = 3 + 4i$ geeft $6 + 6i$

$z = 1 + 4i$ geeft $4 + 6i$

c. de rechthoek is getransleerd over $(3,2)$

**Opgave 48:**




a. $f(-1+2i) = -\frac{1}{2} + 2i$

$f(3+2i) = 1\frac{1}{2} + 2i$

$f(3+3i) = 1\frac{1}{2} + 2\frac{1}{2}i$

$f(-1+3i) = -\frac{1}{2} + 2\frac{1}{2}i$

Dus de rechthoek wordt eerst vermenigvuldigt t.o.v. $O$ met de factor $\frac{1}{2}$ en daarna getransleerd over $(0,1)$

b. $f(x) = \frac{1}{2}z + i = 0$

$\frac{1}{2}z = -i$

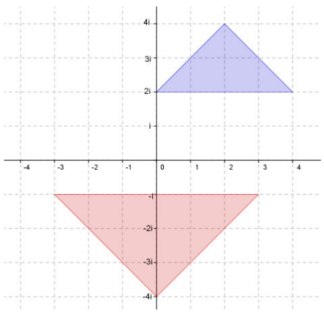
$z = -2i$

c. $f(x) = \frac{1}{2}z + i = z$

$-\frac{1}{2}z = -i$

$z = 2i$

**Opgave 49:**



a. $f(2i) = 3 - i$

$f(4+2i) = -3 - i$

$f(2+4i) = -4i$

1. vermenigvuldiging t.o.v. $O$ met factor $-1\frac{1}{2}$
2. translatie over $(3,2)$

b. $f(z) = -1\frac{1}{2}z + 3 + 2i = 0$

$-1\frac{1}{2}z = -3 - 2i$

$z = 2 + \frac{4}{3}i$

c. $f(z) = -1\frac{1}{2}z + 3 + 2i = z$

$-2\frac{1}{2}z = -3 - 2i$

$z = \frac{6}{5} + \frac{4}{5}i$

**Opgave 50:**

a. nulpunt: $f(z) = 3z + 2 - 4i = 0$

$3z = -2 + 4i$

$z = -\frac{2}{3} + \frac{4}{3}i$

dekpunt: $f(z) = 3z + 2 - 4i = z$

$2z = -2 + 4i$

$z = -1 + 2i$

b. nulpunt: $g(z) = \frac{1}{3}z + 5 = 0$

$\frac{1}{3}z = -5$

$z = -15$

dekpunt: $g(z) = \frac{1}{3}z + 5 = z$

$-\frac{2}{3}z = -5$

$z = 7\frac{1}{2}$

**Opgave 51:**

a. $f(z) = az + 5 - 2i = z$

$az - z = -5 + 2i$

$(a-1)z = -5 + 2i$

$z = \dfrac{-5 + 2i}{a - 1}$ als $a \neq 1$

dus er is geen dekpunt als $a = 1$

b. $f(z) = az + 5 - 2i = 0$

$az = -5 + 2i$

$z = \dfrac{-5 + 2i}{a}$ als $a \neq 0$

dus er is geen nulpunt als $a = 0$

**Opgave 52:**

a. $f(1+2i) = 3(1+2i) + a + bi = 0$

$3 + 6i + a + bi = 0$

$3 + a + (6+b)i = 0$

$$\begin{cases} 3 + a = 0 \\ 6 + b = 0 \end{cases}$$

$a = -3 \quad \wedge \quad b = -6$

b. $f(1+2i) = 3(1+2i) + a + bi = 1 + 2i$

$3 + 6i + a + bi = 1 + 2i$

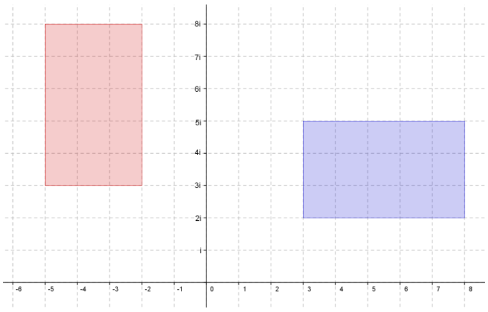
$3 + a + (6+b)i = 1 + 2i$

$$\begin{cases} 3 + a = 1 \\ 6 + b = 2 \end{cases}$$

$a = -2 \quad \wedge \quad b = -4$

**Opgave 53:**

a.



b. $z = 3 + 2i$ geeft $-2 + 3i$

$z = 8 + 2i$ geeft $-2 + 8i$

$z = 8 + 5i$ geeft $-5 + 8i$

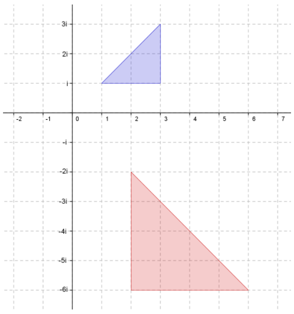
$z = 3 + 5i$ geeft $-5 + 3i$

c. rotatie om $O$ over $90°$

d. $\arg(\frac{1}{2} + \frac{1}{2}i\sqrt{3}) = 60°$ dus rotatie om $O$ over $60°$

**Opgave 54:**

a.



b. $z = 1 + i$ geeft $2 - 2i$

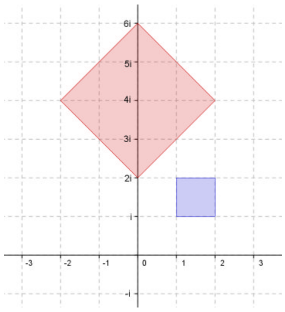
$z = 3 + i$ geeft $2 - 6i$

$z = 3 + 3i$ geeft $6 - 6i$

c. rotatie om $O$ over $-90°$

vermenigvuldiging t.o.v. $O$ met factor 2

**Opgave 55:**



a. $|2 + 2i| = \sqrt{2^2 + 2^2} = 2\sqrt{2}$

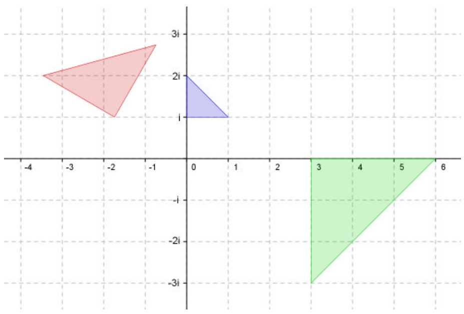
$\arg(2 + 2i) = 45°$

rotatie om $O$ over $45°$

vermenigvuldiging t.o.v. $O$ met $2\sqrt{2}$

b. het bereik is $2\sqrt{2} \leq |z| \leq 4\sqrt{2} \quad \wedge \quad 75° \leq \arg(z) \leq 105°$

**Opgave 56:**



a. $f(i) = -\sqrt{3} + i$

$f(1+i) = 1 - \sqrt{3} + i \cdot (1 + \sqrt{3})$

$f(2i) = -2\sqrt{3} + 2i$

$\arg(1 + i\sqrt{3}) = 60°$

$|1+i\sqrt{3}| = \sqrt{1^2 + (\sqrt{3})^2} = 2$

rotatie om $O$ over $60°$
vermenigvuldiging t.o.v. $O$ met factor 2

b. $g(i) = 3$

$g(1+i) = 3 - 3i$

$g(2i) = 6$

rotatie om $O$ over $-90°$
vermenigvuldiging t.o.v. $O$ met factor 3

**Opgave 57:**

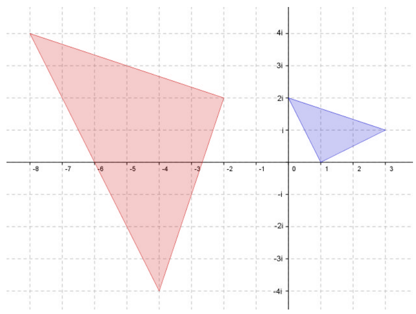
$\arg(\sqrt{3} - i) = -30°$

$|\sqrt{3} - i| = \sqrt{(\sqrt{3})^2 + (-1)^2} = 2$

a. het bereik is $20 \le |z| \le 40 \quad \wedge \quad 60° \le \arg(z) \le 150°$

b. het bereik is $|z| \ge 6 \quad \wedge \quad -30° \le \arg(z) \le 60°$

**Opgave 58:**



a. $f(1) = -2 + 2i$

$f(3+i) = -8 + 4i$

$f(2i) = -4 - 4i$

$\arg(-2 + 2i) = 135°$

$|-2 + 2i| = \sqrt{(-2)^2 + 2^2} = 2\sqrt{2}$

rotatie om $O$ over $135°$
vermenigvuldiging t.o.v. $O$ met factor $2\sqrt{2}$

b. $(-2 + 2i) \cdot z = 3 + 2i$

$z = \dfrac{3+2i}{-2+2i} = \dfrac{3+2i}{-2+2i} \cdot \dfrac{-2-2i}{-2-2i} = \dfrac{-2-10i}{8} = -\tfrac{1}{4} - 1\tfrac{1}{4}i$

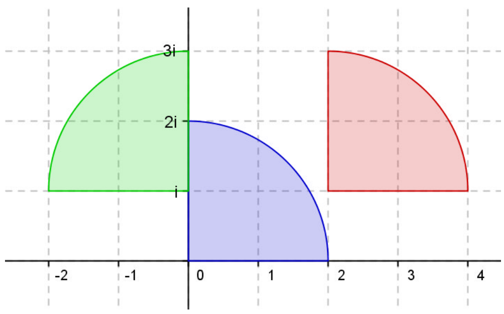
c. $(-2+2i) \cdot z = \dfrac{-2+2i}{z}$

$z = \dfrac{1}{z}$

$z^2 = 1$

$z = 1 \quad \vee \quad z = -1$

**Opgave 59:**




a.

b. translatie over $(2,1)$

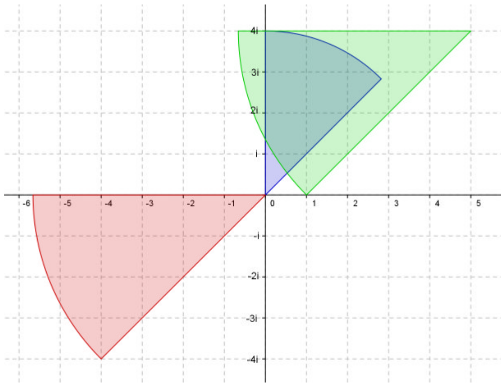
c. rotatie om $O$ over $90°$
translatie over $(0,1)$

**Opgave 60:**

a. $(-1+i)\cdot z+5+4i=10+i$

$(-1+i)\cdot z=5-3i$

$z=\dfrac{5-3i}{-1+i}=\dfrac{5-3i}{-1+i}\cdot\dfrac{-1-i}{-1-i}=\dfrac{-8-2i}{2}=-4-i$



b. $\arg(-1+i)=135°$

$|-1+i|=\sqrt{(-1)^2+1^2}=\sqrt{2}$

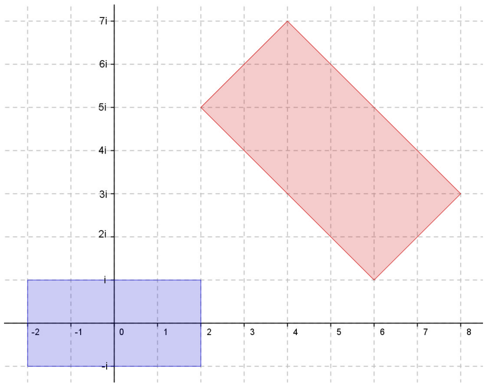
1. rotatie om $O$ over 135°
2. vermenigvuldiging t.o.v. $O$ met factor $\sqrt{2}$
3. translatie over $(5,4)$

c. $f(2+i)=2+5i$

$f(-2+i)=6+i$

$f(-2-i)=8+3i$

$f(2-i)=4+7i$



d. de vermenigvuldigingsfactor is $\sqrt{2}$

dus de oppervlakte wordt $(\sqrt{2})^2=2\times$ zo groot

$Opp(V)=\dfrac{10}{2}=5$

**Opgave 61:**

a. nulpunt: $f(z)=(1+i\sqrt{3})\cdot z-2+i=0$

$(1+i\sqrt{3})\cdot z=2-i$

$z=\dfrac{2-i}{1+i\sqrt{3}}=\dfrac{2-i}{1+i\sqrt{3}}\cdot\dfrac{1-i\sqrt{3}}{1-i\sqrt{3}}=\dfrac{2-\sqrt{3}+i(-1-2\sqrt{3})}{4}=\tfrac{1}{2}-\tfrac{1}{4}\sqrt{3}+(-\tfrac{1}{4}-\tfrac{1}{2}\sqrt{3})\cdot i$

dekpunt: $f(z)=(1+i\sqrt{3})\cdot z-2+i=z$

$(1+i\sqrt{3})\cdot z-z=2-i$

$i\sqrt{3}\cdot z=2-i$

$z=\dfrac{2-i}{i\sqrt{3}}=\dfrac{2-i}{i\sqrt{3}}\cdot\dfrac{i\sqrt{3}}{i\sqrt{3}}=\dfrac{2i\sqrt{3}+\sqrt{3}}{-3}=-\tfrac{1}{3}\sqrt{3}-\tfrac{2}{3}i\sqrt{3}$

b. nulpunt: $g(z)=-2i\cdot z+1-3i=0$

$-2i\cdot z=-1+3i$

$z=\dfrac{-1+3i}{-2i}=\dfrac{-1+3i}{-2i}\cdot\dfrac{i}{i}=\dfrac{-i-3}{2}=-1\tfrac{1}{2}-\tfrac{1}{2}i$

dekpunt: $g(z)=-2i\cdot z+1-3i=z$

$-2i\cdot z-z=-1+3i$

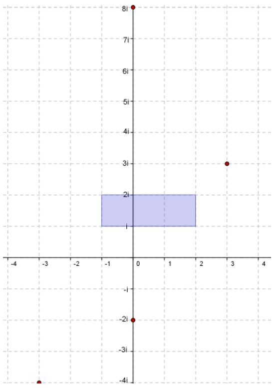
$2i\cdot z+z=1-3i$

$(1+2i)\cdot z=1-3i$

$z=\dfrac{1-3i}{1+2i}=\dfrac{1-3i}{1+2i}\cdot\dfrac{1-2i}{1-2i}=\dfrac{-5-5i}{5}=-1-i$

**Opgave 62:**

a.



b. $f(-1+i)=(-1+i)^2=1-2i+i^2=-2i$

$f(2+i)=(2+i)^2=4+4i+i^2=3+4i$

$f(2+2i)=(2+2i)^2=4+8i+4i^2=8i$

$f(-1+2i)=(-1+2i)^2=1-4i+4i^2=-3-4i$

c. nee

**Opgave 63:**

a. $f(z)=z^2+2=0$

$z^2=-2$

$z=\sqrt{-2}=i\sqrt{2} \quad \vee \quad z=-i\sqrt{2}$

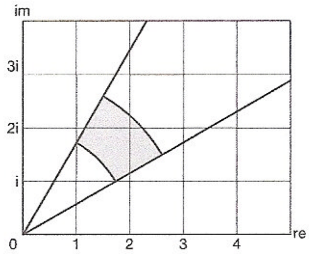
b. $f(z)=z^2+2=z$

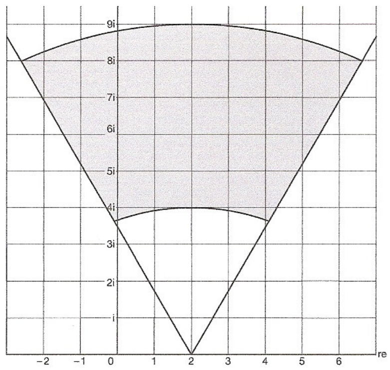
$z^2-z+2=0$

$z=\dfrac{1\pm\sqrt{-7}}{2}=\frac{1}{2}\pm\frac{1}{2}i\sqrt{7}$

$z=\frac{1}{2}+\frac{1}{2}i\sqrt{7} \quad \vee \quad z=\frac{1}{2}-\frac{1}{2}i\sqrt{7}$

c.



d. $f(1-3i)=-6-6i$

$f(1-2i)=-1-4i$

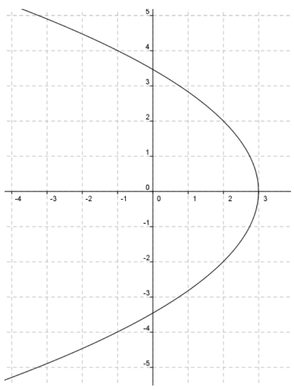
$f(1-i)=2-2i$

$f(1)=3$

$f(1+i)=2+2i$

$f(1+2i)=-1+4i$

$f(1+3i)=-6+6i$



**Opgave 64:**

a. $f(z)=i\cdot z^2-4=0$

$i\cdot z^2=4$

$z^2=\dfrac{4}{i}=\dfrac{4}{i}\cdot\dfrac{i}{i}=\dfrac{4i}{-1}=-4i$

$\left|z^2\right|=4$

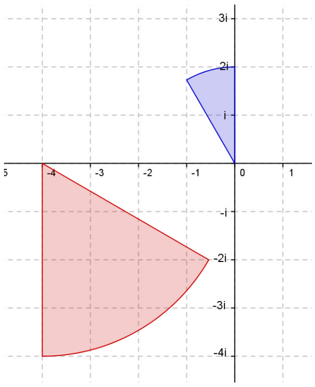
$\left|z\right|=2$

$\arg(z^2)270°+k\cdot 360°$

$\arg(z)=135°+k\cdot 180°$

$z=2(\cos 135°+i\cdot\sin 135°)=2(-\tfrac{1}{2}\sqrt{2}+\tfrac{1}{2}i\sqrt{2})=-\sqrt{2}+i\sqrt{2}$

of $z=2(\cos 315°+i\cdot\sin 315°)=2(\tfrac{1}{2}\sqrt{2}-\tfrac{1}{2}i\sqrt{2})=\sqrt{2}-i\sqrt{2}$




b. 1. $\left|z\right|$ wordt gekwadrateerd

$\arg(z)$ wordt verdubbeld

2. rotatie om $O$ over $90°$
3. translatie over $(-4,0)$

c. $f(1-i)=i\cdot(1-i)^2-4=i\cdot(1-2i-1)-4=-2i^2-4=-2$

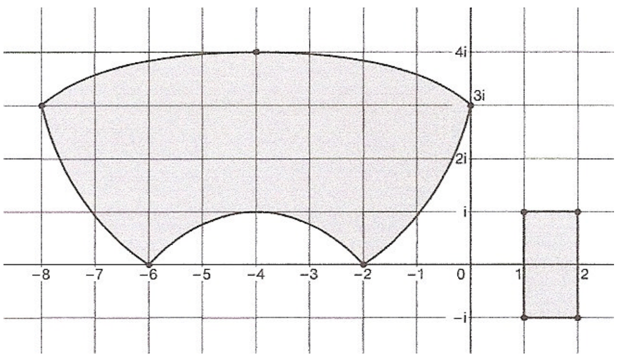
$f(2-i)=i\cdot(2-i)^2-4=i\cdot(4-4i-1)-4=3i-4i^2-4=3i$

$f(2+i)=i\cdot(2+i)^2-4=i\cdot(4+4i-1)-4=3i+4i^2-4=-8+3i$

$f(1+i)=i\cdot(1+i)^2-4=i\cdot(1+2i-1)-4=2i^2-4=-6$

$f(2)=i\cdot 2^2-4=-4+4i$

$f(1)=i\cdot 1^2-4=-4+i$




**Opgave 65:**

a. het beeld van $\text{Re}(z)=1$ en $\text{Re}(z)=-\text{Im}(z)$ gaat door $-2-2i$

dus $z=1-i$

b. $\left|z^3\right|=\left|z\right|^3$

$\arg(z^3)=3\cdot\arg(z)$

Voor ieder punt op een lijn door $O$ wordt het argument $3\times$ zo groot, dus het beeld van een lijn is dus weer een rechte lijn.

c. voor deze punten moet gelden: $\arg(z)=60° \quad\vee\quad \arg(z)=-60°$

dus $z=1+i\sqrt{3} \quad\vee\quad z=1-i\sqrt{3}$